Geodezic Network Check Up Using Distance Measurements

Luciana Oprea, drd. eng., University „1 Decembrie 1918" Alba Iulia;
Eva Magdolna Koncsag, univ. assist. drd. eng., University „1 Decembrie 1918" Alba Iulia.

***Abstract**: The present paper is focused on developing methods for verifing the geodetic network. The main goal of the solving is to check up the x, y coordinates of the geodetic points. These presented methods are based on measurement of the distance between the network points.*

The base of topographic detailing is based on a geodesic network of points equally distributed on surface of the earth with a certain density according to the law. These points will be active from the moment that they'll be established and positioned good referring to a certain reference system.

Replacing the angle measurements with distance measurements in the geodesic network created new methods of solving these networks, this kind of network is called side measured.

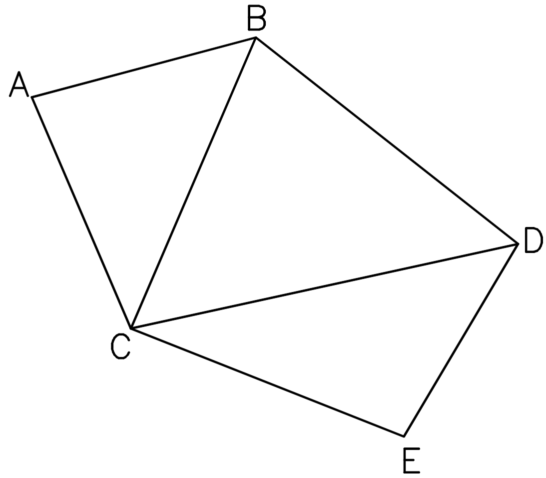

Fig. 1 Side measured network

For the network above in witch the points were given generic names with alphabet letters the distances were measured by reducing them to the horizon to, therefore we have the next field table.

Table 1

St. Pc.	Obsdpt.	Side name	Dist. val. (m)	Correction (m)	Corrected dist. (m)
A	B	AB	1697,271	-2,165	1695,106
	C	AC	1644,236	-6,073	1638,163
B	A	BA	1697,271	-2,165	1695,106
	D	BD	2323,574	-0,727	2322,847
	C	BC	2047,562	-0,752	2046,810
C	A	CA	1644,254	-6,073	1638,181
	B	CB	2047,524	-0,752	2046,772
	D	CD	2876,094	-0,001	2876,093
	E	CE	2116,551	-0,002	2116,549

D	B	DB	2323,524	-0,727	2322,797
	E	DE	1480,505	0,000	1480,505
	C	DC	2876,117	-0,001	2876,116
E	D	ED	1480,505	0,000	1480,505
	C	EC	2116,539	-0,002	2116,537

Each side was calculated by the arithmetic value because we have double measurements with the total station, therefore for each point we have double measurements from each station point and because the distance between the points is relatively small we only need to apply the horizon correction.

Applying the theory of conditioned measurements we can solve the network by side measurements method if we know the number of geometrical conditions for the network.

To establish the number of geometrical conditions we will make the next markings:

-1'=total number of sides from witch we exclude the bases (the bases are known)

-n=number of new points

-p=total number of points

Establishing the excess measurements witch are actually the number of geometrical conditions for the independent and dependent networks, is made with the help of the next mathematical relations:

$$N = l' - 2n \qquad (1)$$

- for dependent networks;

$$N = l' - 2p + 3 \qquad (2)$$

- for independent networks;

For the chosen network we'll have 3 geometrical conditions:

- one for orientation
- two for coordinates(one on x and one on y)

The geometric conditions are referring strictly to the form of the geometric figure from the network from witch it makes part of.

The orientation condition is written starting with one of the network base towards the other side of the network witch is known and called base, thus going through all the network points.

In our case we'll have the following conditions:

$$\Theta_{BA} - (\hat{2}) + (\hat{6}) - (\hat{8}) = \Theta_{DE} \qquad (3)$$

The coordinate's condition is written starting from a stable known point towards another stable known point going through all the new points:

$$\begin{cases} x_D = x_B + [(\Delta x)_B^D] \\ y_D = y_B + [(\Delta y)_B^D] \end{cases} \qquad (4)$$

Based on these conditions we'll go on to writing the error equations for each geometric condition.

The error conditions can be written just if the angular conditions can be calculated with the help of side's corrections. This can be realised by starting of from angular corrections and sides from a triangle with the help of tangent theorem.

$$tg\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}} \qquad (5)$$

unde:

$$p = \frac{a+b+c}{2} \qquad (6)$$

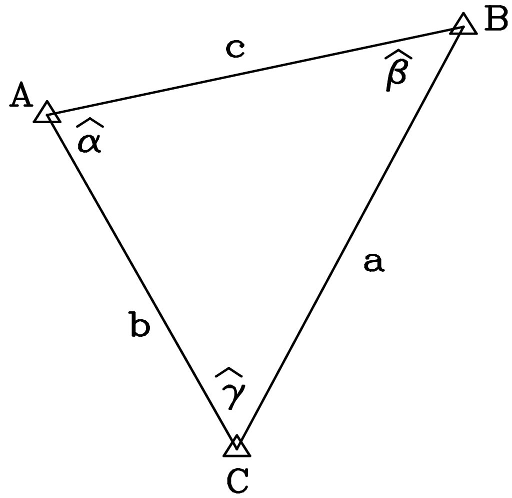

Fig. 2 General notations in the triangle

In order to obtain the errors equations with the help of side's correction after equation no 5 suffered a linearization mathematical calculation will fallow and the result will be the general form for the error equation:

$$v''_{\alpha} = Av_a + Bv_b + Cv_c \qquad (7)$$

The relation above represents the error equations that make a connection between the angular correction and the sides correction.

This relation can be applied in any case no matter the form of condition for the sides measured network analysed.

After making all the calculations the error system will result and it'll look the one below:

$$\begin{cases} -A_2 v_{AC} + A_6 v_{BD} - A_8 v_{CE} + (B_6 - B_8) v_{CD} + (C_2 + C_6) v_{BC} + \dfrac{w_1}{\rho^{cc}} = 0 \\ v_{BC}[\cos\Theta_{BC} + C_2(y_C - y_B) + (C_2 - C_6)(y_D - y_C)] + \\ + v_{CD}[\cos\Theta_{CD} - B_6(y_D - y_C)] + v_{AC}[A_2(y_C - y_B) + A_2(y_D - y_C)] - \\ - v_{BD} A_6 (y_D - y_C) + w_2 = 0 \\ v_{BC}[\sin\Theta_{BC} - C_2(x_C - x_B) + (-C_2 + C_6)(x_D - x_C)] + \\ + v_{CD}[\sin\Theta_{CD} + B_6(x_D - x_C)] + v_{AC}[-A_2(x_C - x_B) - A_2(x_D - x_C)] + \\ + v_{BD} A_6 (x_D - x_C) + w_3 = 0 \end{cases} \qquad (8)$$

In order to obtain the numeric form of the equation system we know the values of the A,B,C coefficients and the coordinates of the fallowing points: A,B, D and E and the temporary coordinates of C point determined by simple linear ahead intersection.

Table 2

Point name	Final coordinates	
	X	Y
A	512942,564	396489,140
B	513326,689	398140,149
D	512028,777	400066,523
E	510798,429	399243,030

Table 3

Point name	Temporary coordinates	
	X	Y
C	511487,543	397241,812

Analysing the error equation system (8) we can see that the unknown elements $v_{i,i+1}$ are 5 and the system has only 3 equations so the system is incompatible and undetermined.

This system can be solved by the smallest square mathematic method from the theory of errors accepting the condition that [vv]=minimum.

Replacing the expressions of v_i according to the k_i correlates we'll obtain a normal equation system (the number of the equation is equal to the number of unknown elements) symmetrical to the basic diagonal where the sums of the coefficients are equal and with apposite signs:

$$\begin{aligned}[aa]K_1 + [ab]K_2 + [ac]K_3 + W_1 &= 0\\ [bb]K_2 + [bc]K_3 + W_2 &= 0\\ [cc]K_3 + W_3 &= 0\end{aligned} \qquad (9)$$

The normal system of equations will be solved by the Gauss method(successive reductions).

After the calculations for the k correlates the fallowing results will have to result:

Table 4

Correlate	Value
k3	-0,007764
k2	-0,005838
k1	-4,103863

Introducing the values above in the relation number (9) we'll obtain the corrections for the network sides analysed.

The sides compensation in the network is made by applying the correspondent correction for each first measurement of each side. The final value of each side will be:

Table5

Side name	Side compensation		
	Initial value	Correction	Final value
AC	1638,163	-0,010481	1638,153
BD	2322,822	0,004038	2322,826
CE	2116,543	0,002867	2116,546
CD	2876,105	-0,005641	2876,099
BC	2046,791	0,017327	2046,808

For the final stage of network analysis well use the compensated values of each side.

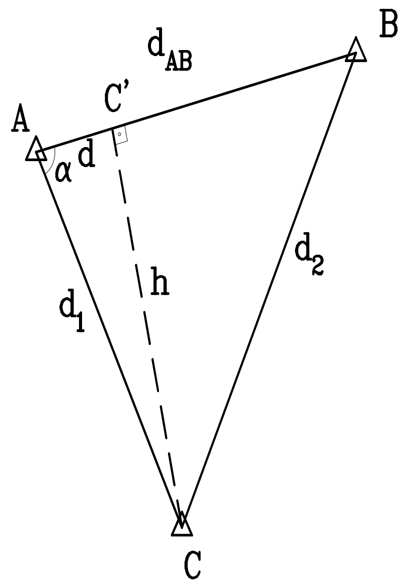

Fig. 3 Network coordinates calculation

In order to calculate the coordinates for the C point we know the fallowing elements:

- coordinates of point A $(x_A; y_A)$;
- coordinates of point B $(x_B; y_B)$;
- value of sides AB, AC and BC;
- Θ_{AB} orientation.

Coordinates of point C, according to figure no. 3 will be:

$$\begin{cases} x_C = x_A + d\cos\Theta_{AB} + h\cos(\Theta_{AB} + 100^g) \\ y_C = y_A + d\sin\Theta_{AB} + h\sin(\Theta_{AB} + 100^g) \end{cases} \quad (10)$$

Making the transition from angles to distances the system (10) will transform into:

$$\begin{cases} x_C = x_A + d\dfrac{x_B - x_A}{d_{AB}} - h\dfrac{y_B - y_A}{d_{AB}} \\ y_C = y_A + d\dfrac{y_B - y_A}{d_{AB}} + h\dfrac{x_B - x_A}{d_{AB}} \end{cases} \quad (11)$$

We can observe in the relations above that the unknown elements are d and h. Their values can be obtained by applying the generalised Pitagora's theorem in the triangle ABC.

In this condition the equation system (11) has no unknown elements so the values of x and y of C point can be calculated resulting the fallowing values:

Table 6

Point name	Final coordinates	
	X	Y
C	511487,558	397241,805

In conclusion about the presented method above there are advantages referring to data collection from the field, reduced time of measuring on the field, the measurements can be done in different atmospheric conditions, small price value because no money need to be spent on signalization pyramid constructions, much simple calculations but with higher precision etc.

Similar to the triangulation networks there will be applied compensations on the measurements – distances measured.

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