Building Observation for Analysis of Displacements and Deformations in Geodetic Networks for Civil Constructions

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Abstract: *The displacements and deformation process, represents an complex aspects of behaviour on buildings, such as bridges, houses or industrial buildings. Using geodetic observations we can calculate with precision the relative or total subsides of building and the horizontal mouvement on the X, Y axis. The geodetic observation are executed from special networks, defined with references points and settlements marks on the buildings, who create an mathematical network, in which can be computed all the variations of the buildings.*

Keywords: *displacements and deformation process, behaviour on buildings, geodetic observations, references points.*

1. Introduction

In this paper are shown the generalized method for identification of relatively stable points, combined with simultaneous determination of deformations in geodetic networks.

The method is based on deterministic relations connecting apparent displacements of any three points/ the three points constitute a triangular element of the network, and the components of translation and rotation vectors of the triangle, togheter with the components of its deformation tensor. The first part of the paper treats the theoretical and practical rules of identification of stable points in horizontal networks.

The second part of the paper is devoted to the final determination of displacements of the network points, on the base of just identified stable points. In this part, I present the account mutual relations among points, making use of their variance-covariance vectors, describing the final determinations of the network.

2. Measurements of Relatively Stable Points

The proposed method is based on the results of adjustment of observed angles, directions, distances in the network, obtained during the original and the actual measurements.

If in both periodic measurements the same quantities were observed, then they can be adjusted simultaneously. In this case I assume stability of two selected points and assign zero displacements to them for the period of interest, obtaining displacements ux, uy of all the remaining points.

I will use the name “apparent displacement” for the computed displacements, distorted by the systematic factors.

Since we a-priori never know anything sure about stability of the two selected points, I have to treat the computed displacements as the apparent ones. Only when the identification process proves their stability, I can treat them as real displacements. Criterions used in the identification of stable points are based on errors of the computed parameters of translation and deformation.

In the case when in the two periodic measurements different elements were observed, what in practice is rather a common case, the two sets of observations have to be adjusted separately.

But in both adjustments I use the same elements of initial information-fix the same two points, or a point and an azimuth, /minimum constraint adjustment/. As the result we obtain two sets of coordinates for points of the net under examination .

Differences of the point coordinates are the apparent displacements of points.

In both of the above mentioned cases I can identify the stable points by the generalized method of examination of deformations and local displacements in a network, defined by its points and periodically observed elements.

I take three points of the network. Their coordinates x, y and their apparent ux ,uy displacements are known. Coordinates of the points are known from the adjustment of the original survey; their apparent displacements are known from adjustment /s/ of two periodic observations, as it was described earlier.

Every such triangle will have a set of six components of apparent translation of points, which can be written as:

$$[v]^T = \left[v_{xi}, v_{yi}, v_{xj}, v_{yj}, v_{xk}, v_{yk}\right] \tag{1}$$

In order to maintain continuity within the neighbouring elements, I will use a six parameter deformaton/displasement model:

$$\begin{aligned} V_X &= p_1 + p_2 x + p_3 y \\ V_Y &= p_4 + p_5 x + p_6 y \end{aligned} \tag{2,3}$$

or, matrix rotation:

$$\begin{bmatrix} V_X \\ V_Y \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \tag{4}$$

The six parameters p_r can be determined solving the folowing two sets of three equations:

$$\begin{aligned} v_{xi} &= p_1 + p_2 x_i + p_3 y_i \\ v_{xj} &= p_1 + p_2 x_j + p_3 y_j \\ v_{xk} &= p_1 + p_2 x_k + p_3 y_k \end{aligned} \tag{5}$$

and

$$\begin{aligned} v_{yi} &= p_4 + p_5 x_i + p_6 y_i \\ v_{yj} &= p_4 + p_5 x_j + p_6 y_j \\ v_{yk} &= p_4 + p_5 x_k + p_6 y_k \end{aligned} \tag{6}$$

In matrix notation equations (5) and (6) are:

$$\begin{aligned} [v_x] &= A \cdot [p_x] \\ [v_y] &= A \cdot [p_y] \end{aligned}$$

In order to solve the system of equations (5) and (6) I have to find the invers of A, which is:

$$A^{-1} = \frac{1}{2P} \begin{bmatrix} a_i & b_i & c_i \\ a_j & b_j & c_j \\ a_k & b_k & c_k \end{bmatrix} \tag{7}$$

where:

$$a_i = x_j y_k - x_k y_j$$
$$b_i = y_j - y_k \quad (8)$$
$$c_i = x_k - x_j$$

The remaining coefficients ae obtained by cyclic change of indices i, j, k and P is the area of the element:

$$2P = \det A \quad (9)$$

Substituting equation (7) into (5) and (6), results:

$$[p_x] = A^{-1}[v_x]$$
$$[p_y] = A^{-1}[v_y]$$

(10)

Now if formula (10) is substitute into (3), so the field of translations within the triangular element will be given by the formula:

$$[V_X] = \begin{bmatrix} A^{-1}[v_x] \\ A^{-1}[v_y] \end{bmatrix}^T \cdot \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \quad (11)$$

or

$$V_X = \frac{1}{2P}(a_i + b_i x + c_i y)v_{xi} + (a_j + b_j + c_j y)v_{xj} + + (a_k + b_k x + c_k y)v_{xk} \quad (12)$$

$$V_y = \frac{1}{2P}(a_i + b_i x + c_i y)v_{yi} + (a_j + b_j + c_j y)v_{yj} + + (a_k + b_k x + c_k y)v_{yk}$$

On the base of translations defined by formula (12) I can find, for every triangular element, six parameters uniquely defining translation and deformation of the element. The parameters are defined as:

$$\begin{bmatrix} \overline{v_{yo}} \\ v_{yo} \\ \omega_{xy} \\ \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_N = \begin{bmatrix} V_X \\ V_Y \\ \frac{1}{2}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) \\ \frac{\partial V_x}{\partial x} \\ \frac{\partial V_y}{\partial y} \\ \left(\frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y}\right) \end{bmatrix}_N \quad (13)$$

where:

v_{xo} = components of the translation vector for the Nth element;

ω_{xy} = rotation of the Nth element in xy plane;

$\varepsilon_x, \varepsilon_y$ = strains of the N[th] element;

γ_{xy} = shape deformation of the N[th] element in the xy plane.

Substituing (13) in (12), obtained formula (14):

$$\begin{bmatrix} v_{yo} \\ v_{yo} \\ \omega_{xy} \\ \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_N = \frac{1}{2P} \begin{bmatrix} a_i & 0 & a_j & 0 & a_k & 0 \\ 0 & a_i & 0 & a_j & 0 & a_k \\ -c_i & b_i & -c_j & b_j & -c_k & b_k \\ b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{bmatrix} \cdot \begin{bmatrix} v_{xi} \\ v_{yi} \\ v_{xj} \\ v_{yj} \\ v_{xk} \\ v_{yk} \end{bmatrix} \qquad (14)$$

Parameters defined and computed in this way are the base for identification of stable points and for analysis of geodetic networks. Of course, the method cannot be applied to triangles which have all their three vertices situated on, or close to, on straight line, becoause their parameters will be undefined, or defined with very poor accuracy. As it was stated earliere, it will treat as relatively stable such points for which:

$$\begin{aligned} \gamma_{xy} &= 0 \\ \varepsilon_x &= 0 \\ \varepsilon_y &= 0 \end{aligned} \qquad (15)$$

The fulfilment of the above criterion is hindered by measurment errors in the network, so for practical reasons, it will be use the criterions:

$$\begin{aligned} \gamma_{xy} &\le 2m_{\gamma_{xy}} \\ \varepsilon_x &\le 2m_{\varepsilon_x} \\ \varepsilon_y &\le 2m_{\varepsilon_y} \end{aligned}$$

(16)

In the case when the criterions (16) would be fulfilled for several triangular elements, it could be checked the stability of their relative position on the base of similarity of their translation and rotation parameters, using the criterions:

$$\begin{aligned} \omega_{xy}^n &= cons\tan ts \pm 3m \\ v_{xo}^n &= cons\tan s \pm 3m_{ux} \\ v_{yo}^n &= cons\tan s \pm 3m_{uy} \end{aligned} \qquad (17)$$

where n is the number of the triangular element.

Assumption of stabilityof two points for the need of the initial adjustment may, in the case when it proves false, leave us with systematic scale error within the network. This systemtic influence will show up in equality of strains:

$$\varepsilon_x = \varepsilon_y \qquad (18)$$

or rather:

$$(\varepsilon_x - \varepsilon_y) \le 3m\varepsilon, \qquad (19)$$

though corresponding criterions in (16) will not hold. Condition of similarity of strains in several triangles can be written as:

$$\varepsilon_y^n = \varepsilon_y^n = const \pm 3m_\varepsilon$$

(20)

In short it can be write that the problem of finding of stable points in a network is reduced to computation, by formula (14), of the six parameters for every triangle of the network, using a simple computer program. Then it look for triangles, for which:

$$\gamma_{xy} \leq 2m\gamma_{xy} \tag{21}$$

is fulfilled, and the remaining parameters are constant:

$$\begin{aligned} \varepsilon_x^n &= \varepsilon_y^n = cons\tan s \pm 3m_\varepsilon \\ \omega_{xy}^n &= cons\tan ts \pm 3m_\omega \\ v_{xo}^n &= cons\tan s \pm 3m_{ux} \\ v_{yo}^n &= cons\tan s \pm 3m_{uy} \end{aligned} \tag{22}$$

In practice, it can be assume as stable only these points, which are members of least two triangular elemets fulfilling formula (21) and (22).

3. Final Vectors Computation of Point Displacemets in the Network

Stable points, found by the method described in the previous chapter, constitute the base, the reference system, for computation of final (free from orientation errors) displacements of the network points.

The computation is done by approximation. In order to find parameters of the transformation of the point displacements, it is set up for every reference point two approximation equations like:

$$\begin{bmatrix} \delta_{xi} \\ \delta_{yi} \end{bmatrix} = \begin{bmatrix} v_{xo} \\ v_{yo} \end{bmatrix} + \begin{bmatrix} \varepsilon & -\omega \\ \omega & \varepsilon \end{bmatrix} \cdot \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} v_{xi} \\ v_{yi} \end{bmatrix} \tag{23}$$

where:

δ_{xi}, δ_{yi} = approximation corrections to components of the translation vector of *i* point;

ε, ω = linear deformation and rotation of the whole network;

x_i, y_i = coordinates of the *i* reference point;

v_{xo}, v_{yo} = components of translation vector of the whole network.

The equation system has four unknows, and can be written as follows:

$$[\delta] = [\alpha] \cdot [\varepsilon] + [v] \tag{24}$$

Knowing the transformation parameters we can compute the final displacements of unstable points, using formula (24) in a modified form:

$$\begin{bmatrix} \overline{v_{xj}} \\ \overline{v_{yj}} \end{bmatrix} = \begin{bmatrix} v_{xj} \\ v_{yj} \end{bmatrix} - \begin{bmatrix} \varepsilon & -\omega \\ \omega & \varepsilon \end{bmatrix} \cdot \begin{bmatrix} x_j \\ y_j \end{bmatrix} - \begin{bmatrix} v_{xo} \\ v_{yo} \end{bmatrix} \tag{25}$$

where:

$\overline{v_{xj}}, \overline{v_{yj}}$ = components of final displacement of the j point;

v_{xj}, v_{yj} = components of apparent displacement of the j point;

ε, ω = transformation parameters (scale change, rotation).

v_{xo}, v_{yo} = components of total translation vector;

x_j, y_j = coordinates of the j point.

The final components of the points displacements, computed along (25), are free from scale and orientation (datum translation and rotation) errors.

4. References

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