Converting Plane Rectangular Coordinates from the Stereographic Projection - 1970 into the Gauss Projection, by Means of the Constant Coefficients' Method

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Abstract: *Coordinates' conversions between the main cartographic projections used in Romania, Gauss-Kruger and Stereographic – 1970, can be made either from the theoretical point of view, as a conversion using the parameters specific to each projection, or in practical circumstances, by the double points' method, in case of some geodesic networks of points with coordinates known in both projection systems. For a rigorous, precise conversion the two-step procedure is known, by direct and reversed transcalculation of the plane rectangular coordinates, using as a transformation mediator, their provisory passing into geographical coordinates on the surface of the Krasovski-1940 reference ellipsoid. For a direct conversion of the plane rectangular coordinates, without intermediating ellipsoidal geographical coordinates, further on algorithms of coordinates' direct conversion are proposed further on, by using formulas with constant coefficients.*

Keywords: *Coordinates conversions, cartographic projections, Gauss-Kruger, Stereographic – 1970, transcalculation.*

1. Introduction

Because both the projections, Gauss – Krüger and Stereographic one – 1970, are defined on the same reference ellipsoid (Krasovski – 1940), we can consider this conventional surface as a mediator of the plane rectangular coordinates' conversion, by their provisory passing, in ellipsoidal geographical coordinates. Therefore, this requires 2 stages:

1) Conversion of the plane rectangular coordinates (x,y) in the Stereographic - 1970 projection system, in geographical coordinates (φ,λ) on the Krasovski – 1940 reference ellipsoid;
2) Conversion of the geographical coordinates (φ,λ) in the Krasovski – 1940 reference ellipsoid, in plane rectangular coordinates (x,y), in Gauss – Krüger projection system.

This procedure implies using into each of the two stages, of the map's equations, by the reversed conversion in case of the Stereographic projection – 1970 and the direct conversion in case of Gauss – Krüger projection. For a direct conversion of the plane rectangular coordinates, with no intermediation from the ellipsoidal geographical coordinates, we propose as follows algorithms of coordinates' direct conversion by using formulas with constant coefficients.

2. Principles of the conversion of the plane rectangular coordinates from the Stereographic projection-1970 into Gauss projection, by constant-coefficient method

We consider the Stereo -1970 plane rectangular coordinates (X_{sec},Y_{sec}) of a point, which shall be passed into the tangent plane of the projection (X_{tg},Y_{tg}), by canceling translation of the axes system of coordinates (X_0 = 500 000,000 m; Y_0 = 500 000,000 m) and multiplication with the coefficient of returning to scale (C'=1,000 250 063): $\mathbf{X_{tg} = C'(X_{sec}-X_0)}$; $\mathbf{Y_{tg} = C'(Y_{sec}-Y_0)}$.

The working operative terms shall be reduced to smaller values, by multiplying the coordinates in the tangent plane with a 10^{-5} factor, according to the model of the reversed conversion in the Stereographic projection – 1970: $\mathbf{X = X_{tg}\,10^{-5}\,;\; Y = Y_{tg}\,10^{-5}}$

Gauss plane rectangular coordinates shall be obtained by applying certain 5-degree polynomial equations, considered as being optimal from the point of view of ensuring an appropriate precision in developing calculus formulas, by 20 constant coefficients:

$$\begin{aligned} X_{Gauss} = {} & A_0 + A_1X + A_2Y + \\ & + A_3X^2 + A_4XY + A_5Y^2 + \\ & + A_6X^3 + A_7X^2Y + A_8XY^2 + A_9Y^3 + \\ & + A_{10}X^4 + A_{11}X^3Y + A_{12}X^2Y^2 + A_{13}XY^3 + A_{14}Y^4 + \\ & + A_{15}X^5 + A_{16}X^4Y + A_{17}X^3Y^2 + A_{18}X^2Y^3 + A_{19}XY^4 + A_{20}Y^5 \end{aligned}$$

$$\begin{aligned} Y_{Gauss} = {} & B_0 + B_1X + B_2Y + \\ & + B_3X^2 + B_4XY + B_5Y^2 + \\ & + B_6X^3 + B_7X^2Y + B_8XY^2 + B_9Y^3 + \\ & + B_{10}X^4 + B_{11}X^3Y + B_{12}X^2Y^2 + B_{13}XY^3 + B_{14}Y^4 + \\ & + B_{15}X^5 + B_{16}X^4Y + B_{17}X^3Y^2 + B_{18}X^2Y^3 + B_{19}XY^4 + B_{20}Y^5 \end{aligned}$$

Where the constant coefficients A0, A1,... and B0, B1,... shall be calculated separately for the spindle 34 and 35 in the Gauss – Krüger projection, owing to the distinct coordinates' systems. The calculating principle has as fundament the combination between the reversed conversion in the Stereographic -1970 projection and the direct conversion in Gauss projection (spindle no. 34/35).

3. Calculus of the constant coefficients for converting the plane rectangular coordinates in the Stereographic – 1970 projection into Gauss – Krüger projection

From the general statements of the reversed conversion in the Stereographic – 1970 projection, only 5-degree terms are kept for the differences in latitude and its afferent powers:

$$\Delta\varphi = (A_{10}\cdot X + A_{20}\cdot X^2 + A_{30}\cdot X^3 + A_{40}\cdot X^4 + A_{50}\cdot X^5 + A_{02}\cdot Y^2 + A_{12}\cdot X\cdot Y^2 + A_{22}\cdot X^2\cdot Y^2 + A_{32}\cdot X^3\cdot Y^2 + A_{04}\cdot Y^4 + A_{14}\cdot X\cdot Y^4$$

$$\Delta\varphi^2 = A_{10}^2\cdot X^2 + 2\cdot A_{10}\cdot A_{20}\cdot X^3 + (A_{20}^2 + 2\cdot A_{10}\cdot A_{30})\cdot X^4 + 2\cdot (A_{10}\cdot A_{40} + A_{20}\cdot A_{30})\cdot X^5 + A_{02}^2\cdot Y^4 + 2\cdot A_{10}\cdot A_{02}\cdot X\cdot Y^2 + 2\cdot (A_{10}\cdot A_{12} + A_{20}\cdot A_{02})\cdot X^2\cdot Y^2 + 2\cdot (A_{10}\cdot A_{22} + A_{20}\cdot A_{12} + A_{30}\cdot A_{02})\cdot X^3\cdot Y^2 + 2\cdot (A_{10}\cdot A_{04} + A_{02}\cdot A_{12})\cdot X\cdot Y^4$$

$$\Delta\varphi^2 = A_{10}^2\cdot X^2 + 2\cdot A_{10}\cdot A_{20}\cdot X^3 + \overline{a_{10}}\cdot X^4 + \overline{a_{15}}\cdot X^5 + A_{02}^2\cdot Y^4 + 2\cdot A_{10}\cdot A_{02}\cdot X\cdot Y^2 + \overline{a_{12}}\cdot X^2\cdot Y^2 + \overline{a_{17}}\cdot X^3\cdot Y^2 + \overline{a_{19}}\cdot X\cdot Y^4$$

...

We act in the same way for the differences in longitude and its afferent powers, only the 5-degree terms being kept after these were previously reduced to the origin meridian of the Stereographic – 1970 projection, by the constant of KL, representing the difference to the axial meridian of the 6° longitude spindle, from Gauss projection:

$$\Delta\lambda = B_{01}\cdot Y + B_{11}\cdot X\cdot Y + B_{21}\cdot X^2\cdot Y + B_{31}\cdot X^3\cdot Y + B_{41}\cdot X^4\cdot Y + B_{03}\cdot Y^3 + B_{13}\cdot X\cdot Y^3 + B_{23}\cdot X^2\cdot Y^3 + B_{05}\cdot Y^5 - KL$$

$$\Delta\lambda^2 = KL^2 - 2\cdot KL\cdot B_{01}\cdot Y - 2\cdot KL\cdot B_{11}\cdot X\cdot Y - 2\cdot KL\cdot B_{21}\cdot X^2\cdot Y - 2\cdot KL\cdot B_{31}\cdot X^3 \cdot Y - 2\cdot KL\cdot B_{41}\cdot X^4\cdot Y + B_{01}^2\cdot Y^2 + 2\cdot B_{01}\cdot B_{11}\cdot X\cdot Y^2 + (B_{11}^2 + 2\cdot B_{01}\cdot B_{21})\cdot X^2\cdot Y^2 + (2\cdot B_{01}\cdot B_{31} + 2\cdot B_{11}\cdot B_{21})\cdot X^3\cdot Y^2 - 2 \cdot KL\cdot B_{03}\cdot Y^3 - 2\cdot KL\cdot B_{13}\cdot X\cdot Y^3 - 2\cdot KL\cdot B_{23}\cdot X^2\cdot Y^3 + 2\cdot B_{01}\cdot B_{03} \cdot Y^4 + (2\cdot B_{01}\cdot B_{13} + 2\cdot B_{11}\cdot B_{03})\cdot X\cdot Y^4 - 2\cdot KL\cdot B_{05}\cdot Y^5$$

$$\Delta\lambda^2 = \mathrm{KL}^2 - 2\cdot KL\cdot B_{01}\cdot Y - 2\cdot KL\cdot B_{11}\cdot X\cdot Y - 2\cdot KL\cdot B_{21}\cdot X^2\cdot Y - 2\cdot KL\cdot B_{31}\cdot X^3\cdot Y - 2\cdot KL\cdot B_{41}\cdot X^4\cdot Y + \mathrm{B}_{01}^2\cdot Y^2 + 2\cdot B_{01}\cdot B_{11}\cdot X\cdot Y^2 + \overline{c_{12}}\cdot X^2\cdot Y^2 + \overline{c_{17}}\cdot X^3 \cdot Y^2 - 2\cdot KL\cdot B_{03}\cdot Y^3 - 2\cdot KL\cdot B_{13}\cdot X\cdot Y^3 - 2\cdot KL\cdot B_{23}\cdot X^2\cdot Y^3 + 2\cdot B_{01} \cdot B_{03}\cdot Y^4 + \overline{c_{19}}\cdot X\cdot Y^4 - 2\cdot KL\cdot B_{05}\cdot Y^5$$

…

Equations of the direct conversion in the Gauss projection shall be written now by replacing the differences in latitude and longitude, expressed above by means of the Stereographic -1970 plane rectangular coordinates.

Consequently, for the differences ΔX and ΔY shall be added three terms for each one, corresponding to the Gauss direct equations. For the difference on the abscissa (X), we get:

$$\Delta X = (a_{00} + a_{10}\,f + a_{20}\,f^2 + a_{30}\,f^3 + a_{40}\,f^4) + (a_{02} + a_{12}\,f + a_{22}\,f^2 + a_{32}\,f^3 + a_{42}\,f^4)\, l^2 + (a_{04} + a_{14}\,f + a_{24}\,f^2 + a_{34}\,f^3)\, l^4 = T_0 + T_2 + T_4$$

where:

$$T_0 = (a_{00} + a_{10}\cdot f + a_{20}\cdot f^2 + a_{30}\cdot f^3 + a_{40}\cdot f^4)$$

$$T_0 = a_{00} + a_{10}\cdot(A_{10}\cdot X + A_{20}\cdot X^2 + A_{30}\cdot X^3 + A_{40}\cdot X^4 + A_{50}\cdot X^5 + A_{02}\cdot Y^2 + A_{12}\cdot X\cdot Y^2 + A_{22}\cdot X^2\cdot Y^2 + A_{32}\cdot X^3\cdot Y^2 + A_{04}\cdot Y^4 + A_{14}\cdot X\cdot Y^4) + a_{20}\cdot(A_{10}^2\cdot X^2 + 2A_{10}\cdot A_{20}\cdot X^3 + \overline{a_{10}}\cdot X^4 + \overline{a_{15}}\cdot X^5 + A_{02}^2\cdot Y^2 + 2\cdot A_{10}\cdot A_{02}\cdot X\cdot Y^2 + \overline{a_{12}}\cdot X^2\cdot Y^2 + \overline{a_{17}}\cdot X^3\cdot Y^2 + \overline{a_{19}}\cdot X\cdot Y^4) + \ldots$$

$$T_2 = (a_{20} + a_{12}f + a_{22}f^2 + a_{32}f^3 + a_{42}f^4)l^2$$
$$= (a_{20} + a_{12}A_{10}X + f_3X^2 + f_6X^3 + f_{10}X^4 + f_{15}X^5 + a_{12}A_{02}Y^2 + f_8XY^2 + f_{12}X^2Y^2 + f_{17}X^3Y^2 + f_{14}Y^4 + f_{19}XY^4)$$

$$(KL^2 - 2\cdot KL\cdot B_{01}\cdot Y - 2\cdot KL\cdot B_{11}\cdot X\cdot Y - 2\cdot KL\cdot B_{21}\cdot X^2\cdot Y - 2\cdot KL\cdot B_{31}\cdot X^3\cdot Y - 2\cdot KL\cdot B_{41}\cdot X^4\cdot Y + B_{01}^2\cdot Y^2 + 2\cdot B_{01}\cdot B_{11}\cdot X\cdot Y^2 + \overline{C_{12}}\cdot X^2\cdot Y^2 + \overline{C_{17}}\cdot X^3\cdot Y^2 - 2\cdot KL\cdot B_{03}\cdot Y^3 - 2\cdot KL\cdot B_{13}\cdot X\cdot Y^3 - 2\cdot KL\cdot B_{23}\cdot X^2\cdot Y^3 + 2\cdot B_{01}\cdot B_{03}\cdot Y^4 + \overline{C_{19}}\cdot X\cdot Y^4 - 2\cdot KL\cdot B_{05}\cdot Y^5) = \ldots$$

$$T_4 = (a_{04} + a_{14}f + a_{24}f^2 + a_{34}f^3)\cdot l^4 =$$
$$= [a_{04} + a_{14}A_{10}X + g_3X^2 + g_6X^3 + g_{10}X^4 + g_{15}X^5 + a_{14}A_{02}Y^2 + g_8XY^2 + g_{12}X^2Y^2 + g_{17}X^3Y^2 + g_{14}Y^4 + g_{19}XY^4][KL^4 - 4\cdot KL^3\cdot B_{01}Y - 4\cdot KL^3\cdot B_{11}XY - 4\cdot KL^3\cdot B_{21}X^2Y - 4\cdot KL^3\cdot B_{31}X^3Y - 4\cdot KL^3\cdot B_{41}X^4Y + 6\cdot KL^2\cdot B_{01}^2Y^2 + 12\cdot KL^2\cdot B_{01}B_{11}XY^2 + \overline{d_{12}}X^2Y^2 + \overline{d_{17}}X^3Y^3 - \overline{d_9}Y^3 - \overline{d_{13}}XY^3 - \overline{d_{18}}X^2Y^2 + \overline{d_{14}}Y^4 + \overline{d_{19}}XY^4 - \overline{d_{20}}Y^5] = \ldots$$

into which the terms were noted down:

$$f_3 = a_{12}\cdot A_{20} + a_{22}\cdot A_{10}^2;\ f_6 = a_{12}\cdot A_{30} + 2\cdot a_{22}\cdot A_{10}\cdot A_{20} + a_{32}\cdot A_{10}^3$$

$$f_{10} = a_{12}\cdot A_{40} + a_{22}\cdot\overline{a_{10}} + 3\cdot a_{32}\cdot A_{10}^2\cdot A_{20} + a_{42}\cdot A_{10}^4;\ f_{15} = a_{12}\cdot A_{50} + a_{22}\cdot\overline{a_{15}} + a_{32}\cdot\overline{b_{15}} + 4\cdot a_{42}\cdot A_{10}^3\cdot A_{20}$$

$$f_8 = a_{12}\cdot A_{12} + 2\cdot a_{22}\cdot A_{10}\cdot A_{02};\ f_{12} = a_{12}\cdot A_{22} + a_{22}\cdot\overline{a_{12}} + 3\cdot a_{32}\cdot A_{10}^2\cdot A_{02}$$

$$f_{17} = a_{12}\cdot A_{32} + a_{22}\cdot\overline{a_{17}} + a_{32}\cdot\overline{b_{17}} + 4\cdot a_{42}\cdot A_{10}^3\cdot A_{02};\ f_{14} = a_{12}\cdot A_{04} + a_{22}\cdot A_{02}^2$$

$$f_{19} = a_{12}\cdot A_{14} + a_{22}\cdot\overline{a_{19}} + 3\cdot a_{32}\cdot A_{10}\cdot A_{02}^2$$

$$g_3 = a_{14}A_{20} + a_{24}A_{10}^2; g_6 = a_{14}A_{30} + 2 \cdot a_{24}A_{10}A_{20} + a_{34}A_{10}^3$$
$$g_{10} = a_{14}A_{40} + a_{24}\overline{a_{10}} + 3 \cdot a_{34}A_{10}^2 A_{20}; g_{15} = a_{14}A_{50} + a_{24}\overline{a_{15}} + a_{34}\overline{b_{15}}$$
$$g_8 = a_{14}A_{12} + a_{24}A_{10}A_{02}; g_{12} = a1_4 A_{22} + a_{24}\overline{a_{12}} + 3 \cdot a_{34}A_{10}^4 A_{02}$$
$$g_{17} = a_{14}A_{32} + a_{24}a_{17} + a_{34}\overline{b_{17}} + a_{44}A_{10}^3 A_{02}; g_{14} = a_{14}A_{04} + a_{24} \cdot A_{02}^2$$
$$g_{19} = a_{14}A_{14} + a_{24}\overline{a_{19}} + 3 \cdot a_{34}A_{10}A_{02}^2$$

For the difference on the ordinate (Y), other three terms shall be added up, corresponding to the Gauss direct equations, resulting the following statement:

$$\Delta Y = (b_{01} + b_{11} f + b_{21} f^2 + b_{31} f^3 + b_{41} f^4) l + (b_{03} + b_{13} f + b_{23} f^2 + b_{33} f^3 + b_{43} f^4) l^3 + (b_{05} + b_{15} f + b_{25} f^2) l^5 = T_1 + T_3 + T_5$$

where:

$$T_1 = (b_{01} + b_{11}f + b_{21}f^2 + b_{31}f^3 + b_{41}f^4) \cdot l$$
$$= [b_{01} + b_{11}A_{10}X + h_3X^2 + h_6X^3 + h_{10}X^4 + h_{15}X^5 + b_{11}A_{02}Y^2 + h_8XY^2 + h_{12}X^2Y^2 + h_{17}X^3Y^3 + h_{14}Y^4 +$$
$$+ h_{19}XY^4] \cdot [B_{01}Y + B_{11}XY + B_{21}X^2Y + B_{31}X^3Y + B_{41}X^4Y + B_{03}Y^3 + B_{13}XY^3 + B_{23}X^2Y^3 + B_{05}Y^5 - KL] = ...$$

$$T_3 = (b_{03} + b_{13}f + b_{23}f^2 + b_{33}f^3 + b_{43}f^4) \cdot l^3$$
$$= [b_{03} + b_{13}A_{10}X + j_3X^2 + j_6X^3 + j_{10}X^4 + j_{15}X^5 + b_{13}A_{02}Y^2 + j_8XY^2 + j_{12}X^2Y^2 +$$
$$+ j_{12}X^2Y^2 + j_{17}X^3Y^3 + j_{14}Y^4 + j_{19}XY^4] \cdot [KL^3 + 3 \cdot KL^2 B_{01}Y - 3 \cdot KL \cdot B_{01}^2 Y^2 +$$
$$+ 3 \cdot KL \cdot B_{11}XY - 6 \cdot KL \cdot B_{01}B_{11}XY^2 + 3 \cdot KL^2 B_{21}X^2Y + 3 \cdot KL^2 B_{31}X^3Y +$$
$$+ 3 \cdot KL^2 B_{41}X^4Y - m_{12}X^2Y^2 - m_{17}X^3Y^2 + m_9Y^3 + m_{13}XY^3 + m_{18}X^2Y^3 -$$
$$- 6 \cdot KL \cdot B_{01}B_{03}Y^4 - m_{19}XY^4 + m_{20}Y^5] = ...$$

$$T_5 = (b_{05} + b_{15}f + b_{25}f^2) \cdot l^5$$
$$= [b_{05} + b_{15}A_{10}X + k_6X^3 + k_{10}X^4 + k_{15}X^5 + b_{15}A_{02}Y^2 + k_8XY^2 + k_{12}X^2Y^2 + k_{17}X^3Y^2 +$$
$$+ k_{14}Y^4 + k_{19}XY^4] \cdot [-KL^5 + 5 \cdot KL^4 B_{01}Y + 5 \cdot KL^4 B_{11}XY + 5 \cdot KL^4 B_{21}X^2Y + 5 \cdot KL^4 B_{31}X^3Y +$$
$$+ 5 \cdot KL^4 B_{41}X^4Y - 10 \cdot KL^3 B_{01}y^2 - 20 \cdot KL^3 B_{01}B_{11}XY^2 - p_{12}X^2Y^2 - p_{17}X^3Y^2 + p_9Y^3 +$$
$$+ p_{13}XY^3 + p_{10}X^2Y^3 - p_{14}Y^4 - p_{19}XY^4 + p_{20}Y^5] = ...$$

into which the terms were noted down:

$$h_3 = b_{11}A_{20} + b_{21} \cdot A_{10}; h_6 = b_{11}A_{30} + 2 \cdot b_{11}A_{10}A_{20} + b_{31}A_{10}^3$$
$$h_{10} = b_{11}A_{40} + b_{21} \cdot \overline{a_{10}} + 3 \cdot b_{31}A_{10}^2 A_{20} + b_{41}A_{10}^4; h_{15} = b_{11}A_{50} + b_{21} \cdot \overline{a_{15}} + b_{31} \cdot \overline{b_{15}} + 4 \cdot b_{41}A_{10}^3 A_{20}$$
$$h_8 = b_{11}A_{12} + 2 \cdot b_{21}A_{10}A_{02}; h_{12} = b_{11}A_{22} + b_{21} \cdot \overline{a_{12}} + 3 \cdot b_{31}A_{10}^3 A_{02}$$
$$h_{17} = b_{11}A_{32} + b_{21}\overline{a_{17}} + b_{31}\overline{b_{17}} + 4 \cdot b_{41}A_{10}^3 A_{02}; h_{14} = b_{11}A_{04} + b_{21}A_{02}^2$$
$$h_{19} = b_{11}A_{14} + b_{21}\overline{a_{19}} + 3 \cdot b_{31}A_{10}A_{02}^2$$

$$j_3 = b_{13}A_{20} + b_{23}A_{10}^2; j_6 = b_{13}A_{30} + 2 \cdot b_{23}A_{10}A_{20} + b_{33}A_{10}^3; j_{10} = b_{13}A_{40} + b_{23}\overline{a_{10}} + 3 \cdot b_{33}A_{10}^2 A_{20} + b_{43}A_{10}^4$$
$$j_{15} = b_{13}A_{50} + b_{23}\overline{a_{15}} + b_{33}\overline{b_{15}} + 4 \cdot b_{43}A_{10}^3 A_{20}; j_8 = b_{13}A_{12} + 2 \cdot b_{23}A_{10}A_{02}; j_{12} = b_{13}A_{22} + b_{23}\overline{a_{12}} + 3 \cdot b_{33}A_{10}^2 A_{02}$$
$$j_{17} = b_{13}A_{32} + b_{23}\overline{a_{17}} + b_{33}\overline{b_{17}} + 4 \cdot b_{43}A_{10}^3 A_{02}; j_{14} = b_{13}A_{04} + b_{23} \cdot A_{02}^2; j_{19} = b_{13}A_{14} + b_{23}\overline{a_{19}} + 3 \cdot b_{33}A_{10}A_{02}^2$$

$k_3 = b_{15}A_{20} + b_{25}A_{10}^2; k_6 = b_{15}A_{30} + 2 \cdot b_{25}A_{10}A_{20}; k_{10} = b_{15}A_{40} + b_{25}\overline{a_{10}}; k_{15} = b_{15}A_{50} + b_{25}\overline{a_{15}}$

$k_8 = b_{15}A_{12} + 2 \cdot b_{25}A_{10}A_{02}; k_{12} = b_{15}A_{22} + b_{25}\overline{a_{12}}; k_{17} = b_{15}A_{32} + b_{25}\overline{a_{17}}; k_{14} = b_{15}A_{04} + b_{25}A_{02}^2$

$k_{19} = b_{15}A_{14} + b_{25}\overline{a_{19}}$

Constant coefficients Ai, Bi (table 1) were determined by convenient grouping of the terms in the expressions of the relative coordinates of abscissae and ordinates (ΔX, ΔY), only two of them being presented out of them, one for abscissa X and the other one for ordinate Y:

$A_0 = 5096175.747 + 10^{-8} (KL^2\ a_{02}) + 10^{-16} (KL^4\ a_{04})$

$A_1 = (a_{10}\ A_{10}) + 10^{-8} (KL^2\ a_{12}\ A_{10}) + 10^{-16} (KL^4\ a_{14}\ A_{10})$

....

$B_0 = 500000.000 - 10^{-4} (KL\ b_{01}) - 10^{-12} (KL^3\ b_{03}) - 10^{-20} (KL^5\ b_{05})$

$B_1 = - 10^{-4} (b_{11}\ A_{10}) - 10^{-12} (KL^3\ b_{13}\ A_{10}) - 10^{-20} (KL^5\ b_{15}\ A_{10})$

....

where:

- For spindle number 34: $KL = KL_{34}$ = -14400'';
- For spindle number 35: $KL = KL_{35}$ = 7200'';
- Coeficients typed a_{ij} and b_{ij} shall be multiplied by 10^{-4i}, in order that the value of the difference of longitude in the system of Gauss projection be observed.

Table 1

Notation of coefficient	Spindle no. 34	Spindle no. 35	Notation of coefficient	Spindle no. 34	Spindle no. 35
A_0	5103962.2303	5098121.2365	B_0	809849.7866	345071.8716
A_1	99991.570829042	99997.950212496	B_1	-5029.728468630	2511.943258761
A_2	5029.728542982	-2511.943261355	B_2	99991.570853096	99997.950225293
A_3	-3.838980553	-0.959326024	B_3	-37.939596286	19.020373846
A_4	75.879219702	-38.040758519	B_4	-7.678030017	-1.918597563
A_5	3.839074806	0.959302702	B_5	37.939616159	-19.020380627
A_6	-2.046041119	-2.047476093	B_6	0.313555516	-0.155900599
A_7	-0.940579319	0.467695516	B_7	-6.138227623	-6.142424559
A_8	6.138234767	6.142429149	B_8	-0.940672749	0.467702025
A_9	0.313583180	-0.155903864	B_9	2.046079726	2.047476789
A_{10}	0.000162971	0.000045482	B_{10}	0.002310988	-0.001157776
A_{11}	-0.009387378	0.004648623	B_{11}	0.001887135	0.000474410
A_{12}	-0.002785231	-0.000701447	B_{12}	-0.014158181	0.006982112
A_{13}	0.009438895	-0.004655122	B_{13}	-0.001889392	-0.000474839
A_{14}	0.000478533	0.000120247	B_{14}	0.002360078	-0.001163911
A_{15}	0.000059162	0.000059284	B_{15}	-0.000016648	0.000008273
A_{16}	-0.000108936	0.000053625	B_{16}	0.000405180	0.000386397
A_{17}	-0.000869406	-0.000787572	B_{17}	0.000187031	-0.000095790
A_{18}	-0.000176451	0.000090615	B_{18}	-0.000905335	-0.000796304
A_{19}	0.000452258	0.000398119	B_{19}	-0.000093181	0.000048023
A_{20}	0.000019469	-0.000010021	B_{20}	0.000091206	0.000080314

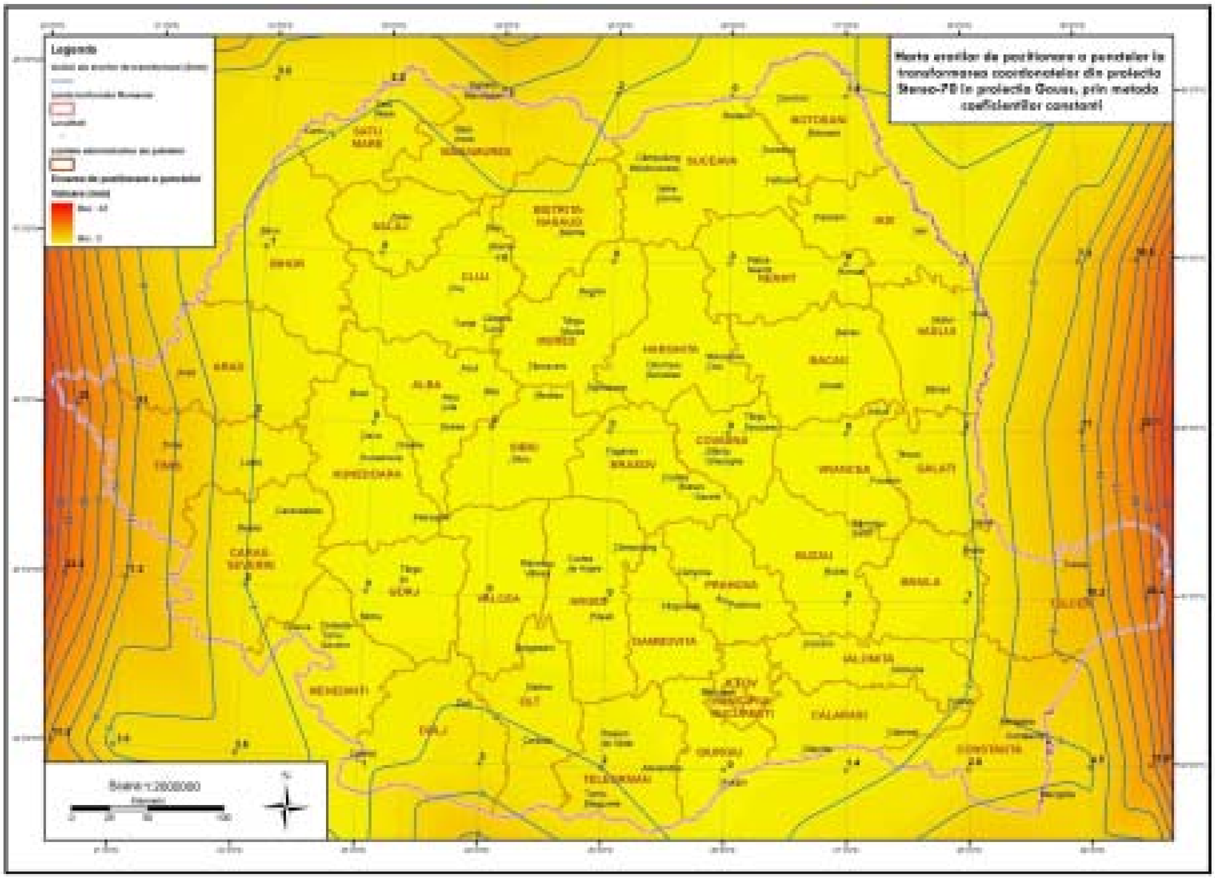

Fig. 1 Map of the points' positioning errors at conversing coordinates from the Stereographic – 1970 projection to the Gauss – Krüger projection, by constant-coefficient method

4. Conclusions

The algorithm elaborated allows a direct conversion of the plane coordinates between the two projections, eliminating the inconvenience of following either an intermediate way by introducing the ellipsoidal geographic coordinates or by appealing to the double-point method by certain polynomial trans-calculus.

Constant coefficients are valid for the whole territory of our country, being distinct for each 6° longitude geographical spindle, specific to the Gauss – Krüger projection (spindle no. 34 and 35). Differences obtained between the coordinates' conversion by constant-coeffcient method and the ones obtained by intermediating the geographical coordinates situate (for the most part of the territory of our country) within the limits of ± 1 – 2 mm, registering maximum values of ± 2 – 3 cm in the eastern and western extemities of Romania (*figure 1*).

5. References

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