Converting Plane Rectangular Coordinates from the Gauss Projection into the Stereographic Projection - 1970, by Means of the Constant Coefficients' Method

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Abstract: *The previously presented cartographic conversion algorithm of the plane rectangular coordinates from the stereographic projection – 1970 into the Gauss projection, underlies the determination of the constant coefficients required for the reversed conversion from the Gauss projection system into the official Romanian one, i.e. the Stereographic – 1970 system. The elaborated algorithm allows a direct conversion of the plane rectangular coordinates between the two projections, eliminating the disadvantage of following either an intermediary way by introducing the ellipsoidal geographical coordinates or by having recourse to the double points' method by polynomial transcalculation procedures.*

Keywords: *Coordinates' conversions, cartographic projections, Gauss-Kruger, Stereographic – 1970, transcalculation.*

1. Introduction

Like in the case of conversion from the Stereographic - 1970 conversion into the Gauss – Krüger projection, the surface of the reference ellipsoid Krasovski – 1940 is used as an intermediary for the conversion, because both the projection have the same ellipsoid as a basis.

In case of the indirect conversion, the algorithm carries 2 stages:

1) Conversion of the plane rectangular coordinates (x,y) in the Gauss- Krüger projection system, in geographical coordinates (φ,λ) on the Krasovski – 1940 reference ellipsoid;
2) Conversion of the geographical coordinates (φ,λ) in the Krasovski – 1940 reference ellipsoid, into plane rectangular coordinates (x,y), in the Stereographic -1970 projection system.

From the practical point of view, the map's equations are applied by reversed conversion in the Gauss-Kruger projection for the first step and by direct conversion in case of the Stereographic – 1970 projection, in the second stage. For an immediate conversion of the plane rectangular coordinates, without intermediation of the ellipsoidal geographic coordinates, there are applied algorithms of coordinates' direct conversion, using formulas with constant coefficients, according the principle of the ones established at the conversion from Stereo – 1970 system to Gauss system.

2. Principles of converting the plane rectangular coordinates from the Gauss projection into the Stereographic – 1970 projection, by constant-coefficient method

We have the Gauss plane rectangular coordinates (X,Y) of a point, which shall reduce themselves of the coordinates of the central point of the Romanian territory corresponding to the spindle of 6° longitude (X_0 = 5096175,747 m; Y_0 = 500000,000 m) and shall be expressed by operative terms (F,L), according to the model of the reversed conversion in the Gauss- Krüger projection system: $\mathbf{F = 10^{-5}\,\Delta X = 10^{-5}\,(X - X_o);\ L = 10^{-5}\,\Delta Y = 10^{-5}\,(Y - Y_o)}$

The Stereo-70 plane rectangular coordinates in the projection's tangent plane shall be obtained by applying some 5-degree polynomial equations, established as an optimum of the development of the constant-coefficient formulas, in order to obtain an appropriate accurateness:

$$\begin{aligned} X_{tg} = {} & C_0 + C_1F + C_2L + \\ & + C_3F^2 + C_4FL + C_5L^2 + \\ & + C_6F^3 + C_7F^2L + C_8FL^2 + C_9L^3 + \\ & + C_{10}F^4 + C_{11}F^3L + C_{12}F^2L^2 + C_{13}FL^3 + C_{14}L^4 + \\ & + C_{15}F^5 + C_{16}F^4L + C_{17}F^3L^2 + C_{18}F^2L^3 + C_{19}FL^4 + C_{20}L^5 \end{aligned}$$

$$\begin{aligned} Y_{tg} = {} & D_0 + D_1F + D_2L + \\ & + D_3F^2 + D_4FL + D_5L^2 + \\ & + D_6F^3 + D_7F^2L + D_8FL^2 + D_9L^3 + \\ & + D_{10}F^4 + D_{11}F^3L + D_{12}F^2L^2 + D_{13}FL^3 + D_{14}L^4 + \\ & + D_{15}F^5 + D_{16}F^4L + D_{17}F^3L^2 + D_{18}F^2L^3 + D_{19}FL^4 + D_{20}L^5 \end{aligned}$$

Where the constant coefficients C_0, C_1,... and D_0, D_1,... shall be calculated separately for the spindle number 34 and 35 in the Gauss – Krüger projection, owing to the distinct coordinates systems, using the same determination principle as in the Stereo-70 → Gauss conversion.

For passing the Stereographic -1970 plane rectangular coordinates from the tangent plane (X_{tg},Y_{tg}) to the secant one (X_{sec},Y_{sec}), the coordinates calculated in the tangent plane are multiplied to the protraction coefficient (*C = 0, 999 750*), after which the coordinates (X_{sec},Y_{sec}) are passed from the system of axes originating in the center or the pole of the Stereo – 70 projection into the translated system of axes ($X_0 = 500\,000{,}000\ m$; $Y_0 = 500\,000{,}000\ m$):

$$X_{sec.70} = X_0 + X_{sec} = X_0 + C\ X_{tg};\quad Y_{sec.70} = Y_0 + Y_{sec} = Y_0 + C\ Y_{tg}$$

3. Calculus of the constant coefficients for converting the plane rectangular coordinates in the Gauss – Krüger projection into the Stereographic – 1970 projection

The constant coefficients (table 1) were determined by replacing the expressions of the differences of latitude and longitude in the algorithm of the coordinates' reversed conversion in the Gauss projection into the algorithm of the coordinates' direct conversion in the Stereographic – 1970 projection, solving the following calculating steps:

- keeping from the general relations of the reverted conversion in the Gauss projection of the 5-degree terms for the differences in latitude and longitude, in the latter situation correspondence between the origin meridian of the Stereographic – 1970 projection and the axial meridian of the spindle of 6° longitude in the Gauss – Krüger projection;
- writing the equations of the direct conversion in the Stereographic – 1970 projection, by replacing the differences of latitude and longitude expressed by the Gauss plane coordinates (as operative terms).

From the calculated constant coefficients, we make an exemplification only for the first two coefficients, that is the one for the abscissa Xtg and the other one for the ordinate Ytg:

$C_0 = 10^{-8}\ (KL^2\ a_{02}) + 10^{-16}\ (KL^4\ a_{04})$

$C_1 = (a_{10}\ C_{10}) + 10^{-8}\ (KL^2\ a_{12}\ C_{10}) + 10^{-16}\ (KL^4\ a_{14}\ C_{10})$

....

$D_0 = -\ 10^{-4}\ (KL\ b_{01}) - 10^{-12}\ (KL^3\ b_{03}) - 10^{-20}\ (KL^5\ b_{05})$

$D_1 = -\ 10^{-4}\ (b_{11}\ C_{10}) - 10^{-12}\ (KL^3\ b_{13}\ C_{10}) - 10^{-20}\ (KL^5\ b_{15}\ C_{10})$

....

where: for spindle number 34: $KL = KL_{34} = 14400''$;

for spindle number 35: KL = KL_{35} = -7200'';

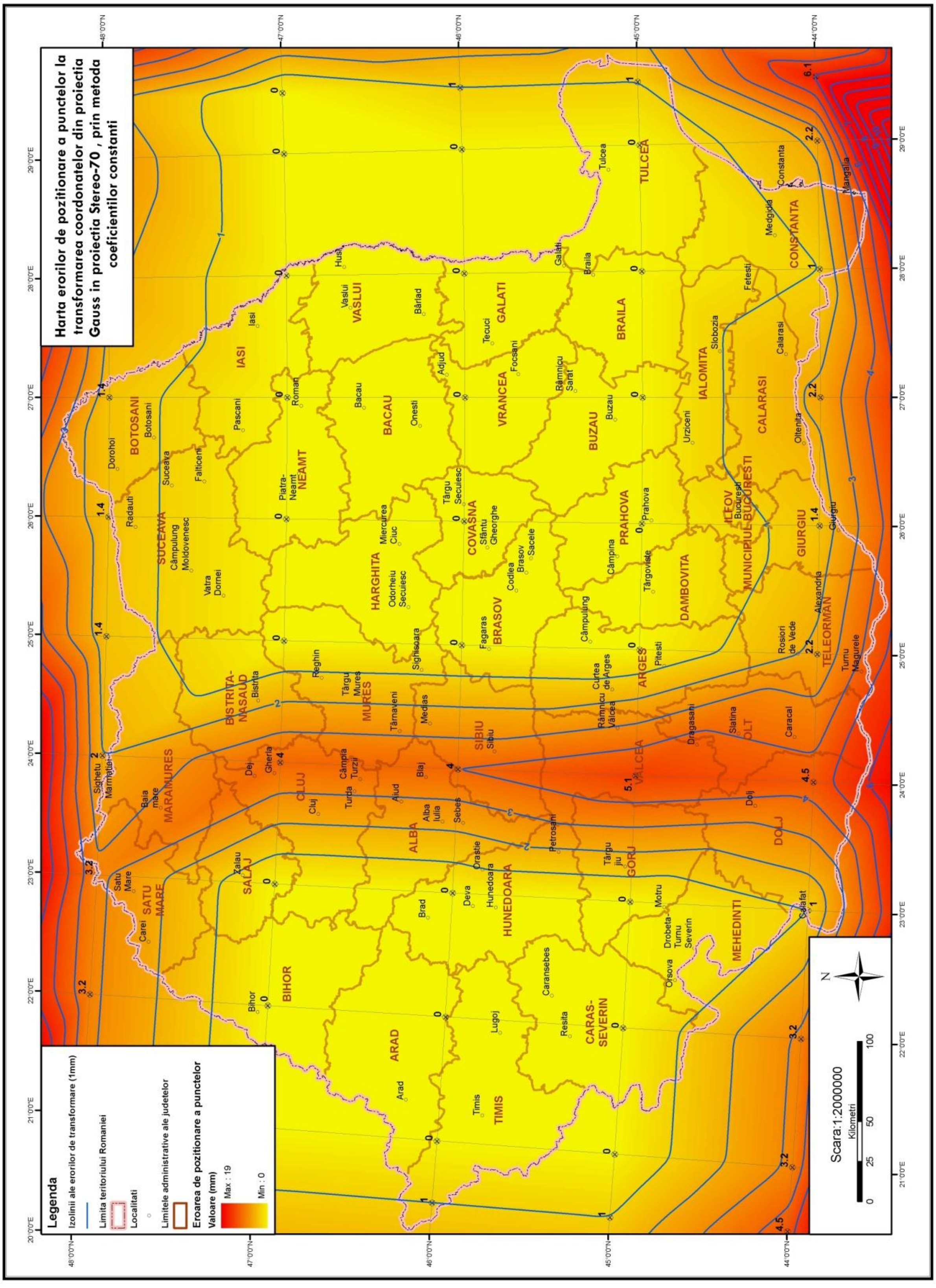

Fig. 1 Map of the points' positioning errors at conversing coordinates from the Gauss – Krüger projection to the Stereographic – 1970 projection, by constant-coefficient method

The constant coefficients C_0, C_1, ... and D_0, D_1, ... are presented separately for the spindle number 34 and 35 in the Gauss – Krüger projection, due to the systems of distinct coordinates, by using the same determination principle as in the previously presented conversion between Stereo 70 projection and the Gauss projection:

Table 1

Notation of coefficient	Spindle no. 34	Spindle no. 35	Notation of coefficient	Spindle no. 34	Spindle no. 35
C_0	7781.8937	1945.2026	D_0	-309788.9962	154920.5168
C_1	99932.802718257	99983.216867358	D_1	5023.802846076	-2511.202631759
C_2	-5023.803729066	2511.202659615	D_2	99932.802729429	99983.216878536
C_3	-1.446440231	-0.361612166	D_3	18.996593432	-9.513538451
C_4	-37.993186720	19.027076895	D_4	-2.892880644	-0.723224516
C_5	1.447152465	0.361656937	D_5	-18.996586202	9.513538220
C_6	2.042690759	2.046635945	D_6	0.099822493	-0.050045360
C_7	-0.299467405	0.150136042	D_7	6.128060502	6.139905739
C_8	-6.128049073	-6.139905023	D_8	-0.299467181	0.150136035
C_9	0.099516108	-0.050007052	D_9	-2.042690719	-2.046635490
C_{10}	-0.000209643	-0.000059166	D_{10}	0.000693944	-0.000346936
C_{11}	-0.002748566	0.001385021	D_{11}	-0.000219963	-0.000054977
C_{12}	0.000322634	0.000074974	D_{12}	-0.004116556	0.002076887
C_{13}	0.002734100	-0.001383160	D_{13}	0.000222568	0.000057755
C_{14}	0.000018378	0.000004093	D_{14}	0.000687248	-0.000346256
C_{15}	0.000057009	0.000056976	D_{15}	-0.000002142	-0.000002211
C_{16}	0.000086582	-0.000043086	D_{16}	0.000271232	0.000273842
C_{17}	-0.000575862	-0.000574106	D_{17}	-0.000016175	0.000008475
C_{18}	0.000015658	-0.000008396	D_{18}	-0.000581988	-0.000575067
C_{19}	0.000294776	0.000288627	D_{19}	0.000008019	-0.000004225
C_{20}	-0.000011082	0.000005619	D_{20}	0.000057976	0.000057465

4. Conclusions

The elaborated algorithm allows us direct conversion of the plane rectangular coordinates between the two projections, using constant coefficients valid for the entire territory of Romania, with different values for each geographical spindle of 6° longitude, specific to the Gauss – Krüger projection (spindle number 34 and 35).

The differences obtained between the trans-calculus of the coordinates by the constant-coefficient method and the ones resulted by intermediating the geographical coordinates fit to the most part of Romanian territory within the limits of ± 1 – 2 mm, recording maximum values of ± 4 – 5 mm in the proximity of the marginal meridian of 24° longitude (figure 1).

5. References

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