Algorithm for direct and inverse coordinate transformation between ETRS89 CRS and S-42 CRS

Neculai AVRAMIUC, PhD. eng., technical adviser, Geodetic Service from Department of Geodesy and Cartography - National Agency for Cadastre and Land Registration, Romania, neculai.avramiuc@ancpi.ro

Petre Iuliu DRAGOMIR, prof. PhD. eng., Director Department of Geodesy and Cartography - National Agency for Cadastre and Land Registration; head Department of Topography and Cadastre from Faculty of Geodesy – Technical University of Civil Engineering Bucharest, Romania, petre.dragomir@ancpi.ro

Tiberiu RUS, lect. PhD. eng., Head Geodetic Service from Department of Geodesy and Cartography - National Agency for Cadastre and Land Registration; lecturer Department of Geodesy and Photogrammetry, from Faculty of Geodesy – Technical University of Civil Engineering Bucharest, Romania, tiberiu.rus@ancpi.ro.

Abstract: *The strategy for coordinate transformation from European Coordinate Reference System (CRS) ETRS89 to national CRS S-42 (Krasovski 1940 – Stereographic 1970 Map Projection) it is based on a knowledge of the pattern of distortion data (due to large errors in the survey control network) and it consists of two main steps:*

1. *Global datum transformation that is accomplished by a conformal transformation*
2. *Interpolation of residual coordinate corrections from a grid of coordinate shifts*

The grid of coordinate shifts was generated using least squares prediction method for the distortion modelling between ETRS89 and S-42 which ensures a continuous transformation process that does not destroy spatial relationships established on the national local datum.

Keywords: *coordinate transformation, distortion modelling, distortion grid, correction interpolation, local datum*

1. Introduction

In order to provide the compatibility and precise georeferencing of spatial data into the new ETRS89 (European Terrestrial Reference System 1989 - ETRS89) for the pan-european products, according to the INSPIRE (Infrastructure for Spatial Information in the European Community) directive of the Europe Parliament from 14.03.2007, Național Agency for Cadastre and Land Registration (NACLR) will provide an Order of the NACLR General Director for adoption of the ETRS89 Coordinate Reference System (CRS) in Romania. The implementation of the ETRS89 in Romania and the actual tendencies of the GNSS satellite technologies applications for the most of the geodetic works, it is required the implementation of an standard algorithm for spatial data transformation from ETRS89 CRS to national CRS (Stereo 1970 projection) and opposite.

ETRS89 CRS it is defined as an geocentric geodetic datum despite S-42 (ellipsoid Krasovski 1940) CRS with Stereographic 1970 projection, defined as a non-geocentric datum (according to the classical astronomical or astro-geodetic orientation of the geodetic datum).

This situation from Romania, similar with other European or World countries, requires serious problems for spatial data transformation from the old CRS to the new CRS (ETRS89), due to large distortions inside the triangulation networks as effect of the classical datum orientation of the S-42 CRS.

In order to underline the distortions between ETRS89 and S-42 CRS from Romania, there was used an conformal orthogonal transformation (2D Helmert), based on a common set of coordintes from both systems. Table 1 presents the statistics of coordinate differences (distortions).

Table 1. Statistics of coordinate differences for common geodetic points after Helmert 2D transformation (before distortions modelling)

Grid step = 15000 m
No of nodes = 2106

Statistic	East	North
Medium:	0.0000	-0.0000
Standard deviation	0.2648	0.3756
Max.:	0.8466	1.3288
Min.:	-0.8632	-1.1928
Total no. of common points	894	894
No. of points above +/-3*(Std.Dev.):	8	3
% points in +/-3*(Std. Dev.):	99.11	99.66

Statistics situation shows that standard deviation of coordinate differences it is about +/- 0.30 m. The value and the surface disposal can be seen in fig.1 (distortions are presented as vectors)

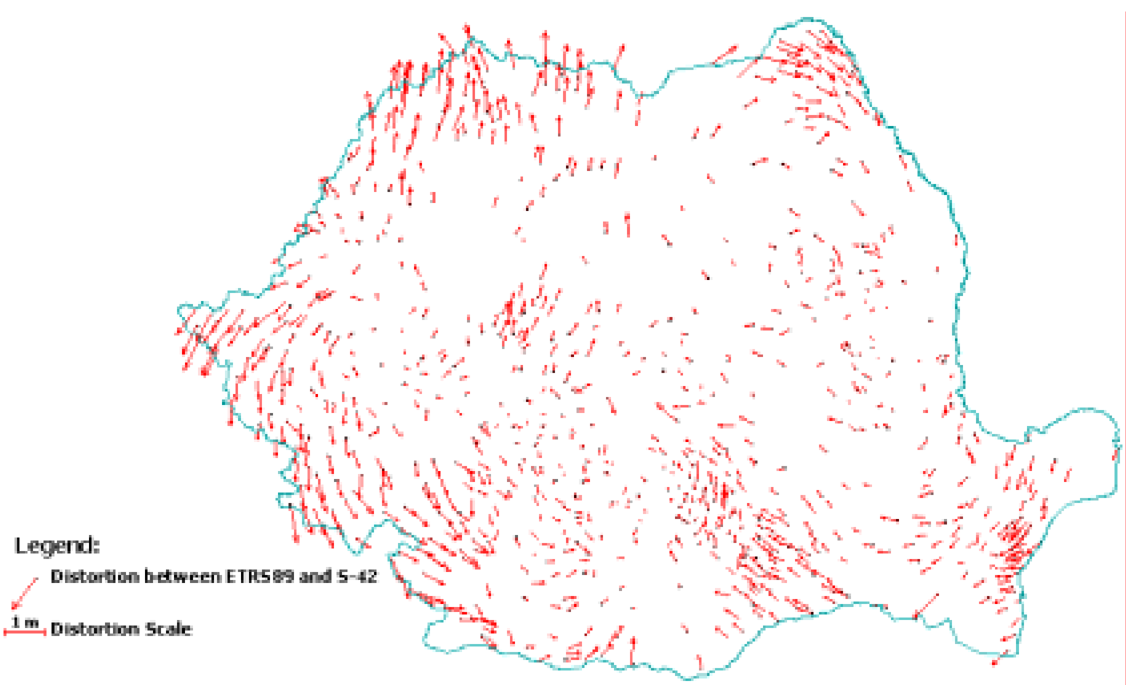

Fig. 1. Distortion situation between ETRS89 and S-42

The big distortions observed in fig.1 should be modelled by a proper technique according to the reality in order to provide a good transformation of spatial data from old datum to the new datum and oposite.

The transformation technique adopted it is similar to the techniques applied in other countries from Europe or abroad and this technique can be implemented also into the GNSS receivers for RTK applications and into the GIS databases for spatial data representation at big scales.

2. General presentation of the transformation algorithm

I. ***Transformation of ellipsoidal coordinates (B, L, h_{el}) from ETRS89 system to plane Stereo 70 coordinates and Black Sea 1975 normal heights (X, Y, H_{MN75}).***

This transformation can be presented as below:

$$(B,L)_{ETRS89} \Rightarrow [1] \Rightarrow (X,Y)_{ObliqueStereographic\_GRS80} \Rightarrow [2] \Rightarrow (X',Y')_{Stereografic1970} \Rightarrow [3] \Rightarrow$$
$$\Rightarrow (Y,X)_{Stereografic1970};$$
$$(h_{el})_{ETRS89} \Rightarrow [4] \Rightarrow (H_{MN75}).$$

where:

[1] Conversion from ETRS89 elipsoidal coordinates (GRS80 ellipsoid) to rectangular coordinates from stereographic (oblique) projection on GRS80 ellipsoid;

[2] 2D Helmert transformation (4 parameters) from rectangular coordinates from stereographic (oblique) projection on GRS80 ellipsoid to rectangular coordinates on Stereo 70 projection;

[3] Interpolation of corrections (distortions) from distortion grid (ETRS89_KRASOVSKI42.GRT file) and apply of them to the previous step coordinates in order to obtain the final plane coordinates (Stereo 70);

[4] Interpolation of quasigeoid anomalies for the Black Sea 1975 datum from the anomalies grid (EGG97_QGR.GRT file) and the substraction of them from ETRS89 ellipsoidal coordinates to obtain the Black Sea 1975 normal heigths.

II. ***Coordinate transformation (X, Y, H_{MN75}) from Stereo 1970 and Black Sea 1975 to ETRS89 ellipsoidal coordinates (B, L, h_{el})***

Similar to the previous transformation algorithm (I), it can be briefly represented the inverse tranformation by the same steps:

$$(Y,X)_{Stereografic1970} \Rightarrow [3'] \Rightarrow (X',Y')_{Stereografic1970} \Rightarrow [2'] \Rightarrow (X,Y)_{ObliqueStereographic\_GRS80} \Rightarrow [1'] \Rightarrow$$
$$\Rightarrow (B,L)_{ETRS89};$$
$$(H_{MN75}) \Rightarrow [4'] \Rightarrow (h_{el})_{ETRS89}.$$

where:

[3’] Interpolation of corrections (distortions) from distortion grid (included into the ETRS89_KRASOVSKI42.GRT file) and obtaining of rectangular coordinates on Stereo 70 projection (without distortions) by substracting of corrections from rectangular coordinates in Stereo 70 projection.

[2'] 2D Helmert transformation with 4 parameters from previous set of coordinates (Stereo 70 without distortions) to rectangular coordinates on stereographic (oblique) projection on GRS80 ellipsoid;

[1'] Conversion from ETRS89 elipsoidal coordinates (GRS80 ellipsoid) to rectangular coordinates from stereographic (oblique) projection on GRS80 ellipsoid;

[4'] Interpolation of quasigeoid anomalies for the Black Sea 1975 datum from the anomalies grid (EGG97_QGR.GRT file) and the adition of them to the normal heights on Black Sea 1975 datum.

3. Direct and invers conversion from ETRS89 (GRS ellipsoid) ellipsoidal coordinates to rectangular oblique stereographic projection (from GRS80 ellipsoid)

Due to the fact that sometime formulas for conversion from Stereo70 projection to the ellipsoid were obtained by Taylor series up to the order necessary for 1-2 cm accuracy, it was necessary to introduce the closed formulas in order to not affect the final accuracy of transformation.

A. ***Direct conversion*** $(B,L)_{ETRS89} \Rightarrow (X,Y)_{ObliqueStereographic\_GRS80}$

There are known:

- Projection pole position $Q_0(\varphi_0 = 46^0, \lambda_0 = 25^0)$ and false Noth and East: x (North) = 500000 m şi y (East) = 500000 m;
- Scale coefficient k_0 = 0,99975 for coordinate conversion from tangent plane to Q_0 pole to secant plane parallel with that;
- GRS80 ellipsoid parameters:
  - Big semi axis *a* = 6378137 m;
  - Flattening *f* = 1:298.257222101;
- Conformal sphere parameters:

$$R = \sqrt{M_0 N_0}$$

$$n = \sqrt{1 + \frac{e^2 \cos^4 \varphi_0}{(1-e^2)}}$$

$$c = \frac{(n + \sin \varphi_0)(1 - \sin \chi_0)}{(n - \sin \varphi_0)(1 + \sin \chi_0)}$$

There are computed:

- Conformal latitude and longitude of point $P(\chi, \Lambda)$ are calculated based on geodetic coordinates of this point $P(\varphi, \lambda)$

- Rectangular coordinates x (N) and y (E) on stereographic oblique plane from GRS80 ellipsoid

$$N = FN + 2 \cdot R \cdot k_0 \left[\sin \chi \cos \chi_0 - \cos \chi \sin \chi_0 \cos(\Lambda - \Lambda_0)\right] / \beta$$

$$E = FE + 2 \cdot R \cdot k_0 \cos \chi \sin(\Lambda - \Lambda_0) / \beta$$

where: FN (false North) = 500000 m, FE (false East) = 500000 m

B. ***Inverse conversion*** $(X,Y)_{ObliqueStereographic\_GRS80} \Rightarrow (B,L)_{ETRS89}$

There are known the same elements as on above paragraph (I).

There are computed:

- Conformal latitude and longitude of point $P(\chi, \Lambda)$ are calculated based on oblique stereographic rectangular (plane) coordinates of this point $P(E,N)$
- Geodetic Longitude λ
- Isometrique latitude ψ
- Geodetic Latitude φ

4. Direct and inverse Helmert transformation (with 4 parameters)

Direct and inverse Helmert transformation it is the first step of combined transformation used in the present algorithm, with a double role: intermediary transformation from one datum to the other and distortion determination for the spatial data, represented with red arrows in fig.2

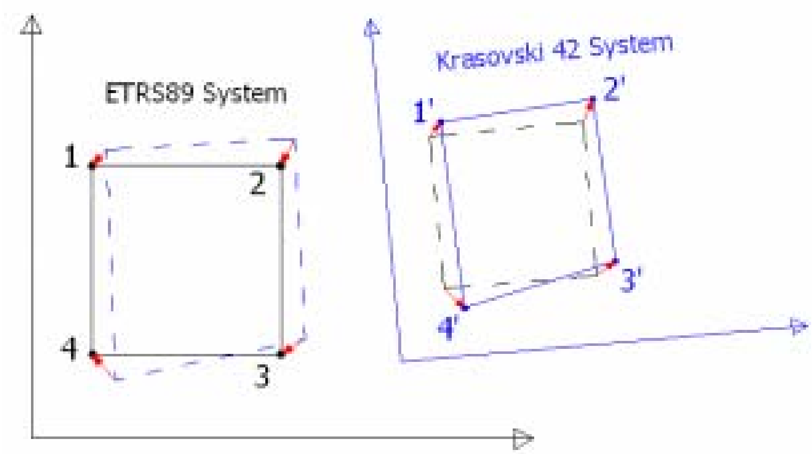

Fig. 2. 2-D Helmert Transformation

A. ***Direct transformation*** $(X,Y)_{ObliqueStereographic\_GRS80} \Rightarrow (X',Y')_{Stereografic1970}$

- There are known the 2D Helmert transformation parameters;
- There are computed the coordinates transformed into national Stereographique 1970 projection as follows::

$$X' = X_0 + X * m * \cos R_Z + Y * m * \sin R_Z$$
$$Y' = Y_0 - X * m * \sin R_Z + Y * m * \cos R_Z$$

where: X=East and Y=North, and X_0=Translation East and Y_0=Translation North, m=scale coefficient and Rz=rotation on (XOY) plane.

*B. **Inverse transformation*** $(X', Y')_{Stereographic1970} \Rightarrow (X, Y)_{ObliqueStereographic\_GRS80}$

Inverse transformation it is similar to the paragraph A.

5. Corrections (distortions) interpolation from distortion grid and rectangular coordinate computation, $(X', Y')_{Stereografic1970} \Rightarrow (X, Y)_{Stereografic1970}$

Correction interpolation from distortion grid it is the second step of the combined transformation used for this algorithm and have the role to correct the coordinates obtained in the previous step. The interpolation of distortion it is done on each component (X and Y). There are presented below the main steps of these computations. More details will be provided for the GNSS or GIS software providers.

We know:

- Distortion grid in a text file with the following header:

```
Minimum East (minE):
 109783.040
Maximum East (maxE):
 904783.040
Minimum North (minN):
 213634.564
Maximum North (maxN):
 783634.564
East grid interval (stepE):
  15000.000
North grid interval (stepN):
  15000.000
Number of grid shift values (rows x columns):
2106
Grid shift values (dEast dNorth) (columns: minE-->maxE; rows: minN-->maxN):
999.000000  999.000000
 999.000000  999.000000
 999.000000  999.000000
…
  -0.218430    1.274732
  -0.203549    0.803709
  -0.204519    0.181353
  -0.218310   -0.104098
  -0.189846   -0.075997
  -0.126835   -0.104002
  -0.065982   -0.263529
  -0.017719   -0.378755
  -0.019843   -0.362699
  -0.074419   -0.270325
  -0.131708   -0.151355
```

-0.099693 -0.053703
0.088060 -0.029743
0.416327 -0.092951
…

Based on this text file can be generated the corespondent binary with direct access to each grid node.

On this algorithm it is ed the spline bicubic interpolation based on smooth interpolation surfaces, accordin to the technique used for computation of distortion grid.

If we have big grid cell including 9 nodes, as in fig. 3:

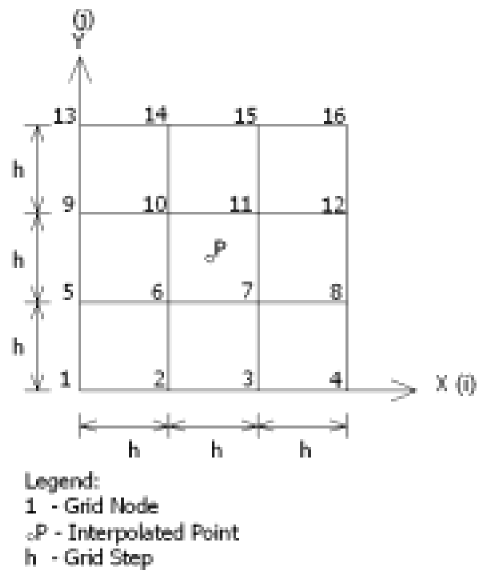

Fig. 3. Grid cell (with 9 nodes) for interpolation of point P

For interpolation it is used the next formula describing a bucubic spline surface:

$$p(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}x^i y^j.$$

It is considered the function $f(x,y) = p(x,y)$ with partial derivatives f_x, f_y and f_{xy} known in the square corners (with h=1) defined by points 6, 7, 11, 10 with coordinates: 6(0,0), 7(1,0), 10(0,1) şi 11(1,1).

Coefficients a_{ij} are computed from the following equations of condition:

1. $f(0,0) = p(0,0) = a_{00}$
2. $f(1,0) = p(1,0) = a_{00} + a_{10} + a_{20} + a_{30}$
3. $f(0,1) = p(0,1) = a_{00} + a_{01} + a_{02} + a_{03}$
4. $f(1,1) = p(1,1) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}$
5. $f_x(0,0) = p_x(0,0) = a_{10}$
6. $f_x(1,0) = p_x(1,0) = a_{10} + 2a_{20} + 3a_{30}$
7. $f_x(0,1) = p_x(0,1) = a_{10} + a_{11} + a_{12} + a_{13}$
8. $f_x(1,1) = p_x(1,1) = \sum_{i=1}^{3}\sum_{j=0}^{3} a_{ij}i$
9. $f_y(0,0) = p_y(0,0) = a_{01}$
10. $f_y(1,0) = p_y(1,0) = a_{01} + a_{11} + a_{21} + a_{31}$
11. $f_y(0,1) = p_y(0,1) = a_{01} + 2a_{02} + 3a_{03}$

12. $f_y(1,1) = p_y(1,1) = \sum_{i=0}^{3}\sum_{j=1}^{3} a_{ij} j$
13. $f_{xy}(0,0) = p_{xy}(0,0) = a_{11}$
14. $f_{xy}(1,0) = p_{xy}(1,0) = a_{11} + 2a_{21} + 3a_{31}$
15. $f_{xy}(0,1) = p_{xy}(0,1) = a_{11} + 2a_{12} + 3a_{13}$
16. $f_{xy}(1,1) = p_{xy}(1,1) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} ij$

where p_x, p_y, p_{xy} are computed as below:

$$p_x(x,y) = \sum_{i=1}^{3}\sum_{j=0}^{3} a_{ij} i x^{i-1} y^j$$
$$p_y(x,y) = \sum_{i=0}^{3}\sum_{j=1}^{3} a_{ij} x^i j y^{j-1}$$
$$p_{xy}(x,y) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} i x^{i-1} j y^{j-1}.$$

Values of derivatives f_x, f_y, f_{xy} are computed for the corners of square 6, 7, 11, 10 based on the neighbour nodes values by use of the finite differences method.

For each square corner it is considered a local coordinate system with origin (i, j) into the mentioned node as in fig. 4:

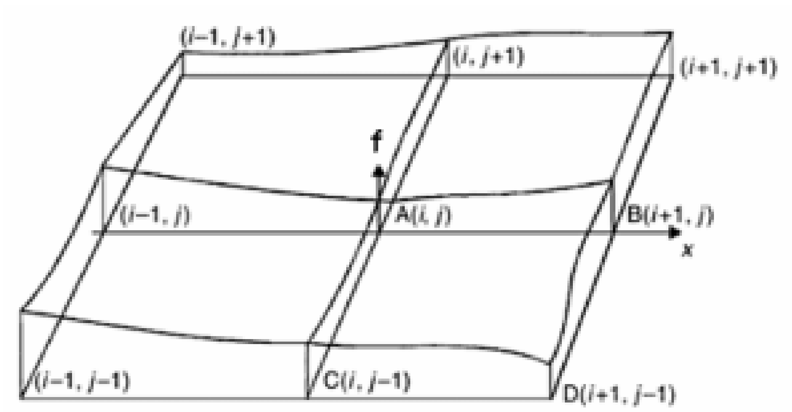

Fig. 4. Local coordinate system (i, j) for derivatives computation

Derivatives of function f are computed with the following formulas:

$$f_x = \frac{-p(i+2,j) + 4*p(i+1,j) - 3*p(i,j)}{2}$$
$$f_y = \frac{-p(i,j+2) + 4*p(i,j+1) - 3*p(i,j)}{2}$$
$$f_{xy} = \frac{p(i-1,j-1) + p(i+2,j+2) - p(i+1,j-1) - p(i-1,j+1)}{4}$$

Ther are computed:

- predicted distortion $p(x, y)$ in a new point P;
- final corrected coordinates with a formula as $X = X' + p(x, y)$

Similar it is interpolated the correction on Y coordinate by use of the same spline bicubic surface, but with other coefficients depending on Y distortions around the point to be interpolated.

6. Conclusions

The existence of common points in a big number and well distributed positions on national surface it is a major requirement for the coordinate transformation from national CRS to the European CRS and oposite. Based on this set of data can be generated the distortion grids and can be predicted the distortions for any interest point in our country. NACLR included in his projects for this year the finalisation of the necessary common set of coordinates by GNSS observations done in triangulation points and of the transformation grid wich will be introduced into the GNSS receivers observing in Romania.

Based on other countries experience in transition from local datums to the new geocetric reference systems (ETRS89, WGS84), we can conclude that the transformation errors and transformation accuracies of points in Romania will be around ±10-15cm, sufficient for the mapping on big scales.

The following table presents the statistic situation of coordinate differences on geodetic common points, available at the present moment, after distorion modelling.

Table 2. Statistics of coordinate differences for common geodetic points after Helmert 2D transformation (after distortions modelling)

Grid step = 11000 m
No of nodes = 3816

Statistic	East	North
Medium:	0.0001	-0.0000
Standard deviation	0.0415	0.0456
Max.:	0.1750	0.1644
Min.:	-0.1729	-0.2022
Total no. Of common points	894	894
No. of points above +/-3*(Std.Dev.):	15	18
% points in +/-3*(Std. Dev.):	98.32	97.99

From this statistic situation analysis it can be deduced that the transformation algorithm adopted it is good and can provide precise and fiducial transformation results for all the users.

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