Sequential Compensation of Geodetic Networks
Least Square Method with Initial Data Obtained from a Previous Adjustment

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Abstract: *The article is concerned with the problem of sequential processing of the geodetic networks, taking in consideration the errors of the initial data. A case study for 4 class structured GPS networks will be presented further ahead.*

Keywords: *adjustment, geodetic, GPS, network, measurement, processing, sequential*

1. Introduction

The classical concept of creating geodetic control networks (triangulation network, trilateration network or leveling network) is the hierarchical one. Control networks are designed (design, measuring and adjusting) based on hierarchical orders from superior to lower level. Even if the design principles have remained the same throughout time, the measurement and adjustment techniques have evolved along with the developing equipment and techniques.

Generally, creating a network design of a certain order on a large territory (a country or even groups of countries) needs long intervals of time during which both the equipment and the measurement and calculation techniques are evolving. Moreover, there are new theories regarding the processing of measurement data appearing. Nevertheless, the emergence of new theories, of new measurement techniques (the electronic measurement of distance) and of high precision devices (gravimeter, automatic precision level, electronic theodolite) didn't determine the renewal of superior level networks, because this implies changing both the numerical (coordinates) and graphical databases (maps and topographic planes), which sums up to a huge material and financial effort. The wide introduction of geodetic satellite positioning technology for national and transnational networks represents an auspicious occasion for implementing the data processing and result interpretation theories developed up to date.

In Romania, the effort of designing a geodetic national network connected to the European network via GPS technology offers us opportunity to bring in actuality a long developed theory, but which hasn't been applied in our country for national network processing up until now, namely: ***processing measurements with errors in the initial data***.

In hierarchical networks we consider as initial data the most frequently determined control point coordinates in a previous adjustment. For example, in the case of adjusting inferior order geodetic networks via the indirect observations method, the initial data is represented by the coordinates of the superior order points.

In the case of network adjustment with initial data obtained from more than one superior network orders (for example the adjustment of the third level network) there are the following processing possibilities:

1. The first order geodetic network is processed independently, and then for second order network adjustment we don't care about errors of the first order points, this representing the classical case of applying the least square method for the second order network adjustment.
2. The first order geodetic network is processed independently and for the second order

network adjustment we apply the ***processing measurements with errors in the initial data*** method, meaning that we take into account about errors of the first order points.

3. Processing the first and second order geodetic networks together.

Generally, free terms and the covariance matrix differ depending on the initial data type used for the third order network adjustment. From here arises the question of what estimations will occur for the coordinates of the third order points with different free terms and from an adjustment with the actual covariance matrix.

When the initial data results from a least square method adjustment, the estimates of the unknown parameters obtained from the ***processing measurements with errors in the initial data*** method***,*** which means from processing with actual covariance matrix of free terms, are identical with the estimation which can be obtain due to adjustment in which initial data are corrected in actual adjustment. In the previous example, this means that if the first and second order networks are adjusted together and for third order network we use the ***processing measurements with errors in the initial data*** method, then the estimated coordinates of the third order points will be identical to those which were determined due to the adjustment of all three orders together.

2. Sequential Compensation of geodetic networks

Next we will present the estimation algorithm of unknown parameters in an adjustment with the actual covariance matrix of free terms and with the condition that those parameters have minimum dispersions. We will analyse only the case of the indirect observation adjustment, without conditions between unknowns, which implies the determination of the estimated values with minimal dispersions $\boldsymbol{x}$, from the $\boldsymbol{l}^o$ observations having the covariance matrix $\boldsymbol{Q}_l$ and initial data $\boldsymbol{y}$ if the observations equations are:

$$\boldsymbol{v} = \boldsymbol{C} \cdot \boldsymbol{x} + \boldsymbol{l} \qquad \text{with weight matrix} \qquad P_c = \mathbf{Q}_l^{-1} \tag{1}$$

with: $$rank(\mathbf{C}) = u \tag{2}$$

where: $$\boldsymbol{l} = \boldsymbol{G} \cdot \boldsymbol{y} - \boldsymbol{l}^0 \tag{3}$$

$$\boldsymbol{y} = \boldsymbol{D} \cdot \boldsymbol{l}_0 \tag{4}$$

This represent the adjustment by indirect observation method without conditions between unknowns with initial data $\boldsymbol{y}$ which are function of initial observations $\boldsymbol{l}_0^0$. We will denote with n_0 the number of initial obsevations $\boldsymbol{l}_0^0$,$\boldsymbol{Q}_0$ represent the variance-covariance matrix of this observations and n_y is the number of initial data. We assume that acctualy observations $\boldsymbol{l}^o$ and initial observations are $\boldsymbol{l}_0^0$ not correlated.

If we determined initial data $\boldsymbol{y}$ with variance-covariance matrix $\boldsymbol{Q}_y$ from a former adjustment ($n_0 > n_y$), as liniar function of $\boldsymbol{l}_0$, we have from equation (4):

$$\boldsymbol{v}_0 = \boldsymbol{S} \cdot \boldsymbol{y} + \boldsymbol{l}_0 \tag{5}$$

The coefficients of $\boldsymbol{D}$ matrix from system of equations (4) are not simply determined, they result from a previous adjustment.

If observations from which results initial data are obtained in more sequences, the $\boldsymbol{S}$ matrix from relation (5), will be in sequence *s*:

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} & \dots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{S}_{s1} & \boldsymbol{S}_{s2} & \dots & \boldsymbol{S}_{ss} \end{bmatrix} \tag{6}$$

Or for previous example, for three control network orders:

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{0} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix} \tag{7}$$

For first and second order we will use following notes:

- The estimate values for unknowns $\boldsymbol{y}_1, \boldsymbol{y}_2$;
- Measured quantities $\boldsymbol{l}_{01}, \boldsymbol{l}_{02}$;
- Variance-covariance matrix:

$$\boldsymbol{Q}_0 = \begin{bmatrix} \boldsymbol{Q}_{01} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_{02} \end{bmatrix} \tag{8}$$

- The weight matrix:

$$\boldsymbol{P}_0 = \begin{bmatrix} \boldsymbol{P}_{01} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_{02} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Q}_{01}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_{02}^{-1} \end{bmatrix} \tag{9}$$

Using this notes the observations equations become:

$$\begin{aligned} \boldsymbol{v}_{01} &= \boldsymbol{S}_{11} \cdot \boldsymbol{y}_1 + \boldsymbol{l}_{01} \\ \boldsymbol{v}_{02} &= \boldsymbol{S}_{21} \cdot \boldsymbol{y}_1 + \boldsymbol{S}_{22} \cdot \boldsymbol{y}_2 + \boldsymbol{l}_{02} \end{aligned} \tag{10}$$

Estimated values for $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ which are initial data for third ordere network adjustment we can have three situations:

1. The first order geodetic network is processed independently, and then for second order network adjustment we don't care about errors of the first order points, this representing the classical case of applying the least square method for the second order network adjustment.

If we note:

$$\begin{aligned} \boldsymbol{N}_{11} &= \boldsymbol{S}_{11}^{\boldsymbol{T}} \cdot \boldsymbol{P}_{01} \cdot \boldsymbol{S}_{11} \\ \boldsymbol{N}_{22} &= \boldsymbol{S}_{22}^{\boldsymbol{T}} \cdot \boldsymbol{P}_{02} \cdot \boldsymbol{S}_{22} \end{aligned} \tag{11}$$

$$\begin{aligned} \boldsymbol{y}_1^{(\boldsymbol{a})} &= (\boldsymbol{S}_{11}^{\boldsymbol{T}} \cdot \boldsymbol{Q}_{01}^{-1} \cdot \boldsymbol{S}_{11})^{-1} \cdot \boldsymbol{S}_{11}^{\boldsymbol{T}} \cdot \boldsymbol{Q}_{01}^{-1} \cdot \boldsymbol{l}_{01} \\ \boldsymbol{y}_2^{(\boldsymbol{a})} &= \left(\boldsymbol{S}_{22}^{\boldsymbol{T}} \boldsymbol{Q}_{02}^{-1} \cdot \boldsymbol{S}_{22}\right)^{-1} \cdot \boldsymbol{S}_{22}^{\boldsymbol{T}} \cdot \boldsymbol{Q}_{02}^{-1} \cdot \left(\boldsymbol{l}_{02} - \boldsymbol{S}_{21} \cdot \boldsymbol{y}_1^{(\boldsymbol{a})}\right) \end{aligned} \tag{12}$$

2. The first order geodetic network is processed independently and for the second order network adjustment we apply the ***processing measurements with errors in the initial data*** method, meaning that we take into account about errors of the first order points (first order points coordinates):

If we note:

$$\begin{aligned} \boldsymbol{N}_{22} &= \boldsymbol{S}_{22}^{\boldsymbol{T}} \cdot {}^{\boldsymbol{S}}\boldsymbol{P}_{21} \cdot \boldsymbol{S}_{22} \\ \boldsymbol{P}_{21} &= \left[\boldsymbol{Q}_{02} + \boldsymbol{S}_{21} \cdot \boldsymbol{N}_{11}^{-1} \cdot \boldsymbol{S}_{21}^{\boldsymbol{T}}\right]^{-1} \end{aligned} \tag{13}$$

$$\begin{aligned} & \boldsymbol{y}_1^{(b)} = \boldsymbol{N}_{11}{}^{-1} \cdot \boldsymbol{S}_{11}^{T} \cdot \boldsymbol{P}_{01} \cdot \boldsymbol{l}_{01} \\ & \boldsymbol{y}^{(b)}{}_2 = \boldsymbol{N}_{22}{}^{-1} \cdot \boldsymbol{S}_{22}^{T} \cdot \boldsymbol{P}_{21} \cdot \left(\boldsymbol{l}_{02} - \boldsymbol{S}_{21} \cdot \boldsymbol{y}_1^{(a)}\right) \end{aligned} \tag{14}$$

3. Processing the first and second order geodetic networks together.

In this case we will obtain the unknown parameters ($\boldsymbol{y}_1^{(c)}$ and $\boldsymbol{y}_2^{(c)}$) by solving the next equations system:

$$\begin{bmatrix} \boldsymbol{N}_{11} + \boldsymbol{N}_{21} & \boldsymbol{N}_{12} \\ \boldsymbol{N}_{12}^{T} & \boldsymbol{N}_{22} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{y}_1^{(c)} \\ \boldsymbol{y}_2^{(c)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_{11}^{T} \cdot \boldsymbol{P}_{01} \cdot \boldsymbol{l}_{01} + \boldsymbol{S}_{21}^{T} \cdot \boldsymbol{P}_{02} \cdot \boldsymbol{l}_{02} \\ \boldsymbol{S}_{22}^{T} \cdot \boldsymbol{P}_{02} \cdot \boldsymbol{l}_{02} \end{bmatrix} \tag{15}$$

Were we noted:

$$\begin{aligned} & \boldsymbol{N}_{12} = \boldsymbol{S}_{21}^{T} \cdot \boldsymbol{P}_{02} \cdot \boldsymbol{S}_{22} \\ & \boldsymbol{N}_{21} = \boldsymbol{S}_{21}^{T} \cdot \boldsymbol{P}_{02} \cdot \boldsymbol{S}_{21} \end{aligned} \tag{16}$$

From the adjustment of the third order network with initial data, in cases 1, 2 or 3, the estimated values of the parametres and covariance matrixes are identical to the results obtained from adjusting the three orders in case 3 alone.

The properties of the estimated values $\underline{x}$, from an adjustment with the known covariance matrix of the free terms, can be determined based on the coeficients of matrix D from system (4) obtained from a previous adjustment. In this case, considering the observations regarding the current order $\boldsymbol{l}^0$ and the initial data $\boldsymbol{l}_0$, (case of $n_0 = r$), system (1) has a unique solution, with the following covariance matrix:

$$\boldsymbol{Q}_l = \boldsymbol{Q}_{l^0} + \boldsymbol{G} \cdot \boldsymbol{Q}_y \cdot \boldsymbol{G}^{T} \tag{17}$$

Thus, the problem reduces itself to adjusting the indirect measurements, corellated with the equations given by relation (1) and covariance matrix (17).

The estimated values $\boldsymbol{x}$ are liniar functions of the observations $\boldsymbol{l}$:

$$\boldsymbol{x} = \boldsymbol{A} \cdot \boldsymbol{l} \tag{18}$$

Where we noted:

$$\begin{aligned} & \boldsymbol{A} = \boldsymbol{N}_C^{-1} \cdot \boldsymbol{C}^{T} \cdot \boldsymbol{P}_C \\ & \boldsymbol{N}_C = \boldsymbol{C}^{T} \cdot \boldsymbol{P}_C \cdot \boldsymbol{C} \\ & \boldsymbol{P}_C = \boldsymbol{Q}_l^{-1} \end{aligned} \tag{19}$$

Establishing the conditions:

$$\begin{aligned} & M(\boldsymbol{x}) = \boldsymbol{x} \\ & D(\boldsymbol{x}_i) = min \qquad (i = 1, 2, \ldots, m) \end{aligned} \tag{20}$$

The following estimated values of the unknowns $\boldsymbol{x}$ are obtained:

$$\boldsymbol{x} = \boldsymbol{N}_C^{-1} \cdot \boldsymbol{C}^{T} \cdot \boldsymbol{P}_C \cdot \boldsymbol{l} \tag{21}$$

And also their variance-covariance matrix:

$$\boldsymbol{Q}_x = \boldsymbol{N}_C^{-1} \tag{22}$$

The same results are obtained if we establish the minimum condition for equations (1):

$$\boldsymbol{v}^{T} \cdot \boldsymbol{P}_C \cdot \boldsymbol{v} = min \tag{23}$$

Which means that we adjust by applying the ***measurements with errors in the initial data*** method with the known variance-covariance matrix of the free terms.

We have to stress the fact that the estimated values $\boldsymbol{x}$ have minimum dispersions regarding observations $\boldsymbol{l}$ and not observations $\boldsymbol{l}^0$ and $\boldsymbol{l}_0$ and we will show that only in some cases they are equal with the estimated values with minimum dispersions regarding the initial observations. The estimated values $\boldsymbol{x}$ and the covariance matrix are obtained as a function of the coeficients of the matrix $\boldsymbol{D}$ from equations (4).

In order to decide when the estimated values $\boldsymbol{x}$ have minimum dispersions regarding the initial observations $\boldsymbol{l}^0$ and $\boldsymbol{l}_0$, we will consider the coeficients of $\boldsymbol{D}$ as being unknown.

Taking into account relations (3) and (4), we can write the equations of observations (1):

$$\boldsymbol{v} = \boldsymbol{C} \cdot \boldsymbol{x} + \boldsymbol{l} - \boldsymbol{G} \cdot \boldsymbol{D} \cdot \boldsymbol{l}_0 \tag{24}$$

In every compensation we obtain $\boldsymbol{x}$, as a liniar function with free terms:

$$\boldsymbol{x} = \boldsymbol{A} \cdot (\boldsymbol{l} - \boldsymbol{G} \cdot \boldsymbol{D} \cdot \boldsymbol{l}_0) \tag{25}$$

or: $$\boldsymbol{x} = \boldsymbol{A} \cdot \boldsymbol{l} - \boldsymbol{A} \cdot \boldsymbol{G} \cdot \boldsymbol{D} \cdot \boldsymbol{l}_0 \tag{26}$$

With the notation: $$\boldsymbol{A} \cdot \boldsymbol{G} \cdot \boldsymbol{D} = \boldsymbol{B} \tag{27}$$

We will have: $$\boldsymbol{x} = \boldsymbol{A} \cdot \boldsymbol{l} - \boldsymbol{B} \cdot \boldsymbol{l}_0 \tag{28}$$

Establishing the condtions:
$$\begin{aligned} &\boldsymbol{B} \cdot \boldsymbol{S} + \boldsymbol{A} \cdot \boldsymbol{G} = \underline{0} \\ &\boldsymbol{A} \cdot \boldsymbol{C} = \boldsymbol{E} \\ &\boldsymbol{D}(\boldsymbol{x}_i) = \min \end{aligned} \tag{29}$$

with the notations:
$$\begin{aligned} &\boldsymbol{P}_0 = \boldsymbol{Q}_0^{-1} \\ &\boldsymbol{N}_{0S} = \boldsymbol{S}^T \cdot \boldsymbol{P}_0 \cdot \boldsymbol{S} \\ &\boldsymbol{P}_S = \left[\boldsymbol{Q}_l + \boldsymbol{G} \cdot \boldsymbol{N}_{0S}^{-1} \cdot \boldsymbol{G}^T\right]^{-1} \\ &\boldsymbol{N}_{CS} = \boldsymbol{C}^T \cdot \boldsymbol{P}_S \cdot \boldsymbol{C} \end{aligned} \tag{30}$$

we obtain for $\boldsymbol{B}$: $$\boldsymbol{B} = \boldsymbol{N}_{CS}^{-1} \cdot \boldsymbol{C}^T \cdot \boldsymbol{P}_S \cdot \boldsymbol{G} \cdot \boldsymbol{N}_{0S}^{-1} \boldsymbol{S}^T \boldsymbol{P}_0 \tag{31}$$

And with the notation: $$\boldsymbol{N}_{SS} = \boldsymbol{S}^T \boldsymbol{P}_S \boldsymbol{S} \tag{32}$$

we obtain for $\boldsymbol{A}$: $$\boldsymbol{A} = \boldsymbol{N}_{SS}^{-1} \cdot \boldsymbol{C}^T \cdot \boldsymbol{P}_C \tag{33}$$

Replacing $\boldsymbol{A}$ and $\boldsymbol{B}$ given by relations (31) and (33) in relation (28), for the estimated values $\boldsymbol{x}$ we obtain:

$$\boldsymbol{x} = \boldsymbol{N}_{CS}^{-1} \cdot \boldsymbol{C}^T \cdot \boldsymbol{P} \cdot \boldsymbol{S} \cdot \left[\boldsymbol{l} - \boldsymbol{G} \boldsymbol{N}_{0S}^{-1} \boldsymbol{S}^T \boldsymbol{P}_0 \boldsymbol{l}_0\right] \tag{34}$$

With the variance-covariance matrix:

$$\boldsymbol{Q}_x = \left\{\boldsymbol{C}^T \left[\boldsymbol{Q}_l + \boldsymbol{G}\left(\boldsymbol{S}^T \cdot \boldsymbol{Q}_0^{-1} \cdot \boldsymbol{S}\right)^{-1} \cdot \boldsymbol{G}^T\right]^{-1} \cdot \boldsymbol{C}\right\}^{-1} \tag{35}$$

or: $$\boldsymbol{Q}_x = \boldsymbol{N}_{CS}^{-1}$$

Therefore, we have an unique solution with minimum dispersions of the estimated values $\boldsymbol{x}$ regarding the initial observations $\boldsymbol{l}_0^0$ and $\boldsymbol{l}^0$.

We verify which are the given conditions for the initial data which derive from the mininum dispersion conditions of values $\boldsymbol{x}$.

Replacing $\boldsymbol{B}$ given by relation (31) in (27) we obtain:

$$\boldsymbol{A} \cdot \boldsymbol{G} \cdot \boldsymbol{D} = \boldsymbol{N}_{CS}^{-1} \cdot \boldsymbol{C}^T \cdot \boldsymbol{P}_S \cdot \boldsymbol{G} \cdot \boldsymbol{N}_{0S}^{-1} \boldsymbol{S}^T \boldsymbol{P}_0 \tag{36}$$

And taking into account relation (33), it results:

$$\boldsymbol{A} \cdot \boldsymbol{G} \cdot \boldsymbol{D} = \boldsymbol{A} \cdot \boldsymbol{G} \cdot \boldsymbol{N}_{0S}^{-1} \cdot \boldsymbol{S}^T \cdot \boldsymbol{P}_0 \tag{37}$$

If in equation (37) we consider matrix $\boldsymbol{D}$ as unknown, by comparing the right member with the left, we observe that:

$$\boldsymbol{D} = \boldsymbol{N}_{0S}^{-1} \cdot \boldsymbol{S}^T \cdot \boldsymbol{P} \tag{38}$$

represents a solution of this matrix equation. Matrix equation (37) cand be written as n_0 liniar systems of u equations with r unknowns. For verifying if equation (37) also has other solutions, we have to examine the rank of matrix $\boldsymbol{A}\cdot\boldsymbol{G}$. Depending on the rank of matrix $\boldsymbol{A}\cdot\boldsymbol{G}$ there are the following possibilities:

$$\text{If: } \operatorname{rank}\left(\boldsymbol{A}\cdot\boldsymbol{G}\right)=\mathrm{r} \tag{39}$$

then the matrix equation (37) has the unique solutiuon given by relation (38).

$$\text{If: } \operatorname{rank}\left(\boldsymbol{A}\cdot\boldsymbol{G}\right)>\mathrm{r} \tag{40}$$

then the matrix equation (37) accepts more solutions; every solution with the exception of the one given by relation (38) cannot have a practical significance, because they depend according to relation (37) primarily on $\boldsymbol{G}$, meaning the mathematical conditions imposed to the initial data and secondly on the desired quantities.

Replacing the coeficients $\boldsymbol{D}$ given by relation (38) in equation (4) we obtain for the initial data the solutions:

$$\boldsymbol{y}=\boldsymbol{N}_{0S}^{-1}\cdot\boldsymbol{S}^{T}\cdot\boldsymbol{P}_{0}\cdot\boldsymbol{l}_{0} \tag{41}$$

which means that (relation (8), inital data $\boldsymbol{y}$ appear as estimated values in the sens of the least squares method and having minimum dispersions regarding observations $\boldsymbol{l}_0$.

For the above given example, of three order triangulation networks, this means that when adjusting the third order network, the initial data (the coordinates of the first and second orders), have to be indetical with those obtained from the joint adjustment of the first and second order networks.

If we replace solutions $\boldsymbol{y}$ given by (41) in relations (3) we will have:

$$\boldsymbol{l}=\boldsymbol{l}^{0}-\boldsymbol{G}\cdot\boldsymbol{N}_{0S}^{-1}\cdot\boldsymbol{S}^{T}\cdot\boldsymbol{P}_{0}\cdot\boldsymbol{l}_{0} \tag{42}$$

and therefore:

$$\boldsymbol{Q}_{l}=\boldsymbol{Q}_{l^{0}}+\boldsymbol{G}\cdot\boldsymbol{N}_{0S}^{-1}\cdot\boldsymbol{G}^{T} \tag{43}$$

and if we establish for unknowns $\boldsymbol{x}$ the minimum dispersion condition regarding observations $\boldsymbol{l}^0$, we will obtain the same estimated values given by relation (34) with the variance-covariance matrix (35), which means that the estimated values have minimum dispersions regarding to $\boldsymbol{l}_0^0$ and $\boldsymbol{l}^0$.

In conclusion, we stress that in the general case of adjusting indirect matrixes, with initial data obtained from a previous adjustment, if in the adjust the ***measurements with errors in the initial data*** method is used, with the variance-covariance matrix of the free terms calculated with relation 17, the estimated values of the unknown parameters calculated with relation 34, will have minimum dispersions regarding the errors of the free terms $\boldsymbol{l}^0$. If the initial data $\boldsymbol{y}$ have minimum dispersions regarding the observations $\boldsymbol{l}_0^0$ from which they were obtained, then the estimated values of the unknown parameters $\boldsymbol{x}$ have minimum dispersions regarding the initial observations $\boldsymbol{l}_0^0$.

According to what was shown previously concerning the sequential adjustment of the geodetic networks of at least three different orders with the current variance-covariance matrix of the free terms $\boldsymbol{Q}_{l^0}$, it results that the estimated values of the unknown, in general, do not have minimum dispersions regarding the initial observations from networks of different orders. With the exception of the first order network, adjustment is made through the indirect measurements method without conditions between the unknowns, resulting estimated values of the unknowns, these becoming themselves initial data in the inferior order network. In general, the estimated values of the unknowns beginning with the second order points will have minimum dispersions only regarding the errors of the free terms obtained from the observations made in the same order.

3. Case study

In recent years in Romania, there have been conducted measurements with the purpose of realising national networks with the help of *GPS* technology, connected to the *ETRS* European network, with the ultimate purpose of bringing our country to the European reference system. The network is created as a precision network, developed hierarchically, on a number of classes and in accordance with international standards. Up until now the National Agency of Cadastre and Real-estate Publicity (ANCPI), has created a network of permanent stations considered a class ***A*** network, in which point *BUCU* is part of network *IGS*, and points *ALBA*, *BAIA*, *COST*, *BACA* and *DEVA* are part of the *EUREF* network. Based upon these points the B class network has been developed, and based upon the class A and B points, the C class network is currently under development, and the particular users are developing the networks that are considered to be part of class ***D***. Because the change to the European reference system has not yet been legalised, even though there are sufficient reference points determined in the ***EUREF-89***, users that determine control points for the different measurements, points considered as class ***D*** points, are forced to transforme the coordinates of these points in the national ***stereographic 1970*** coordinate system.

Because the internal norms have not been made public, based on which the measurements were made and the observations have been obtained, the programs and the algorhytms that these use, we cannot make asumptions regarding to these adjustments and the accuracy of the coordinates that *ANCPI* has in its database and serves its users. We do not know if when determining the accuracy of the points in the inferior class network the ***measurements with errors in the initial data*** method was used.

For giving examples of the differences between the obtained results from the same observations in the different approaches of the adjustment of the hierarchically developed networks, we will present a case study.

We considered a four class sequentially developed network, starting from the permanent station ***bucu***, control point from the IGS network, class ***AA***. We considered points ***ploi***, ***lehl*** and ***tggc*** as class ***A*** points, ***buc1***, ***sokk*** and ***tggb*** as class ***B*** points, ***sysc***, ***bran*** and ***gp2*** as class ***C*** points and ***gp1*** and ***bsl*** from the traverse network as class ***D*** points. For points from classes ***AA***, ***A*** and ***B*** we used recordings made every 30 seconds for a period of 8 hours, for the class ***C*** points we used recordings made every 30 seconds for a period of approximately 5 hours and for the class ***D*** points we used recordings made every 30 seconds for a period of one hour. The lenghts of the bases had values ranging from under 1 km between the class ***D*** points to over 100 km between the class ***A*** points. The observations were calculated with the *Trimble Business Center* programm of the company *Trimble*, in more than one variant. The resulting data and results volume being very large, in the issue we will present for every variant only the coordinates adjusted in the global system on ellipsoid *WGS 84*, the errors of coordinates projected on the North and East direction, the error of the ellipsoidal height and the plane components of the error ellipse.

The results of the measurement processing in the four classes are presented in tables 1-4, and the network sketch in figures 1-3. In every class the coordinates of the points in the immediate upper class were considered without error. In table 5 and in sketch 4 the result of the joint processing of the four classes is presented, where only the coordinates of point ***bucu*** from network ***IGS*** were considered without error.

Table 1. Adjusted Geodetic Coordinates and Errors – Class A

Point	Latitude	Longitude	H (m)	Errors (m)			Error Ellipse		
				H	E	N	a	b	azim
bucu	44°27'50,19198"	26°07'32,65012"	143,100	0	0	0	0	0	0
lehl	44°26'31,99176"	26°51'03,88137"	99,376	0,022	0,005	0,006	0,008	0,006	155°
ploi	44°56'01,54192"	25°59'23,51232"	222,873	0,017	0,004	0,005	0,006	0,004	164°
tggc	45°07'27,44728"	25°44'26,21704"	474,284	0,017	0,003	0,005	0,006	0,004	165°

Table 2. Adjusted Geodetic Coordinates and Errors – Class B

Point	Latitude	Longitude	H (m)	Errors (m)			Error Ellipse		
				H	E	N	a	b	azim
buc1	44°27'50,08341"	26°07'32,99177"	142,848	0,153	0,004	0,004	0,005	0,005	172°
lehl	44°26'31,99176"	26°51'03,88137"	99,376	0	0	0	0	0	0
ploi	44°56'01,54192"	25°59'23,51232"	222,873	0	0	0	0	0	0
sokk	44°27'02,00338"	26°04'32,71719"	129,139	0,170	0,004	0,004	0,005	0,005	173°
tggb	44°26'51,71074"	26°07'29,57565"	131,309	0,160	0,004	0,004	0,005	0,005	173°
tggc	45°07'27,44728"	25°44'26,21704"	474,284	0	0	0	0	0	0

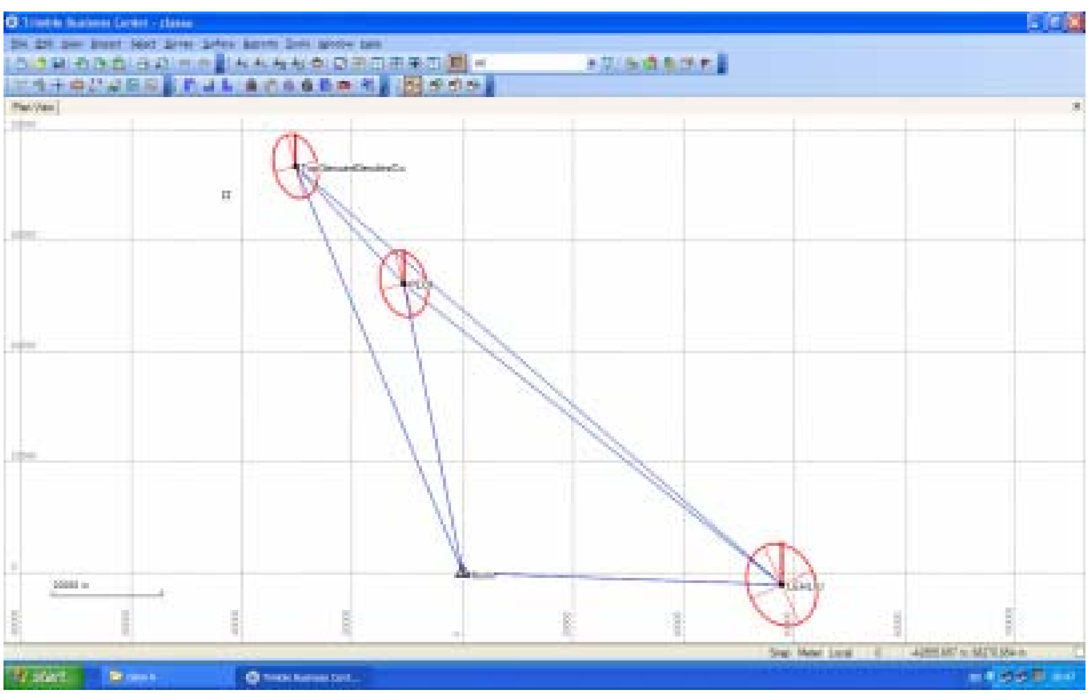

Fig 1. A class network sketch

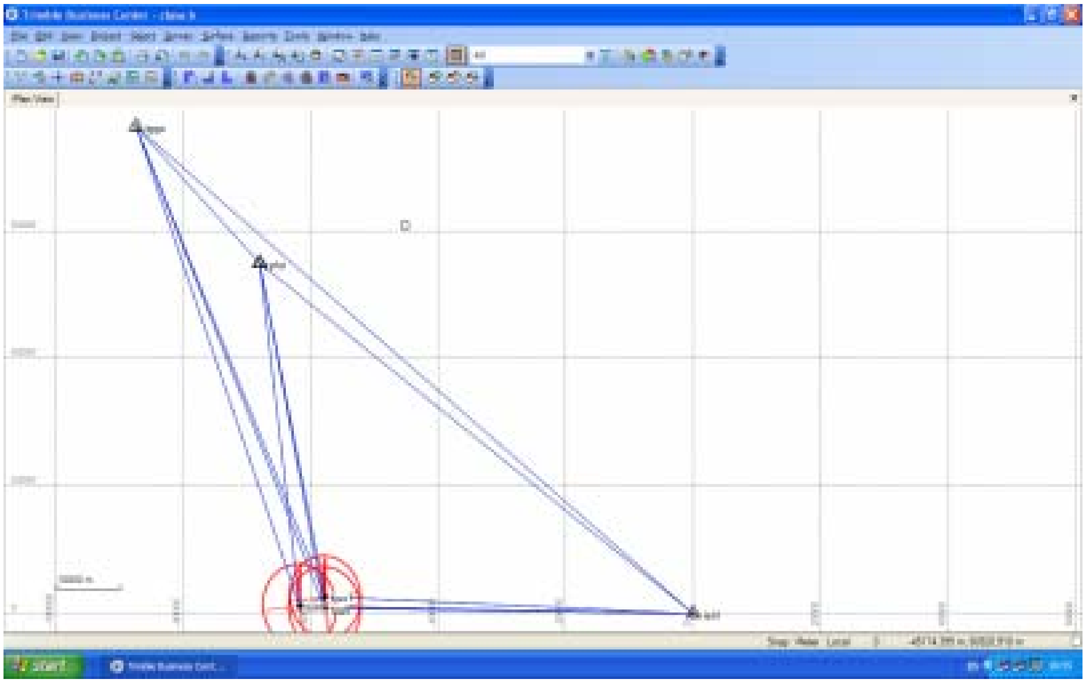

Fig. 2- B class network sketch

Table 3. Adjusted Geodetic Coordinates and Errors – Class C

Point	Latitude	Longitude	H (m)	Errors (m)			Error Ellipse		
				H	E	N	a	b	azim
bran	44°27'12,09613"	26°19'36,25044"	106,606	0,029	0,005	0,005	0,007	0,006	172°
buc1	44°27'50,08341"	26°07'32,99177"	142,848	0	0	0	0	0	0
gp2	44°32'32,43767"	26°23'05,47748"	110,250	0,036	0,006	0,007	0,009	0,008	160°
sokk	44°27'02,00338"	26°04'32,71719"	129,139	0	0	0	0	0	0
sysc	44°25'10,62895"	26°01'44,76345"	155,178	0,011	0,002	0,002	0,003	0,002	156°
tggb	44°26'51,71074"	26°07'29,57565"	131,309	0	0	0	0	0	0

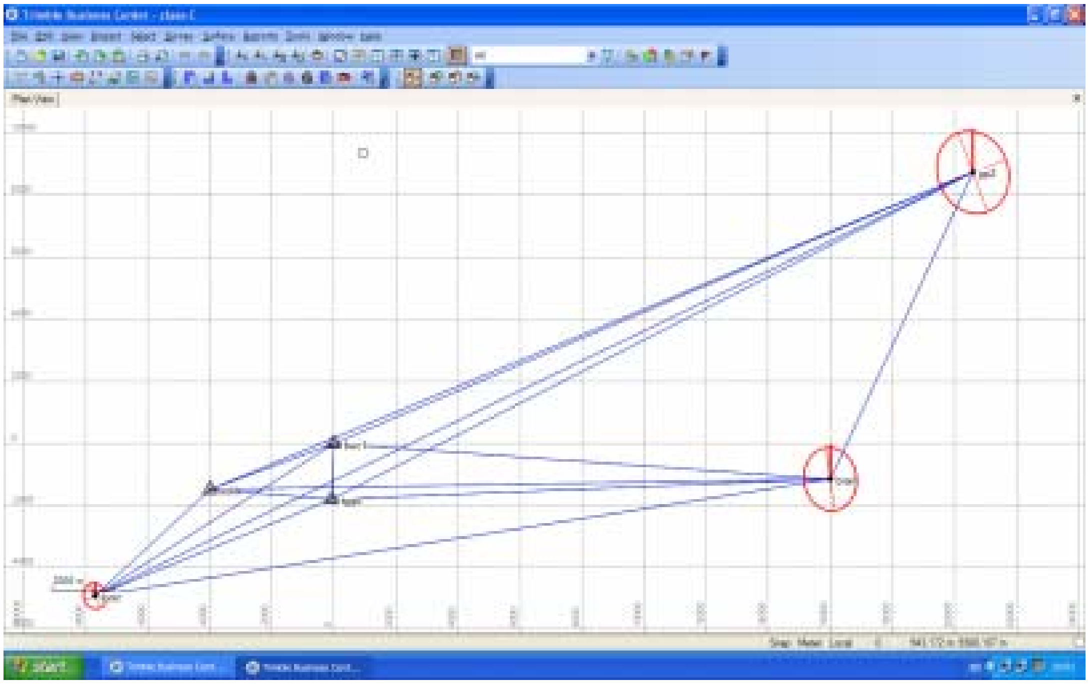

Fig. 3- C class network sketch

Table 4. Adjusted Geodetic Coordinates and Errors – Class D

Point	Latitude	Longitude	H (m)	Errors (m)			Error Ellipse		
				H	E	N	a	b	azim
bran	44°27'12,09613"	26°19'36,25044"	106,606	?	?	?	0	0	0
bsl	44°30'48,48545"	26°19'11,01563"	107,973	0,002	0,001	0,001	0,001	0,001	157°
gp1	44°32'27,29046"	26°23'12,26944"	110,070	0,001	0,000	0,000	0,001	0,000	173°
gp2	44°32'32,43767"	26°23'05,47748"	110,250	0	0	0	0	0	0
sysc	44°25'10,62895"	26°01'44,76345"	155,178	0	0	0	0	0	0
bran	44°27'12,09613"	26°19'36,25044"	106,606	0	0	0	0	0	0

Table 5 Adjusted Geodetic Coordinates and Errors – Classes A, B, C, D

Point	Latitude	Longitude	H (m)	Errors (m)			Error Ellipse		
				H	E	N	a	b	azim
bran	44°27'12,09621"	26°19'36,25114"	106,624	0,008	0,002	0,002	0,003	0,002	164°
bsl	44°30'48,48576"	26°19'11,01634"	107,986	0,008	0,003	0,004	0,005	0,004	159°
buc1	44°27'50,08350"	26°07'32,99177"	142,861	0,002	0,001	0,001	0,001	0,001	165°
bucu	44°27'50,19198"	26°07'32,65012"	143,100	0	0	0	0	0	0
gp1	44°32'27,29086"	26°23'12,27032"	110,086	0,008	0,002	0,003	0,003	0,002	171°
gp2	44°32'32,43807"	26°23'05,47835"	110,266	0,008	0,002	0,002	0,003	0,002	168°
lehl	44°26'31,99177"	26°51'03,88405"	99,389	0,019	0,004	0,005	0,007	0,005	161°
ploi	44°56'01,54371"	25°59'23,51184"	222,853	0,018	0,004	0,005	0,006	0,005	164°
sokk	44°27'02,00342"	26°04'32,71701"	129,156	0,005	0,001	0,001	0,001	0,001	168°
sysc	44°25'10,62888"	26°01'44,76312"	155,192	0,004	0,001	0,001	0,001	0,001	173°
tggb	44°26'51,71079"	26°07'29,57562"	131,323	0,001	0,000	0,001	0,001	0,001	178°
tggc	45°07'27,44950"	25°44'26,21593"	474,262	0,018	0,004	0,005	0,007	0,005	166°

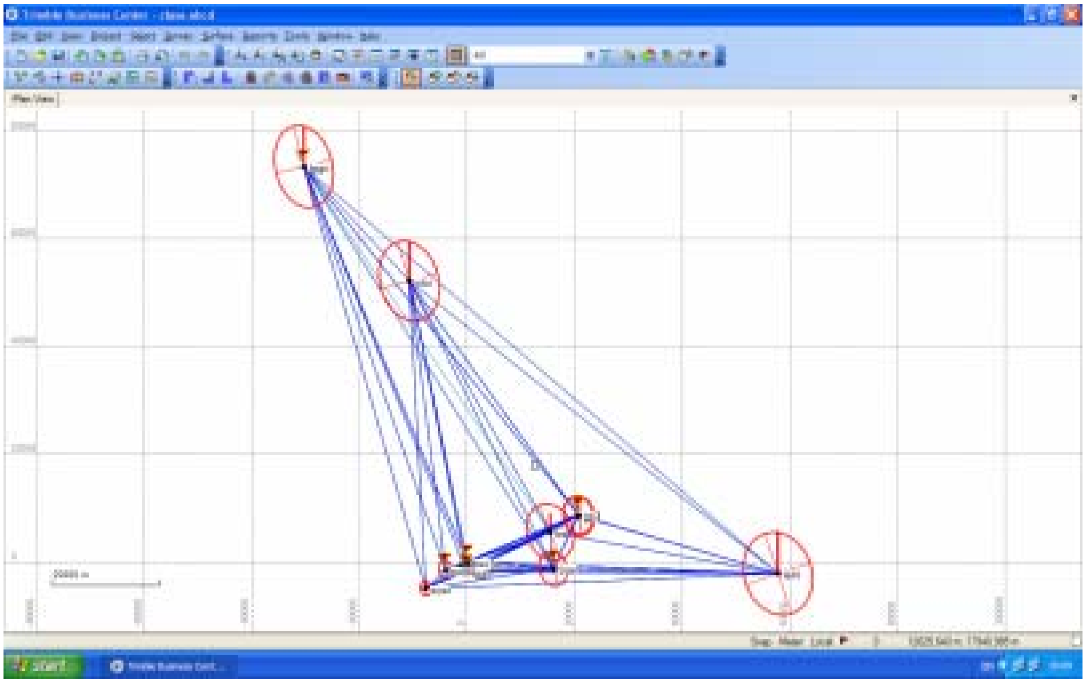

Fig. 4- A, B, C, D classes network sketch

4. Conclusions and proposals

Analyzing the adjustment results it is plain to see the fact that when the errors of the initial data are not taken into consideration, the results of the adjustment of the observations from the inferior classes are different from the results obtained from the joint adjustment of all the observations from all the classes. Furthermore, it is easily observed that the errors of the class ***D*** points are smaller when only the observations in this class are adjusted, than when they are adjusted along with the observations from all of the superior classes. This is explained by the fact the class ***D*** point errors are relative to the class immediately superior. They are due only to the measurements made in class ***D***. It is recommended that the class ***B*** and ***C*** adjustment of the GPS networks in our contry be redeveloped by applying the method described in the article.

5. References

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