Conversion of the plane rectangular coordinates between the cartographical projections Stereo-70 and U.T.M., defined on the international reference ellipsoid W.G.S. - 84

Constantin CHIRILĂ, Lecturer, Ph.D., Engineer, „Gh. Asachi" Technical University of Iaşi, e-mail: tynelro@yahoo.com
Maximilian DIAC, Assistant Lecturer, Ph.D. Student, Engineer, „Gh. Asachi" Technical University of Iaşi, e-mail: maximiliandiac@yahoo.com

Abstract: *Taking into consideration the necessity of approaching a new geodesic datum in România, this implying the change of the reference ellipsoid and the creation of a new supporting geodesic network, it is also to be considered the establishment of the official cartographic projection, which should the best adapt itself to the requirements specific to the cadastre and, in general, to the terrestrial measurements. By virtue of the opinion to maintain Stereographic projection – 1970 as the official projection for România, however, redefined on an international geocentric ellipsoid, there are presented algorithms for rapid conversion of the plane rectangular coordinates by formulas with constant coefficients, in comparison with U.T.M. projection, recommended by the European Commission with the purpose of the cartographic representation of the Romanian territory at great scales.*

Keywords: *rectangular coordinates, cartographical projections Stereo-70 and U.T.M., reference ellipsoid W.G.S. - 84*

1. Introduction

One of the fundamental criteria for adopting a countrywide cartographical projection, in order that our country should be represented in large scales, is represented by the deformations' character, both by the recorded values and their regional distribution.

For this purpose it was carried out a comparative study of the deformations between the two cartographical projections adapted to the present Romania, Stereo – 70 and U.T.M., by making the maps of the relative linear deformations, using the technology offered by ArcGIS software [5].

The comparative analysis of the distribution of the relative linear deformations in the two cartographical projections for three intervals (0...±15 cm/km; ±15...±30 cm/km; ± 30 cm/km...) pointed out a percentage of 60,4 % of the Romanian territory where the deformations of the Stereo – 70 projection are within the admissible limit for the geodesic and cadastre workings (±15 cm/km), while for the U.T.M. projection this percent is only 30,7 % (*table 1*).

Table 1. The comparative analysis of the distribution of the relative linear deformations

Cartographic projections	[0...±15 cm/km]	[±15...±30 cm/km]	[±30 cm/km ...]
STEREO – 70	60,4 %	32,7 %	6,9 %
U.T.M.	30,7 %	30,4 %	38,9 %

Considering the importance of the topographic-cadastral measurements in our country's localities and towns, where the relative linear deformations must not exceed ± 5 cm/km, we extracted their values for 91 localities, both for the Stereo – 70 and U.T.M. projections. By comparing the obtained data, we calculated (for the whole localities) a value of 71,4 % favorable cases for the Stereo – 70 projection in regard to only 28,6 % for the U.T.M. projection. On basis of these values of the relative linear deformations in the two presented cartographic projections we were also able to render evident the statistic repartition percents, on the values' intervals of the deformations, adapted to the urban requirements (table 2).

Table 2. The statistic repartition percents adapted to the urban requirements

Cartographic projections	**[0...± 5 cm/km]**	**[± 5...±15 cm/km]**	**[± 15 cm/km ...]**
STEREO – 70	16,5 %	42,9 %	39,6 %
U.T.M.	9,9 %	19,8 %	70,3 %

The percentage ratio for the first two values' intervals, with an important significance for the urban cadastre and topological-geodesic measurements, is 62,5 % to 37,5 % (interval 0...± 5 cm/km), respectively 68,4% to 31,6 % (interval ±5...±15 cm/km), in favor of the official Stereo – 70 projection.

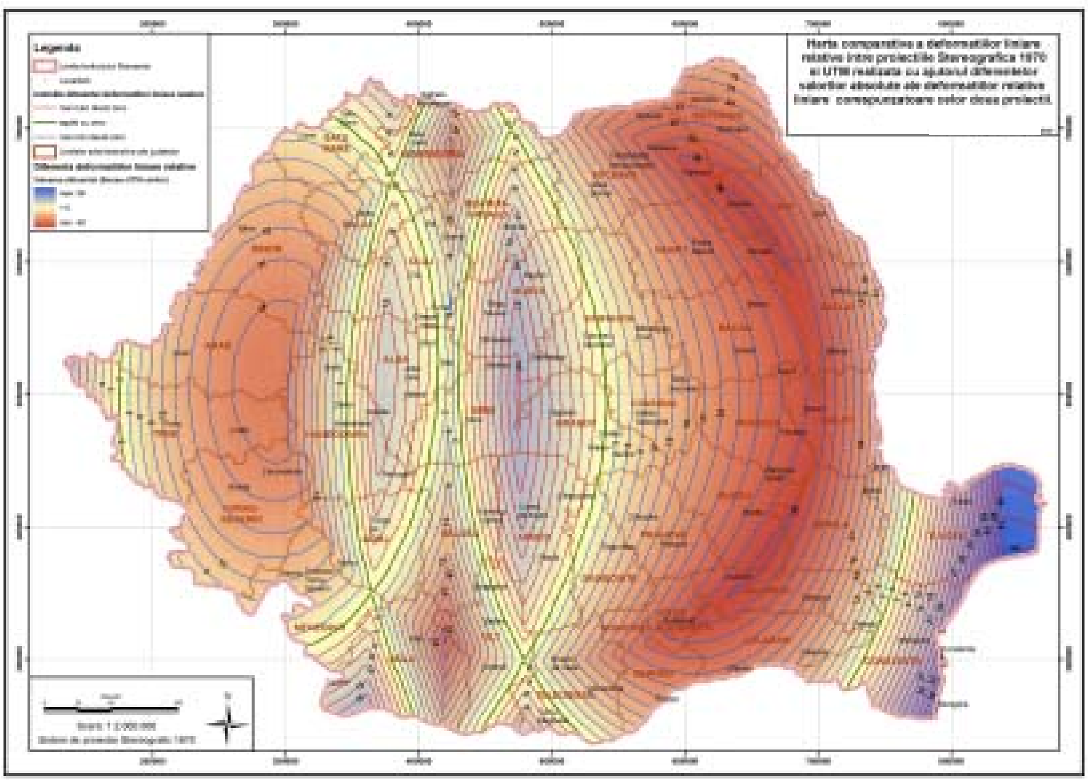

Fig. 1. Comparative map of the relative linear deformations between Stereo - 70 and U.T.M. projections, drawn up on basis of the differences in absolute value

As a result of the complex analysis with the help of G.I.S technology and of the deformations appeared in the Stereo – 70 and U.T.M. cartographic projections, both on the entire territory of Romania and the main localities of our country, we can conclude that the Stereo – 70 projection is superior from the point of view of the deformations' character. Taking also into consideration one of the major disadvantages of the U.T.M. projection, that is representation of our country's territory in different coordinate systems, the idea of the continuity in using the Stereographic – 1970 projection as the official projection for Romania, but redefined on an international geocentric ellipsoid specific to the satellite measurements (WGS – 84 or GRS – 80) should remain in use.

2. Conversion of the plane rectangular coordinates between the cartographical projections Stereo-70 and U.T.M., by means of the constant coefficients' method

On basis of the conversion algorithms elaborated for Gauss and Stereographic – 1970 projections [3; 4], both of them being defined by the Krasovski – 1940 reference ellipsoid, there were also calculated the constant coefficients of direct conversion of the plane rectangular coordinates, without the help of the ellipsoidal geographical coordinates, for the two cartographical projections defined on the international WGS-84 reference ellipsoid.

Conversion of the coordinates between the two systems of cartographical projections is simple owing to the fact that the projections are defined on the same reference ellipsoid. UTM projection is based on the equations of the map into Gauss projection, defined on WGS-84 reference ellipsoid, and Stereo – 70 / WGS-84 projection, can be a solution for the future as we previously showed, once a new ellipsoid is adopted as reference for the satellitary measurements.

- Converting plane rectangular coordinates from the Stereo-70 / WGS-84 into the UTM projection, by means of the constant coefficients' method
- We consider the Stereo – 70 / WGS-84 plane rectangular coordinates (Xsec,Ysec) of a point, which shall be passed into the tangent plane of the projection (Xtg,Ytg), by canceling translation of the axes system of coordinates (X_0 = 500 km; Y_0 = 500 km) and multiplication with the coefficient of returning to scale (C'=1,000 250 063):

$$Xtg = C'(Xsec-X_0)\ ;\ \ Ytg = C'(Ysec-Y_0) \qquad (1)$$

- The working operative terms shall be reduced to smaller values, by multiplying the coordinates in the tangent plane with a 10^{-5} factor, according to the model of the reversed conversion in the Stereo – 70 projection:

$$X = Xtg\ 10^{-5}\ ;\ Y = Ytg\ 10^{-5} \qquad (2)$$

- Gauss / WGS-84 plane rectangular coordinates shall be obtained by applying certain 5-degree polynomial equations, considered as being optimal from the point of view of ensuring an appropriate precision in developing calculus formulas, by 20 constant coefficients:

$$\begin{aligned} X_{Gauss/WGS-84} = {} & A_0 + A_1X + A_2Y + \\ & + A_3X^2 + A_4XY + A_5Y^2 + \\ & + A_6X^3 + A_7X^2Y + A_8XY^2 + A_9Y^3 + \\ & + A_{10}X^4 + A_{11}X^3Y + A_{12}X^2Y^2 + A_{13}XY^3 + A_{14}Y^4 + \\ & + A_{15}X^5 + A_{16}X^4Y + A_{17}X^3Y^2 + A_{18}X^2Y^3 + A_{19}XY^4 + A_{20}Y^5 \end{aligned}$$

$$\begin{aligned} Y_{Gauss/WGS-84} = {} & B_0 + B_1X + B_2Y + \\ & + B_3X^2 + B_4XY + B_5Y^2 + \\ & + B_6X^3 + B_7X^2Y + B_8XY^2 + B_9Y^3 + \\ & + B_{10}X^4 + B_{11}X^3Y + B_{12}X^2Y^2 + B_{13}XY^3 + B_{14}Y^4 + \\ & + B_{15}X^5 + B_{16}X^4Y + B_{17}X^3Y^2 + B_{18}X^2Y^3 + B_{19}XY^4 + B_{20}Y^5 \end{aligned} \qquad (3)$$

Where the constant coefficients A_0, A_1,... şi B_0, B_1,... shall be calculated separately for the spindle 34 and 35 in the Gauss / WGS-84 projection, owing to the distinct coordinates' systems. The calculating principle has as fundament the combination between the reversed conversion in the Stereo–70/WGS-84 projection and the direct conversion in Gauss / WGS-84 projection (spindle no. 34/35).

- ❖ The constant coefficients required for transformation (*table 3*) realize firstly passing the coordinates from the plane of the Stereo-70 / WGS-84 projection to the plane of the Gauss / WGS-84 one; then, they are multiplied with the scale factor (k_o = 0,9996), specific to the UTM projection:

$$N = k_o X_{Gauss/WGS-84};\ E = k_o Y_{Gauss/WGS-84} + Y_0 (1- k_o), \text{ where } Y_0 = 500 \text{ km} \quad (4)$$

Table 3. Conversion Stereo-70 / WGS-84 - Gauss / WGS-84

Notation of coefficient	Spindle no. 34	Spindle no. 35	Notation of coefficient	Spindle no. 34	Spindle no. 35
A_0	5103872.2794	5098031.383	B_0	809844.6172	345074.4564
A_1	99991.570883628	99997.950226236	B_1	-5029.728476949	2511.943259802
A_2	5029.728551310	-2511.943262396	B_2	99991.570907694	99997.950239040
A_3	-3.839048190	-0.959342938	B_3	-37.940246783	19.020700041
A_4	75.880520708	-38.041410912	B_4	-7.678165281	-1.918631365
A_5	3.839142446	0.959319603	B_5	37.940266665	-19.020706824
A_6	-2.046110300	-2.047545359	B_6	0.313566511	-0.155906067
A_7	-0.940612290	0.467711919	B_7	-6.138435165	-6.142632353
A_8	6.138442312	6.142636945	B_8	-0.940705735	0.467718429
A_9	0.313594180	-0.155909332	B_9	2.046148908	2.047546054
A_{10}	0.000162981	0.000045486	B_{10}	0.002311102	-0.001157834
A_{11}	-0.009387844	0.004648853	B_{11}	0.001887233	0.000474435
A_{12}	-0.002785369	-0.000701483	B_{12}	-0.014158883	0.006982459
A_{13}	0.009439363	-0.004655353	B_{13}	-0.001889491	-0.000474864
A_{14}	0.000478559	0.000120254	B_{14}	0.002360195	-0.001163969
A_{15}	0.000059164	0.000059285	B_{15}	-0.000016649	0.000008273
A_{16}	-0.000108944	0.000053628	B_{16}	0.000405208	0.000386424
A_{17}	-0.000869465	-0.000787626	B_{17}	0.000187044	-0.000095796
A_{18}	-0.000176462	0.000090620	B_{18}	-0.000905397	-0.000796359
A_{19}	0.000452289	0.000398146	B_{19}	-0.000093187	0.000048027
A_{20}	0.000019460	-0.000010016	B_{20}	0.000091010	0.000080117

In the expressions of the above coefficients, there were also used the values of the constant coefficients calculated for the direct conversion into Gauss / WGS-84 projection and the reverse conversion into Stereo – 70 / WGS-84 projection [2].

- ➢ Converting plane rectangular coordinates from the UTM into the Stereo-70 / WGS-84 projection, by means of the constant coefficients' method

- ❖ There are to be considered the plane rectangular coordinates UTM (N, E) which, after cancelation of the translation of the axes system's origin (0,000 m; 500000,000 m) will be transcalculated into Gauss / WGS-84 projection by means of the scale coefficient (k_o):

$$X_{Gauss/WGS-84} = N / k_o;\ Y_{Gauss/WGS-84} = [E - Y_0 (1 - k_o)] / k_o \quad (5)$$

- ❖ Coordinates' conversion from the Gauss / WGS-84 projection into the Stereo-70 / WGS-84 projection will be made on basis of the constant coefficients, calculated according to the same algorithm elaborated for the Krasovski ellipsoid [3;4], considering that in the Gauss / WGS-84 projection the coordinates of the central point of Romanian territory corresponding to the time zone of 6° longitude have the following values:

$$X_0 = 5096085{,}926 \text{ m}; \; Y_0 = 500000{,}000 \text{ m}. \quad (6)$$

- The Gauss / WGS-84 plane rectangular coordinates (X,Y) of a point, shall reduce themselves of the coordinates of the central point of the Romanian territory corresponding to the spindle of 6° longitude and shall be expressed by operative terms (F,L), according to the model of the reversed conversion in the Gauss / WGS-84 projection system:

$$F = 10^{-5}\,\Delta X = 10^{-5}\,(X_{Gauss/WGS\text{-}84} - X_o);\; L = 10^{-5}\,\Delta Y = 10^{-5}\,(Y_{Gauss/WGS\text{-}84} - Y_o) \quad (7)$$

- The Stereo-70 / WGS-84 plane rectangular coordinates in the projection's tangent plane shall be obtained by applying some 5-degree polynomial equations, established as an optimum of the development of the constant-coefficient formulas, in order to obtain an appropriate accurateness:

$$\begin{aligned} X_{tg} = {} & C_0 + C_1F + C_2L + \\ & + C_3F^2 + C_4FL + C_5L^2 + \\ & + C_6F^3 + C_7F^2L + C_8FL^2 + C_9L^3 + \\ & + C_{10}F^4 + C_{11}F^3L + C_{12}F^2L^2 + C_{13}FL^3 + C_{14}L^4 + \\ & + C_{15}F^5 + C_{16}F^4L + C_{17}F^3L^2 + C_{18}F^2L^3 + C_{19}FL^4 + C_{20}L^5 \end{aligned}$$

$$\begin{aligned} Y_{tg} = {} & D_0 + D_1F + D_2L + \\ & + D_3F^2 + D_4FL + D_5L^2 + \\ & + D_6F^3 + D_7F^2L + D_8FL^2 + D_9L^3 + \\ & + D_{10}F^4 + D_{11}F^3L + D_{12}F^2L^2 + D_{13}FL^3 + D_{14}L^4 + \\ & + D_{15}F^5 + D_{16}F^4L + D_{17}F^3L^2 + D_{18}F^2L^3 + D_{19}FL^4 + D_{20}L^5 \end{aligned} \quad (8)$$

Where the constant coefficients $C_0, C_1, \ldots$ şi $D_0, D_1, \ldots$ shall be calculated separately for the spindle number 34 and 35 in the Gauss / WGS-84 projection, owing to the distinct coordinates systems, using the same determination principle as in the Stereo-70 → Gauss conversion.

The resulted constant coefficients (*table 4*) are based on the coefficients calculated for the reverse conversion in the Gauss / WGS-84 projection and for the direct conversion in the Stereo-70 / WGS-84 projection.

Table 4. Conversion Gauss / WGS-84 - Stereo-70 / WGS-84

Notation of coefficient	Spindle no. 34	Spindle no. 35	Notation of coefficient	Spindle no. 34	Spindle no. 35
C_0	7781.7639	1945.1702	D_0	-309783.8278	154917.9321
C_1	99932.802745810	99983.216874241	D_1	5023.802851611	-2511.2026324507
C_2	-5023.803734602	2511.202660307	D_2	99932.802756989	99983.2168854255
C_3	-1.446466820	-0.361618814	D_3	18.996919311	-9.5137016275
C_4	-37.993838478	19.027403248	D_4	-2.892933823	-0.7232378104
C_5	1.447179066	0.361663585	D_5	-18.996912081	9.5137013972
C_6	2.042759836	2.046705184	D_6	0.099825488	-0.0500468614
C_7	-0.299476391	0.150140548	D_7	6.128267732	6.1401134569
C_8	-6.128256303	-6.140112741	D_8	-0.299476167	0.1501405406
C_9	0.099519093	-0.050008553	D_9	-2.042759796	-2.0467047294
C_{10}	-0.000209655	-0.000059171	D_{10}	0.000693967	-0.0003469476
C_{11}	-0.002748657	0.001385068	D_{11}	-0.000219977	-0.0000549802

C_{12}	0.000322653	0.000074976	D_{12}	-0.004116693	0.0020769563
C_{13}	0.002734190	-0.001383207	D_{13}	0.000222584	0.0000577598
C_{14}	0.000018377	0.000004093	D_{14}	0.000687271	-0.0003462672
C_{15}	0.000057014	0.000056981	D_{15}	0.000000046	-0.0000000230
C_{16}	0.000086588	-0.000043089	D_{16}	0.000271253	0.0002738642
C_{17}	-0.000575911	-0.000574154	D_{17}	-0.000016173	0.0000084743
C_{18}	0.000015656	-0.000008395	D_{18}	-0.000582037	-0.000575116
C_{19}	0.000294801	0.000288651	D_{19}	0.000008018	-0.0000042241
C_{20}	-0.000011082	0.000005619	D_{20}	0.000057981	0.0000574699

❖ Having obtained the plane rectangular coordinates of the Stereo-70 / WGS-84 in the projection's tangent plane (X_{tg},Y_{tg}), and then having them multiplied to the scale coefficient (*C = 0, 999 750*) and applied the translation constant of the coordinates' axis system (*X_0 = 500 000,000 m; Y_0 = 500 000,000 m*), it results finally in the plane rectangular coordinates of the Stereo-70 / WGS-84 in the secant plane of the projection:

$$X_{sec.70} = X_0 + X_{sec} = X_0 + C\,X_{tg};\;\; Y_{sec.70} = Y_0 + Y_{sec} = Y_0 + C\,Y_{tg} \qquad (9)$$

The differencies obtained between the coordinates' conversion by means of the constant coefficients' method and the differences resulted by means of the geographical coordinates are aproximatively equal to the ones obtained if the two cartographical projections are defined on the Krasovski-1940 ellipsoid.

4. Conclusions and proposals

The algorithm presented in here allows a direct transformation of the plane coordinates between the Stereo-70 / WGS-84 and U.T.M. cartographical projections, by using constant coefficients valid for the entire Romanian territory, which are distinct for each 6° longitude geographical spindle, specific to the U.T.M. projection (spindle no. 34 and 35).

Differences obtained between the coordinates' conversion by constant-coeffcient method and the ones obtained by intermediating the geographical coordinates situate (for the most part of the territory of our country) within the limits of ± 1 – 2 mm, registering maximum values of ± 2 – 3 cm in the eastern and western extemities of Romania

Operating with these two cartographic projections raise no problems because the fast conversion of the coordinates by the method of the formulas with constant coefficients, determined for the whole Romanian territory is a rapid and efficient means of transferring the geodesic and cartographic information.

5. References

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