Considerations regarding the realization of a traverse control network necessary to delimitate some real estates

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Abstract: *Taking into account the accuracy imposed by the standards and the limited number of the terrain points known, the present paper approaches the rigorous processing of the polygonal paths. The elements measured (aiming directions and distances) and the topographic coordinates of points will be processed using the method of indirect measuring. According to this method, an observational equation corresponds to each measured element. After solving the set of equations, one will obtain the corrections of the provisional coordinates and their precision indices.*

Keywords: *traverse control network, real estates, rigorous processing*

1. General

In the cadastral survey works, in order to delimitate some real estate bodies, one often elaborate suspended (floating) polygonal paths with sights, sometimes to triangulation points located in the area.

When solving the topographical path, the long aims do not provide any control on the bearings and the floating polygonal path does not provide any control on the coordinates. Still, the long aims will give the possibility to elaborate equations on the aim directions to solve the polygonal path by indirect measurements.

Taking into consideration the polygonal path from figure 1, where the elements known are the coordinates of points P and R and the measured sizes are the aim directions r_i (i=1,2…11) and the distances $d_{P,101}$, $d_{101,102}$, $d_{102,103}$, $d_{103,104}$., in order to obtain the values of the provisional coordinates of points one uses the relations (1):

$$X_i = X_{i-1} + d_{i-1,i} \cos\theta_{i-1,i}$$
$$Y_i = Y_{i-1} + d_{i-1,i} \sin\theta_{i-1,i} \ (i = 101,102...104) \quad (1)$$

The most probable values of the coordinates of the polygonal path points will be calculated using the relations:

$$(x_i) = x_i + \Delta x_i$$
$$(y_i) = y_i + \Delta y_i \quad (2)$$

Where: (x_i), (y_i) – the most probable values of the polygonal path coordinates;

x_i, y_i – the provisional coordinates of the polygonal path;

Δx_i, Δy_i – the probable corrections of the provisional coordinates.

The system of equations elaborated for the measured aim directions and distances, after eliminating the variation of the points of station modulus and after applying the equivalence equation, is presented as follows:

$$a_2\Delta x_{101} + b_2\Delta y_{101} + l_2 = v_2$$

$$\frac{a_2}{\sqrt{2}}\Delta x_{101} + \frac{b_2}{\sqrt{2}}\Delta y_{101} = v_2^{'}$$

$$-a_3\Delta x_{101} - b_3\Delta y_{101} + l_3 = v_3$$

$$-a_4\Delta x_{101} - b_4\Delta y_{101} + a_4\Delta x_{102} + b_4\Delta y_{102} + l_4 = v_4$$

$$-\frac{[a]}{\sqrt{2}}\Delta x_{101} - \frac{[b]}{\sqrt{2}}\Delta y_{101} + \frac{a_4}{\sqrt{2}}\Delta x_{102} + \frac{b_4}{\sqrt{2}}\Delta y_{102} = v_2^{'}$$

$$a_5\Delta x_{101} + b_5\Delta y_{101} - a_5\Delta x_{102} - b_5\Delta y_{102} + l_5 = v_5$$

$$-a_6\Delta x_{102} - b_6\Delta y_{102} + a_6\Delta x_{103} + b_6\Delta y_{103} + l_6 = v_6$$

$$-a_7\Delta x_{102} - b_8\Delta y_{102} + l_8 = v_8$$

$$\frac{a_5}{\sqrt{3}}\Delta x_{101} + \frac{b_5}{\sqrt{3}}\Delta y_{101} - \frac{[a]}{\sqrt{3}}\Delta x_{102} - \frac{[b]}{\sqrt{3}}\Delta y_{102} + \frac{a_6}{\sqrt{3}}\Delta x_{103} + \frac{b_6}{\sqrt{3}}\Delta y_{103} = v_2^{'}$$

$$a_8\Delta x_{102} + b_8\Delta y_{102} - a_8\Delta x_{103} - b_8\Delta y_{103} + l_8 = v_8$$

$$-a_9\Delta x_{103} - b_9\Delta y_{103} + a_9\Delta x_{104} + b_9\Delta y_{104} + l_9 = v_9 \qquad (3)$$

$$\frac{a_8}{\sqrt{2}}\Delta x_{102} + \frac{b_8}{\sqrt{2}}\Delta y_{102} - \frac{[a]}{\sqrt{2}}\Delta x_{103} - \frac{[b]}{\sqrt{2}}\Delta y_{103} + \frac{a_9}{\sqrt{2}}\Delta x_{104} + \frac{b_9}{\sqrt{2}}\Delta y_{104} = v_9^{'}$$

$$a_{10}\Delta x_{103} + b_{10}\Delta y_{103} - a_{10}\Delta x_{104} - b_{10}\Delta y_{104} + l_{10} = v_{10}$$

$$-a_{11}\Delta x_{104} - b_{11}\Delta y_{104} + l_{11} = v_{11}$$

$$\frac{[a]}{\sqrt{2}}\Delta x_{103} + \frac{[b]}{\sqrt{2}}\Delta y_{103} - \frac{[a]}{\sqrt{2}}\Delta x_{104} - \frac{[b]}{\sqrt{2}}\Delta y_{104} = v_{11}^{'}$$

$$a_2^{'}\Delta x_{101} + b_2^{'}\Delta y_{101} + l_2^{'} = v_2^{''}$$

$$-a_4^{'}\Delta x_{101} - b_4^{'}\Delta y_{101} + a_4^{'}\Delta x_{102} + b_4^{'}\Delta y_{102} + l_4^{'} = v_4^{''}$$

$$-a_6^{'}\Delta x_{102} - b_6^{'}\Delta y_{102} + a_6^{'}\Delta x_{103} + b_6^{'}\Delta y_{103} + l_6^{'} = v_6^{''}$$

$$-a_9^{'}\Delta x_{103} - b_9^{'}\Delta y_{103} + a_9^{'}\Delta x_{104} + b_9^{'}\Delta y_{104} + l_9^{'} = v_9^{''}$$

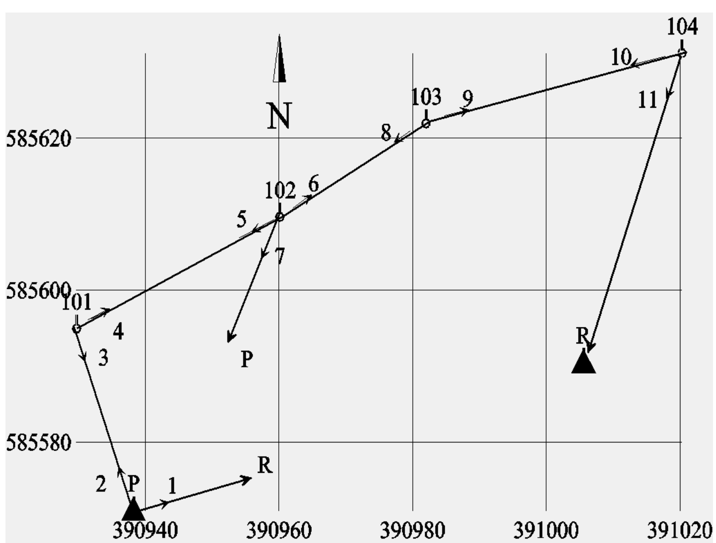

Fig.1

The system obtained is composed of 19 equations with 8 unknowns, where the unknown terms are the corrections Δx and Δy belonging to the points 101, 102, 103, 104.

In the system of equations (3), one noted with a_i, b_i the direction coefficients and with l_i the free terms for the measured directions:

$$a_i = -\rho^{cc} \frac{\sin\theta_i}{d_i}; b_i = \rho^{cc} \frac{\cos\theta_i}{d_i}; \qquad (4)$$
$$l_i = \theta_i^c - \theta_i^m; \quad i = 1, 2...11$$

Respectively, one noted with a'_i, b'_i the direction coefficients and with l'_i the free terms for the measured distances:

$$a_i^{'} = \frac{\Delta x_{ij}}{d_i} = \cos\theta_i; b_i^{'} = \frac{\Delta y_{ij}}{d_i} = \sin\theta_i; \qquad (5)$$
$$l_i^{'} = d_i^c - d_i^m; \quad i = 1, 2...11$$

When processing in block the measurements of the directions and distances, the coefficients and the free terms of the written equations for distances must be calculated in similar units of measure as the directions ones, so the relation (5) becomes:

$$a_i^{'} = \rho^{cc} \frac{\cos\theta_i}{d_i}; b_i^{'} = \rho^{cc} \frac{\sin\theta_i}{d_i}; \qquad (6)$$
$$l_i^{'} = \rho^{cc} \frac{d_i^c - d_i^m}{d_i}; \quad i = 1, 2...11$$

The equations of the system (3) have different weights, as follows:

- the directions equations: $p_{dir} = \frac{1}{m_{dir}^2} = \pm 1$,

- the distances equations: $p_{dis} = \frac{1}{m_{dis}^2}$

$$\text{or} \quad \frac{p_{dis}}{p_{dir}} = p_{dis} = \frac{m_{dir}^2}{m_{dis}^2} \qquad (7)$$

The system of equations for the corrections (3) written as a matrix is presented as follows:

$$Ax - l = v \qquad (8)$$

where:

- A – the matrix of the coefficients

$$A=\begin{pmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & f_1 & g_1 & h_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 & f_2 & g_2 & h_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{19} & b_{19} & c_{19} & d_{19} & e_{19} & f_{19} & g_{19} & h_{19} \end{pmatrix} \quad (9)$$

- x, l, v, p- the matrix of the unknown terms, free terms, corrections and weights

$$x=\begin{pmatrix} \Delta x_{101} \\ \Delta y_{101} \\ \dots \\ \Delta x_{104} \\ \Delta y_{104} \end{pmatrix}; \quad l=-\begin{pmatrix} l_1 \\ l_2 \\ l_3 \\ \dots \\ l_{19} \end{pmatrix}; \quad v=\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \dots \\ v_{19} \end{pmatrix}; \quad p=\begin{pmatrix} p_1 & & & & \\ & p_2 & & & \\ & & p_3 & & \\ & & & \dots & \\ & & & & p_{19} \end{pmatrix} \quad (10)$$

The values of the unknown terms are calculated using the relation:

$$X=(A^* pA)^{-1} A^* pl \quad (11)$$

The indicators of accuracy are calculated using the following relations:

$$m_o=\pm\sqrt{\frac{v^* pv}{n-k}}; \quad m_{X_i}=\pm m_o\sqrt{Q_{ii}} \quad (12)$$

where:

m_0 – the error of the weight unit;
v – the apparent error of measurements;
p – the weight of the observations;
n – the number of the equations;
k – the number of the unknown terms;
m_{X} – the mean square error of the unknown terms;
Q_{ii} - the coefficients of weight.
The coefficients of weight are located on the main diagonal line of the matrix

$$Q_{xx}=(A^* pA)^{-1}.$$

2. Case study:

On the line presented in figure 1, afferent to a polygonal path supported on the coordinates of two points, with long aims, one considered the point 104 as floating point (without any control on orientations and coordinates).

The numerical values, obtained after processing the data resulted from measurements, using the relations (9), (10) and (11), - the matrix of the coefficients (A), the matrix of the weights (P) and the matrix of the unknown terms (X), are presented below.

On the basis of a rigorous processing, one intended to enhance the accuracy of the floating polygonal path coordinates, with long aims to the known points, comparatively to the method of the floating polygonal path (the classic method).

To that effect, in the case study, one observed that the values of the coordinates obtained through the rigorous method, are closer to the values of the coordinates obtained through a supported polygonal path, which indicates higher results accuracy. The small variation of the coordinates is due to the short distances between points, fact that impose a review of the method for long distances. The comparative values of the processing are presented in table 1.

$$
A = \begin{pmatrix}
-87,26 & 25,27 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 \\
-61,70 & 17,87 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 \\
83,72 & 235,51 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 \\
169,88 & -81,76 & -169,88 & 81,76 & 0,00 & 0,00 & 0,00 & 0,00 \\
179,33 & 108,72 & -120,12 & 57,81 & 0,00 & 0,00 & 0,00 & 0,00 \\
169,88 & -81,76 & -169,88 & 81,76 & 0,00 & 0,00 & 0,00 & 0,00 \\
0,00 & 0,00 & 220,84 & -124,14 & -220,84 & 124,14 & 0,00 & 0,00 \\
0,00 & 0,00 & -70,69 & 124,75 & 0,00 & 0,00 & 0,00 & 0,00 \\
98,08 & -47,20 & -11,39 & 47,55 & -127,50 & 71,67 & 0,00 & 0,00 \\
0,00 & 0,00 & 220,84 & -124,14 & -220,84 & 124,14 & 0,00 & 0,00 \\
0,00 & 0,00 & 0,00 & 0,00 & 156,90 & -38,10 & -156,90 & 38,10 \\
0,00 & 0,00 & 156,16 & -87,78 & -45,21 & 60,84 & -110,95 & 26,94 \\
0,00 & 0,00 & 0,00 & 0,00 & 156,90 & -38,10 & -156,90 & 38,10 \\
0,00 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 & -50,11 & 137,97 \\
0,00 & 0,00 & 0,00 & 0,00 & 110,95 & -26,94 & -146,38 & 124,50 \\
235,51 & -83,72 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 & 0,00 \\
-81,76 & -169,88 & 81,76 & 169,88 & 0,00 & 0,00 & 0,00 & 0,00 \\
0,00 & 0,00 & -124,14 & -220,84 & 124,14 & 220,84 & 0,00 & 0,00 \\
0,00 & 0,00 & 0,00 & 0,00 & -38,10 & -156,90 & 38,10 & 156,90
\end{pmatrix}
$$

$$P = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2{,}25 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2{,}25 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2{,}25 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2{,}25
\end{pmatrix}$$

$$X = A*pA)-1A*pl = \begin{pmatrix} 0{,}00009 \\ 0{,}00038 \\ -0{,}00152 \\ 0{,}00115 \\ -0{,}00258 \\ 0{,}00175 \\ -0{,}00508 \\ 0{,}00236 \end{pmatrix}$$

The indicators of accuracy, according the relation (12), have the following values:

$$m_O = \pm 5{,}2 \qquad \begin{array}{l} m_{X_{101}} = 0{,}02, \quad m_{Y_{101}} = 0{,}04 \\ m_{X_{102}} = 0{,}04, \quad m_{Y_{102}} = 0{,}06 \\ m_{X_{103}} = 0{,}08, \quad m_{Y_{103}} = 0{,}07 \\ m_{X4} = 0{,}17, \quad m_{Y_{104}} = 0{,}09 \end{array}$$

Table 1

The comparative values of the horizontal absolute coordinates (X, Y) of points from the polygonal path

Point	Rigorous resolution		Classical resolution		Supported polygonal path		Difference between rigorous and classical resolution		Difference between rigorous resolution and supported polygonal path	
	X	Y	X	Y	X	Y	ΔX	ΔY	ΔX	ΔY
101	585594,979	390929,719	585594,969	390929,721	585594,969	390929,716	0,010	-0,003	-0,010	-0,003
102	585609,621	390960,147	585609,609	390960,140	585609,607	390960,126	0,012	0,007	-0,014	-0,020
103	585621,933	390982,052	585621,917	390982,036	585621,914	390982,017	0,016	0,016	-0,019	-0,035
104	585631,234	391020,368	585631,219	391020,341	585631,212	391020,313	0,016	0,026	-0,022	-0,054

3. Conclusions

- The rigorous processing of the floating polygonal path provides a better coordinates accuracy, when long aim (control aim) to the points of trigonometrical control network are given.
- The small values of the differences resulted from calculations, for the case study presented, are determined by the small length of the polygonal path sides, therefore imposing a review of the method for grater lengths.
- The more the number of the sighted points of the support network is higher, the better accuracy of results is obtained.
- When processing in block the measurements, the free terms of the distance equations are equal to zero, in the case when the provisional coordinates are the ones obtained in the calculus of the floating polygonal path (classical resolution).
- The matrix resolution of the systems of equations is comfortably solved using a spreadsheet tool.

4. References

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