The general presentation of the coordinate reference systems

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Abstract: *This paper presents a few elements that facilitate the understanding of the basic notions on a global level regarding Coordinate Reference Systems.*

Keywords: *coordinates, coordinates systems, reference system, datum, terrestrial reference system, reference coordinate systems, composed reference coordinate system, coordinate operations*

1. Introduction

The idea of this article's publishing started from many elements, the most important of these are the following:

- In a very short time, in Romania, like in other European countries, it will adopted the European Terrestrial Reference System 1989, (**ETRS89**), that will serve to realization of the Spatial National Geodetic Network (**RGNS**) and of the pan-European cartographical products. This fact requires the presentation of a few elements, such as the coordinate systems, the reference systems, coordinates operations that must facilitate the understanding of the basic notions utilized at a global level and a new concept of Coordinate Reference System: CRS.
- Today, there are a multitude of coordinate systems, local or global, used for geo-referencing operations. These systems are necessary for the precise positioning of the spatial points: navigations, GIS, etc. The knowing of these coordinate systems is absolutely necessary for the positioning operation and for the safe navigation.
- The coordinate systems used today are based on a large variety of geodetic datums, units, projections and reference systems.

2. The fundamental coordinate systems

An important issue is the answer of this question: Do we have need of the coordinate systems? The answer is of course affirmative because the position, no matter the space type, is represented by a coordinates set that belongs to one coordinate system and, the positioning operation (static, cinematic, relative or absolute) is one of the most important objectives of the geodesy.

There are, as we know form geometry and trigonometry, more coordinate systems for point spatial representation: two dimensions (2D) and three dimensions (3D). René Descartes (1596-1650) introduced the coordinate systems based on a right angle named the orthogonal coordinate systems or the Cartesian coordinate systems (after the name of the discoverer). In a similar way, there are coordinate systems based also on a right angle but with a reference line, named polar coordinate systems. In the following paragraphs are presented, shortly, the coordinate systems used frequently, function of the space type.

2.1. *Coordinate systems in one dimensional space.*

In geometry, this space is less used, but in geodesy is the exactly the opposite: it is used frequently because it give us the vertical position of a point. That means that in this space, as we expected, the position is represented by a single coordinate. For the positioning operation, we must have an origin and a reference line that represent the space. After dividing the reference line in multiple equal intervals and choosing of a measure unit for a single interval, we can define the position in the one dimensional space.

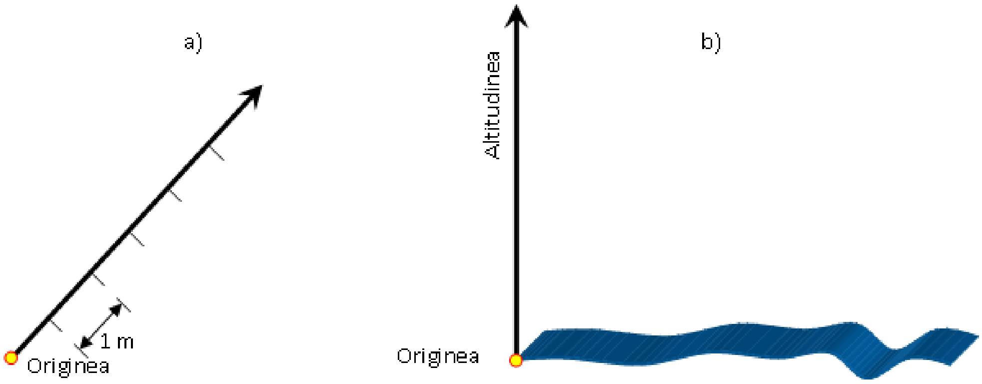

Figure 2: The one dimensional coordinate system 1D
(a-mathematical representation, b-geodetic representation)

2.2. *Two-dimensional coordinate systems.*

2.2.1. *Cartesian coordinate systems.*

In mathematics, the Cartesian coordinate system is used for defining uniquely the position of a point through two numbers that represent the coordinates of the point (one coordinate, noted as a rule with x – the abscissa, and the second coordinate, noted as a rule with y – the ordinate). In plane, a Cartesian system or a rectangular system is defined by two lines, reciprocal perpendicular, the intersection point constitutes the origin of the system and usually is noted with O. Once the coordinate axis are well known, a length unit is chosen. This is marked on each axis so we will obtain a rectangular grid. The unique position of a point in the Cartesian coordinate system is defined by the two coordinates: **x** and **y**. The two axis that intersect each other, separate the two dimensional space in quadrants, as we can see in the Figure 2 – left. In the Figure 2 – right, it is presented an orthogonal coordinates plane used in geodesy.

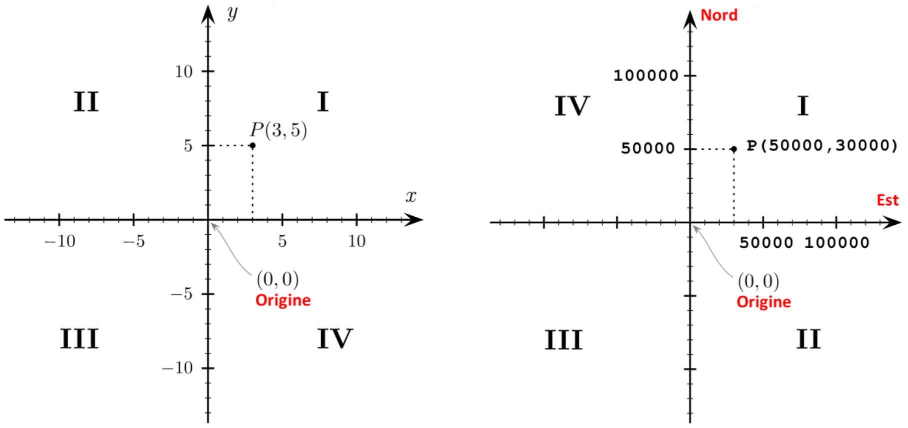

Figure 3: Rectangular coordinate system in 2D space (a-mathematical, b-geodetic)

It must be specified the fact that the Cartesian plane coordinates systems are used in geodesy as local coordinate systems and never as global (that refer to the entire terrestrial surface).

2.2.2. Polar coordinate systems.

In mathematics, the polar coordinate system (Figure 3) is a bi-dimensional system: to each point from a plane are allocated both an angle and a distance. The polar coordinate system is useful especially in situations where the two points relationship is easier expressed in distances and directions (angles) terms; in the Cartesian or orthogonal system, a such relationship can be found only with the mathematical formulae help.

Due to the fact that the coordinate system is bi-dimensional, each point is determined by two polar coordinates: the radial coordinate and the angular coordinate. The radial coordinate (noted usually with **r**) represent the distance of a point with respect of a central point, named *pole* (similar to the origin at Cartesian systems). The angular coordinate (known as the polar angle, or orientation, and noted with θ) represent the angle, measured trigonometrically (or anti clockwise in geodesy). It is also measured from the reference axis (the zero direction) - the polar axis (in geodesy, usually this axis is chosen to be the North axis).

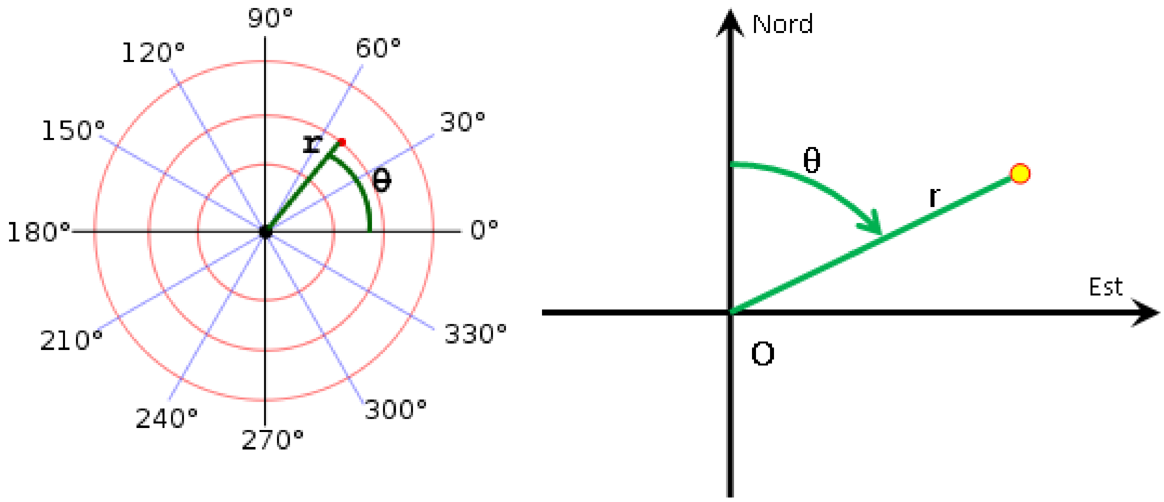

Figure 4: Polar coordinate system in the 2D Space (left-mathematical, right-geodetic)

2.2.3. Geodetic Coordinate Systems (Geographical Coordinate Systems)

The Geodetic Coordinate System (Figure 4) is a curved orthogonal coordinate system in the 2D space. The position of a point located on the ellipsoid considered as reference (the surface that approximates The Earth at one moment) it is given through geodetic coordinates: geodetic latitude and geodetic longitude. The geodetic latitude $\mathcal{B}$ is defined as the angle formed by the equatorial plane with the normal at ellipsoid through the point. The geodetic longitude $\mathcal{L}$ is defined as the dihedral angle formed by origin meridian and the meridian of the point (the both planes contains the rotation axis of the ellipsoid, the angle is measured from the prime meridian (Greenwich meridian) to the meridian of the considered point.

Unlike the Cartesian plane coordinate system that has a local character, the geodetic coordinate system has a global character, every point of the terrestrial surface can be positioned by this system.

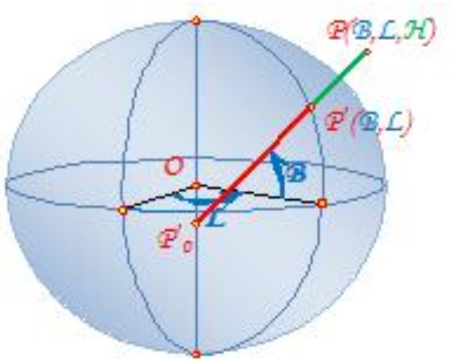

Figure 5: Geodetic coordinate system in 2D space.

2.3. Coordinate systems in 3D space. (in the three-dimensional space)

2.3.1. Cartesian coordinate systems

A Cartesian three-dimensional coordinate system offers the three dimensions of the physical space: length, width and height (Figure 5 – left). The three axes that define the system are perpendicular one on each other. For geodesy (Figure 5 - right) the coordinate system must be positioned (for example the system origin is located in the mass center of the Earth) and the axis must be orientated (for example the OX axis is situated in the Greenwich meridian plane, the OZ axis coincides with the Earth's rotation axis and the OY axis is oriented to the East and also perpendicular to the XOZ plane).

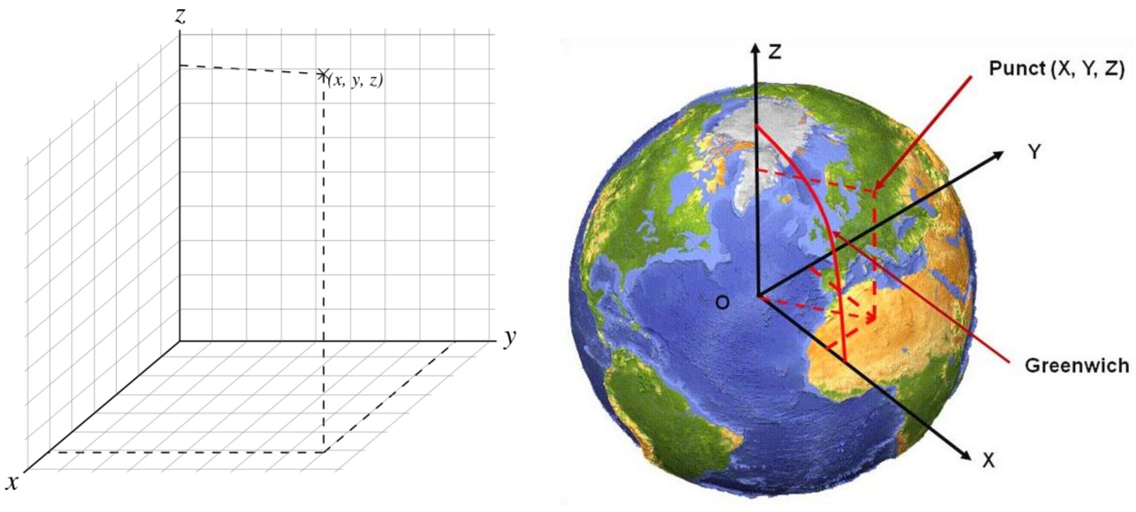

Figure 6: Cartesian coordinate systems in 3D space.

It must be specified that, as a rule, a such three-dimensional system (in geodesy) is a geocentric one (the origin of the system is located in the mass center of the Earth).

2.3.2. Spherical Coordinate System.

The polar coordinates can be extended in the 3D space using for this the coordinates (ρ, φ, θ), where ***ρ*** is the distance from origin, ***φ*** is the angle formed with ***Z*** axis (named also co-latitude) and ***θ*** is the angle measured in rapport with the ***X*** axis (like in the polar coordinates case). The spherical coordinate system is similar to the system given by latitude and longitude, used for the

Earth, with origin situated in the mass center of the Earth (geocentric), the latitude being the complement of the co-latitude.

2.3.3. Compound coordinate systems

If to the geodetic coordinate system from 2D space, we add the ellipsoidal altitude (Figure 4) it is obtained a three-dimensional coordinate system. Another coordinate system in the 3D space can be obtain from a Cartesian coordinate system in 2D space on which it is added a system with one dimension (altitude). These two coordinate systems, each one being obtained by considering of two other systems that come from different spaces (2D and 3D), have a common characteristic: the three coordinates that define the position, don’t have the same origin.

3. Reference ellipsoids.

3.1. The Earth from geometrical standpoint.

During time, there was many opinions about the Earth shape. At first, it was assumed that the Earth is flat, but this idea was historically fast changed due to some obvious phenomena: eclipses, the fact that a ship that displace itself farther from the shore, does not disappear instantly (normal thing in the case of a flat Earth), but gradually: first the hull and later the mast, etc.. The first geometrical body modeled by the Earth was the sphere for many reasons: at first it was hard to realize that the Earth was somehow flat at poles; the sphere is a body with many advantages from geographic and geodetic standpoints: the calculations can be realized in a very simple way, etc. The first man that effectuated measurements for establishing the dimensions of the Earth approximated by a sphere, was Eratosthenes (276-195 i.e.), the manager of the famous library from Alexandria, a man with large knowledge in various domains. This approximation survived long time because the sphere is a perfect geometrical body so the calculations made on it was very easy. The sphere is also used today for the calculation less pretentious from precision standpoint. Once the great expedition organized by French Academy (1735), when many measurements was effectuated in the meridian and parallel arcs in the both hemispheres, it was established that, from the geometric standpoint, the Earth can be better approximated with a rotation ellipsoid derived from a ellipse rotation around its small axis.

Today, for problems resolving that appear in the mathematical geodesy it is used the rotation ellipsoid as a good approximation of the shape and dimensions of the Earth. A such ellipsoid is completely defined through two parameters, under the condition that one of them must be linear. Usually, the two parameters are ***semi major axis*** and the inverse of the flat ***f***.

3.2. The Earth from physical standpoint.

Nowadays, geodesy evolved and the range of its concerns expanded: along with the geometric shape study, it also studies the determination of the gravitational field of the Earth that influences in a significant way the geodetic measurements. The shape determination of the Earth means also a progress from the directly determination of the terrestrial surface to the geoid studies. The terrestrial surface is very irregular (the highest point is about 10 Km. above the sea and the lowest point is at about 11 Km. below the sea) in comparison with the geoid surface (±110 m. in respect with a geocentric revolution ellipsoid).

From the internal structure analysis of the Earth, it can be said that the density of the material components varies very much and that the more we go to the inside of the Earth the determination of the specific geodetic values is realized only on the assumptions basis. The main objective of the physical geodesy is the determination of the level surfaces of the Earth's gravitational field, therefore the potential function determination. Also, it must be acknowledged the basic idea of the physical geodesy that from physical determinations it can be outlined geometric conclusions about the shape and the dimensions of the Earth, principle proclaimed by Claireaut through his theorem about the connection between the geometric flattening and gravimetric flattening with the gravity variations with the latitude.

From physical standpoint, about the Earth, Laplace named the equipotent surfaces as level surfaces. A level surface is an equipotent surface that has the property that in any point, the heaviness force has the same direction with the normal of this surface in the same point, the horizontal components of this force being nil. For Geodesy discipline, among all of these level surfaces, an important one is the zero level surface named geoid and proposed by Gauss as the mathematical figure of the Earth. It can be said about: a reference geoid , that is an equipotent surface that identify itself with the medium sea level on an imaginary rotating Earth, in which all the masses are uniformly distributed (a mathematical construction); real geoid, that is an equipotent surface that identify itself with the medium sea level on a real Earth and that differ from the reference geoid due to the irregularities in the masses distribution.

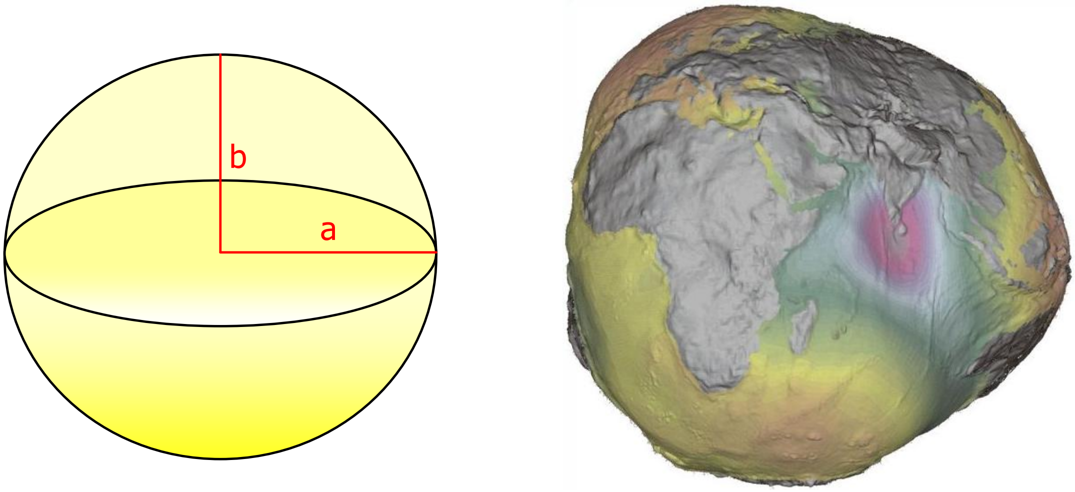

Figure 7: Rotation ellipsoid, the geoid undulations.

3.3. Equipotent ellipsoid.

As it is already known from the gravity potential development, in a first approximation, the Earth can be considered a sphere that rotate itself. In a second approximation, the Earth can be seen as an equipotent revolution ellipsoid. A special revolution ellipsoid is the so called "Normal Earth" that has the same angular speed and the same mass with the real Earth, the (normal) potential on the ellipsoid's surface being equal with the geoid's surface potential and its geometric center is the mass center of the Earth. As "normal Earths" examples, it can be reminded: Geodetic Reference System 1967 (**GRS 67 - G**eodetic **R**eference **S**ystem, 19**67**), 1980 (**GRS 80 - G**eodetic **R**eference **S**ystem, 19**80**, **W**orld **G**eodetic **S**ystem 19**84** (**WGS 84**).

Although the Earth is not exactly an ellipsoid, the equipotent ellipsoid furnishes a simple, consistent and uniform reference system for all the geodesy purposes: a reference surface for geometrical purposes (such as cartographic projections and the satellites navigation) and a gravity's normal field on the Earth surface and in space, as a reference for gravimetry and satellites geodesy. The gravity's field on the ellipsoid is very important from a practical standpoint because it is easy to be used from a mathematical point of view and for the fact that the real field deviations of the gravity from normal field are (theoretical) very smalls. The dividing ofe the gravity's terrestrial field in a normal field and a small disturbing one or a field of anomalies, simplify the resolving of many issues such as the geoid determination.

Although an equipotent ellipsoid has many geometrical and physical parameters, it can be defined completely with only four independent parameters, the others could be determined through these ones. One of the main objectives of a global geodetic system is to replace the local datums, developed for satisfying the requests of the cartography and navigation in different regions of the world.

4. Geodetic Datum

4.1. Generalities

The datum is defined as being any numerical or geometrical quantity (an amount) or a set of such quantities that serves as reference or basis for another quantities (amounts). From the above definition, results that are many activity domains that use one or more of these datums.

The Earth has a very complex shape and its surface is disturbed by high mountains and deep oceans. For cartographic purposes it is needed a reference model that can furnish the recording modality of all the irregularities from the terrestrial surface. A such model must be: simple, to be easy in its application; to include a coordinate system that can permit a unique identification of the element's position that must be represented; to be easy associate with the physical world, so its utilization to be intuitive.

A reference model with the "curved Earth" is known as a geodetic datum, or, in other words, as a datum that defines the size and shape of the Earth and also the origin and the orientation of the coordinate systems utilized for the mapping of the terrestrial surface. The principal characteristics of a Geodetic Datum are the following:

- It is a simplified mathematical representation of the shape and the size of the Earth's dimensions;
- It presents itself as a spheroid shape that is obtained by rotating of an ellipse around its semi minor axis.
- It is vital for all activities that use spatial data; The spheroid furnishes a simple mathematical surface for the measurements and for the navigation calculations on large terrain surfaces and offers a reference surface for cartography and GIS systems.
- The spheroid surface is positioned in a such way that it can approximate optimally the planetary ocean's surface, that is the geoid one. An exact framing it's not possible due to the anomalies in the gravitational field of the Earth. These anomalies, caused by the density and the masses distribution variations in the Earth's interior, produces irregularities of the sea level surfaces. This fact make the sea level to be an inconvenient choice for consider it an horizontal reference surface for mapping activities.
- The sea level is utilized on a large scale as a reference surface for height measurements. On a map, usually, it will be displayed the height above the sea level. Besides that, their positions is given as a spheroid function.

It must be underlined the fact that the datum notion and the TRS - Terrestrial Reference System are synonyms.

4.2. The datum's classification.

There is, like in any other situation, more criteria that lead at diverse datum's classifications. The most usual one, divides the datums in local geodetic datums and geocentric geodetic datums (Figure 7).

The local geodetic datum approximates better only one portion of the sea level surface (geoid). Till recently, the countries that worked with spatial information systems utilized such datums. At these datums, the spheroid center did not coincided with the mass center of the Earth.

The geocentric geodetic datum approximates better the entire terrestrial surface (without areas more or less better approximated). At these datums the geometric spheroid's center coincides with the mass center of the Earth. The Global Positioning Systems uses geocentric datums.

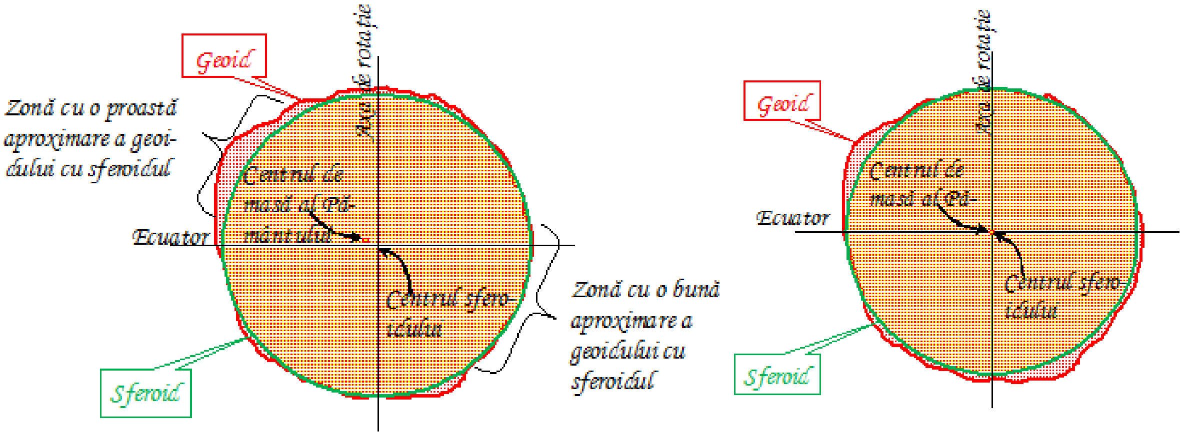

Figure 8: Geodetic Datum and Geocentric Datum.

Another classification of datums can be made in function of the space type. From this standpoint, it can be said about: vertical geodetic datums (used for defining the vertical position of the terrestrial surface's objects); horizontal geodetic datums (absolutely necessary for mapping purposes); complete geodetic datums (these datums must allow the object' position definition in a three-dimensional space.

In conformity with international standards (ISO 19111) the datums are divided in: geodetic datums (this expresses the liaison connections between coordinate system and Earth, most of the time this datum includes also the ellipsoid definition); vertical datum (describes the relation between the Earth's gravity and altitude) and special (technical) datum or local datum (describes the relations between a coordinate system and a local reference).

Another classification of the datums would be the following: static datum (defined by the coordinates of some key stations that are kept fixed); semi-dynamic (defined through liaison relations with a global dynamic reference network at one epoch, the datum definition being frozen at that epoch an not being dependent of time); dynamic (the datum being defined in a continuous way through its relations with a global dynamic reference system such as **ITRS** – **I**nternational **T**errestrial **R**eference **S**ystem).

4.3. The position identification on a geodetic datum.

Two geodetic datums are implicitly associated with a geodetic datum:

A coordinate geodetic datum is the system associated in a natural way to a spheroid. This system allow the describing of a point position on the terrestrial surface through spherical latitude, longitude and altitude (see the coordinate systems).

A three-dimensional Cartesian system can be also associated with a geodetic datum. The position of a point from terrestrial surface it is given by the coordinate triplet: X, Y and Z.

A third coordinate system is the cartographic projection used for representation on a plane surface of the curved spheroid's surface. It must be underlined the fact that no matter the projection used, the spheroid surface representation it is made in a distortional way, the size of the distortions being eligible for calculations.. To every projection we could associate an orthogonal coordinate system where the position (the coordinates) is given in North, East terms.

4.4. The geodetic datum definition.

The process through a geodetic datum is formulated with the aim of a better approximation of the physical surface of the Earth is known as the datum definition.

It is possible that the final datum to be the result of an iteration process, to each iteration corresponding a intermediary definition. Every single intermediary definition is the result of thousand and thousand of measurements. These measurements could include the both classical observations (angular directions and distances) and gravimetric observations. To these, we can add the new techniques that use the artificial satellites of the Earth.

In the past the definitions of the local datums (that are not geocentric)was dependent in a large way by the astronomical observations that furnished the astronomical latitude and longitude for a number of sites (selected by some specific principle)well distributed in the target area. A correspondent set of latitudes and longitudes was processed through angular directions and distances measurements effectuated in the geodetic network, the processing (the adjustment) being realized on the surface of a certain ellipsoid, considering one of the sites as origin. The coordinates obtained are function by the chosen surface and the system's origin.

On each level (iteration) in datum's defining, are obtained coordinate sets that differ due to the irregularities that exist in the gravitational Earth's field (the difference is given by the vertical's deviations). The differences are analyzed for establishing the moment when a revision is necessary, through the utilization of another ellipsoid or of another origin, being possible the obtaining of a smaller values for the vertical’s deviations. When the annalists find the smallest differences in iterations, they establish according with the values obtained, the datum by reporting the values for the latitude, the longitude and the ellipsoidal height of the origin station, together with the parameters that define the ellipsoid.

The defining of a geocentric datum is a more complex process, that needs measurements on entire surface of the Earth. On the basis of these measurements, the coordinates of all the participant sites are generated. The totality of these reference sites composes a network named Terrestrial Reference Frame. This network is used in the reference system realization and to furnish coordinates to the users, and, in this way, it is possible the determination of other points of the terrestrial surface.

The conceptual difference between a datum (terrestrial reference system) and a terrestrial reference frame (TRF) is the following: that the datum is characterized by the absence of errors (it is chosen a set of parameters) and the TRF is the subject of inherent errors (errors of the measurement process, process through the network was realized).

4.5. The multiple datum problems.

A geodetic datum is, as it can be observed, a mathematical concept, that means that it can be defined an infinity of datums that can cover any surface. The ideal situation is, for a country, a region, to define itself a single datum, and in this way all the measurements and specific information can be referred at a single coordinate system. Many times, in practice is necessary to be established, for various motives (military or technical motives), one or two datums (for example: one local datum for the existent maps and one geocentric datum for satellites navigation).

It is important to be understood that the coordinates values of a point are dependent by the datum used. The latitude, longitude and altitude of a point defined by the datum 1 differ by the latitude, longitude and altitude defined by the datum 2 for many reasons:

- The shapes of the ellipsoids differ;
- The centers of the two ellipsoids are positioned differently.

- The axis of the coordinate systems used are not parallel or it is possible the existence of a scale factor.

It is very important for all the people involved in the geodesy domain to know very well the datum or datums they used. The incorrect combining of the coordinates provided from different datums will lead at obtaining of a erroneous values, and these also can lead to obtaining of a catastrophic results.

5. Coordinate reference systems

According to ISO 19111, "The location or position on/or near the terrestrial surface can be described by the coordinates". The coordinates are considered without ambiguities only then the reference coordinate system - to which these coordinates are referred - was defined completely. Each position will be described by a set of coordinates that it will be referred at one reference coordinate system. According to the same standard: "a Coordinate Reference System - CRS is composed by a datum and a coordinate system. A Coordinate Reference System is a class that contains two components: the datum and the coordinate system. Also, the class named datum contains the subclasses: geodetic datum, vertical datum and engineering specific datum.

The horizontal and vertical components used in the describing of the spatial position can be derived from **CRS** (this way is the most often used nowadays). This fact (mentioned above) lead the scientific world to talk about a **C**ompound **C**oordinate **R**eference **S**ystem: **CCRS**. In other words, **CCRS** describes the position of a point by two coordinate reference systems that are independent.

6. Coordinate operations.

The coordinate operations are referred at the changing of the coordinate values from one coordinate reference system to another. The information regarding coordinate operations can be given if the data sets that has coordinates and use different coordinate reference systems are compatible. There are defined the following coordinate operations: the coordinate conversion (the values of the coordinates are changed from one coordinate reference system to another, but both systems has basically the same datum); the coordinate transformation (it is realized the transition from one datum to another, each one having different CRS’s and keeping the coordinate system type); the coordinate concatenation (the coordinates modification from one coordinate reference system to another can be accompanied by a series of operations that consist in one or more coordinate transformations and/or one or more coordinate conversions).

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