STEREOGRAPHIC PROJECTIONS FOR ROMANIA

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Abstract: *Having adopted the geodetic datum ETRS 89 in Romania, the necessity of studying the implications on the official projection used in our country has emerged. Taking into account the shape of the land and the geographical position, we have studied the possibilities of representing the ellipsoid onto a plane through direct projection (which is the case of the actual The Stereographic Projection 1970 used in our country today) or double projection, by mapping it to a sphere and then onto the plane.*

Keywords: *oblique stereographic projection, ellipsoid, sphere, distortion*

1. Introduction

Taking into account the approximately round shape of our country and its geographical position, multiple variants of the stereographic projection were the preferred methods of representation for many years.

Thus we must remind ***the Budapest tangent plane stereographic projection***, used in the provinces Crişana, Banat and Maramureş, which were first under the domination of the Habsburg Empire and then the Austro-Hungarian Empire. The Bessel ellipsoid was used as the reference ellipsoid and was represented on the sphere through a conformal transformation. The pole of this projection is the *Gellerthegy* fundamental geodetic point, bearing the following coordinates on the Bessel ellipsoid:

$$\varphi_o = 47^o 29'09'',6380$$

$$\lambda_o = 36^o 42'53'',5733 \cdot est \cdot Ferro$$

and on the sphere of radius R= 6 378 512.966 m:

$$\varphi'_o = 47^\circ 26'21'',1372$$

$$\lambda'_o = 0^\circ 00'00'',0000$$

In Transilvania, province of the Habsburg Empire, which was later annexed to the Hungarian Kingdom, the ***Targu Mureş tangent plane stereographic projection*** was used.

The Bessel 1841 ellipsoid was used as the reference.

The pole of the projection is the *Kesztej* fundamental geodetic point bearing the following coordinates on the ellipsoid:

$$\varphi_o = 46^\circ 33'08'',8500$$

$$\lambda_o = 42^\circ 03'20'',9550 \cdot est \cdot Ferro$$

The geographical coordinates of the projection pole on the sphere are:

$$\varphi'_o = 46^\circ 30'25'',408$$

$$\lambda'_o = 5^\circ 20'41'',829 \cdot est \cdot Gellerthegy$$

In these projections, the rectangular coordinate sistem was defined as following: the origin is the image of the projection pole, the *OX* axis is the line that represents the meridian image which passes through the projection pole with the positive direction towards the south, and the *OY* axis chosen such that it forms a straight angle with the *OX* axis, having the positive direction towards the west.

At the beginning of the '30s the ***sterographic projection on a secant plane in Braşov (1930/1933)*** was adopted**.** The reference ellipsoid used was the Hayford 1910, and coordinates of the projection pole are:

$$\varphi_o = 51^G 00^c 00^{cc},000 = 45^o 54'00''.0000$$

$$\lambda_o = 28^G 21^c 38^{cc}.510 = 25^o 23'32''.8772 \cdot est \cdot Greenwich$$

The image of the projection pole Q_0 *(*φ_0, λ_0*) is the origin of* the rectangular coordinate sistem. The *OX* axis has the positive direction towards the east and the *OY* axis has the positive direction towards the north.

The whole country is mapped on a secant plane, which intersects the ellipsoid resulting a circle of a *232.965 km* radius, in the centre of this circle, the linear scale distortion being -33.33 cm/km.

In 1971 a new stereographic projection named ***Stereographic projection 1970*** was adopted for the civil sector in Romania. The reference ellipsoid was the Krsovski 1940. The projection pole Q_o, also named the „center of projection" has the geographical coordinates: φ_o=46°, λ_o=25°.

The whole country is represented on a single secant projection plane, which has a zero scale distortion circle, with the radius $\rho_o = 201.718 km$, the linear scale distortion in the center of the circle being –25 cm/km. The rectangular axis system has the origin in the central point of projection, the *OX* axis has the positive direction towards the north and the *OY* axis towards the east.

2. Stereographic projections

Representing an ellipsoid in a stereographic projection can be made using two methods:

a) Through a ***double projection***, meaning the ellipsoid is firstly represented on a sphere through a conformal transformation and then the sphere is represented in the mapping plane.
b) Trough a ***quasi-stereographic transformation***, meaning the ellipsoid is directly represented in the mapping plane, rigorously respecting only one of the properties of mapping a sphere on a plane. This is the case of stereographic projection 1970, which respects the properties of mapping a sphere stereographically to a plane only for the points situated on the meridian of the projection pole (λ_o=25°). The coordinates are calculated with the following relation:

$$x_m = 2R_o \tan \frac{\beta}{2R_o} \qquad (1)$$

where: x_m is the coordinate of any point situated on the meridian of the pole;
R_o is the mean radius of the ellipsoid at latitude $(\varphi_o = 46^o)$;
β is the length of the meridian arch from latitude $\varphi_o = 46^o$ to latitude φ of the considered point;

In this paper we have studied the double stereographic projection with four cases of usage comparing it with the quasi-stereographic projection applied for ellipsoid GRS 80.

a) ***Double stereographic projection***

Spherical latitude φ'and longitude λ' are computed from geodetic latitude φ and longitude λ with formulas (Soloviev 1954):

$$\varphi' = 2\left\{\arctan\left[\frac{1}{K}\tan\left(45^o + \frac{\varphi}{2}\right)\left(\frac{1-e\sin\varphi_o}{1+e\sin\varphi_o}\right)^{e/2} - 45^o\right]\right\} \qquad (2)$$

$$\lambda' = \alpha \cdot \lambda$$

In which constants α and K are calculated with:

$$\alpha = \frac{\sin\varphi_o}{\sin\varphi'_o} \qquad (3)$$

$$tg\varphi'_o = \sqrt{\frac{M_o}{N_o}}tg\varphi_o \qquad (4)$$

$$K = \frac{\tan^{\alpha}\left(45^o + \frac{\varphi_o}{2}\right)\left(\frac{1-e\sin\varphi_o}{1+e\sin\varphi_o}\right)^{\alpha e/2}}{\tan\left(45^o + \frac{\varphi'_o}{2}\right)} \qquad (5)$$

For the studying of distortions we use linear scale factors along meridian, parallel and area scale factors:

$$m = n = \alpha\frac{R\cos\varphi'}{N\cos\varphi} \qquad (6)$$

$$p = m^2$$

In this case we studied four conformal mapping cases of the ellipsoid onto a sphere:

i) Case 1 – *the sphere is considered tangent to the ellipsoid on the equator*.

In this case the meridians of the sphere coincide with the meridians on the ellipsoid, meaning that the ellipsoidal and spherical longitudes are equal: $\lambda' = \lambda$

Because the two equatorial planes coincide, the radius of the sphere is equal to the semimajor axis of the ellipsoid: $R = a$. The constants of projection are:

$$\alpha = 1$$
$$K = 1 \qquad (7)$$

The spherical latitude is determined from the ellipsoidal one using relation (2) with the constants from relations (3), (4), (5).

ii) Case 2 – *the intersections of the terrestrial sphere and the ellipsoid are the parallels of latitudes* $\varphi_k = \pm 46^o$

In this case the meridians on the sphere coincide with the meridians on the ellipsoid, so that the ellipsoidal and spherical longitudes are equal: *λ'=λ.*

The parallels on the ellipsoid are represented without distortion on the sphere, resulting in the following relation for the radius of the sphere:

$$R = \frac{N_k \cos\varphi_k}{\cos\varphi'_k} \tag{8}$$

The projection constants are in this case:

$$\alpha = 1$$
$$K = 1 \tag{9}$$

Substituting these values in relation (2) for the spherical latitude yields:

$$\varphi' = 2\left\{\arctan\left[\tan\left(45^o + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi_o}{1 + e\sin\varphi_o}\right)^{e/2} - 45^o\right]\right\} \tag{10}$$

iii) **Case 3 –** *the terrestrial sphere is tangent to the ellipsoid on the parallel of latitude* $\varphi_o = +46^o$, therefore: $\varphi'_o = \varphi_o$

Moreover, the meridians on the sphere coincide with the meridians on the ellipsoid, meaning the ellipsoidal and spherical longitudes are equal: $\lambda' = \lambda$

The radius of the sphere is calculated through the following relation: $R = N_o$

The projection constants are in this case:

$$\alpha = 1$$

$$K = \left(\frac{1 - e\sin\varphi_o}{1 + e\sin\varphi_o}\right)^{e/2} \tag{11}$$

The spherical latitude is determined with the relation:

$$\varphi' = 2\left\{\arctan\left[\frac{1}{K}\tan\left(45^o + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi_o}{1 + e\sin\varphi_o}\right)^{e/2} - 45^o\right]\right\} \tag{12}$$

iv) **Case 4 –** *the ellipsoid is tangent to the terrestrial sphere in projection pole*

The radius of the sphere is calculated with the relation:

$$R = \sqrt{M_o N_o} \tag{13}$$

The projection constants α and K are calcultated with relations (3), (4) and (5).

In this case the meridians of the ellipsoid no longer coincide with the meridians of the sphere and the spherical coordinates are calcultated with the relations (2):

The coordinates from the sphere are transformed on the mapping plane using the relations:

$$x = \frac{2R(\cos\varphi'_o \sin\varphi' - \sin\varphi'_o \cos\varphi' \cos(\lambda' - \lambda'_o))}{1 + (\sin\varphi' \sin\varphi'_o + \cos\varphi' \cos\varphi'_o \cos(\lambda' - \lambda'_o))}$$
$$y = \frac{2R\cos\varphi' \sin(\lambda' - \lambda'_o)}{1 + (\sin\varphi' \sin\varphi'_o + \cos\varphi' \cos\varphi'_o \cos(\lambda' - \lambda'_o))} \tag{14}$$

For determining the distortions we use the relations:

$$\mu_1 = \mu_2 = \frac{2}{1 + \sin\varphi'_o \sin\varphi' + \cos\varphi'_o \cos\varphi' \cos(\lambda' - \lambda'_o)}$$
$$p = \mu_1 \mu_2 \tag{15}$$
$$\omega = 0$$

b. *The quasi-stereographic projection*

In this projection, which has been applied in Romania up until now, but for the Krasovski 1940 ellipsoid, we have determined the values of the constant coefficients for the GRS 80 ellipsoid, coefficients which are part of the calculation formulas of the coordinates in the mapping plane tangent to the ellipsoid at origin:

$$
\begin{aligned}
x_{tg} &= \left(a_{00} + a_{10} f + a_{20} f^2 + a_{30} f^3 + a_{40} f^4 + a_{50} f^5 + a_{60} f^6 + a_{70} f^7 + \ldots\right) + \\
&+ \left(a_{02} + a_{12} f + a_{22} f^2 + a_{32} f^3 + a_{42} f^4 + \ldots\right) \cdot l^2 + \left(a_{04} + a_{14} f + \ldots\right) \cdot l^4 + \left(a_{06} + \ldots\right) \cdot l^6 \\
y_{tg} &= \left(b_{01} + b_{11} f + b_{21} f^2 + b_{31} f^3 + b_{41} f^4 + b_{51} f^5 + \ldots\right) \cdot l + \\
&+ \left(b_{03} + b_{13} f + b_{23} f^2 + b_{33} f^3 + \ldots\right) \cdot l^3 + + \left(b_{05} + b_{15} f + \ldots\right) \cdot l^5
\end{aligned}
\tag{16}
$$

Coordinates in the secant plane are:

$$
\begin{aligned}
x &= x_{tg} \cdot c \\
y &= y_{tg} \cdot c
\end{aligned}
\tag{17}
$$

Where *c=const.=0.999 750 000*

The values of these coefficients are presented in Table 1.

In all four cases analyzed we have calculated the spherical coordinates (φ',λ') and the rectangular coordinates *(x, y)* of the map graticule which vary from latitude $\varphi_S = 44^\circ$ to $\varphi_N = 48^\circ$ and from longitude $\lambda_V = 21^\circ 30'$ to $\lambda_E = 29^\circ$(tab. 4). We have also studied the influence of changing the ellipsoid (GRS-80 versus Krasovski 1940) on the coordinates of the points by calculating the lengths and orientations of the vectors determined by the image of the points in quasi-stereographic projections (tab3, fig. 2). For comparing the quality of the analyzed projections we have conducted a study of distortions. All the stereographic projections are conformal meaning that the angles are represented undistorted in the mapping plane. The obtained results are listed in Table 2.

Table 1. Constant coeficients for calculating the x, y coordinates

a_{00}= 0	a_{02}=+3752.083111289852	a04=+ 0.335923134383	a06=-0.000064962727
a_{00}=0.000000000	a_{02}=+3752.083111289852	a_{04}=+0.335923134383	a_{06}= -0.000064962727
a_{10}=+308753.662509386486	a_{12}= -99.926335176515	a_{14}= -0.064339659184	a_{16}= -0.000005157411
a_{20}= +75.370427646071	a_{22}= -6.674419721378	a_{24}= +0.000382610090	
a_{30}= +60.214993686898	a_{32}= -0.068376791286	a_{34}= 0.000073424202	
a_{40}= -0.014952397386	a_{42}= -0.002605399990		
a_{50}= +0.014096938640	a_{52}= -0.000007250470		
a_{60}= +0.000001183300			
a_{70}= -0.000000005068			

b_{01}=+215175.830840683659	b_{03}= -23.213470596855	b_{05}= -0.008967857118	b_{06}= -0.000000511806
b_{11}= -10767.653580090855	b_{13}= -1.928051001237	b_{15}= +0.000559489287	
b_{21}= -128.658279238281	b_{23}= +0.135928415327	b_{25}= +0.000048545149	
b_{31}= -2.106816412385	b_{33}= +0.003100511105		
b_{41}= -0.050029680696	b_{43}= +0.000100929173		
b_{51}= -0.000510398265	b_{61}= -0.000032291115		

Table 2. Distortions of length and distortions of area in quasi-stereographic projection

Distance to pole [km]	μ	D[cm/km]	p	[mp/ha]
0.0	0.999750000	-25.000	0.999500063	-4.999
20.0	0.999752458	-24.754	0.999504977	-4.950
40.0	0.999759831	-24.017	0.999519719	-4.803
60.0	0.999772119	-22.788	0.999544290	-4.557
80.0	0.999789323	-21.068	0.999578690	-4.213
100.0	0.999811443	-18.856	0.999622922	-3.771
120.0	0.999838480	-16.152	0.999676985	-3.230
140.0	0.999870433	-12.957	0.999740883	-2.591
160.0	0.999907304	-9.270	0.999814617	-1.854
180.0	0.999949094	-5.091	0.999898190	-1.018
200.0	0.999995803	-0.420	0.999991605	-0.084
220.0	1.000047431	4.743	1.000094865	0.949
240.0	1.000103981	10.398	1.000207973	2.080
260.0	1.000165453	16.545	1.000330934	3.309
280.0	1.000231849	23.185	1.000463752	4.638
300.0	1.000303169	30.317	1.000606430	6.064
320.0	1.000379416	37.942	1.000758975	7.590
340.0	1.000460590	46.059	1.000921391	9.214
360.0	1.000546693	54.669	1.001093684	10.937
380.0	1.000637727	63.773	1.001275861	12.759
400.0	1.000733694	73.369	1.001467926	14.679

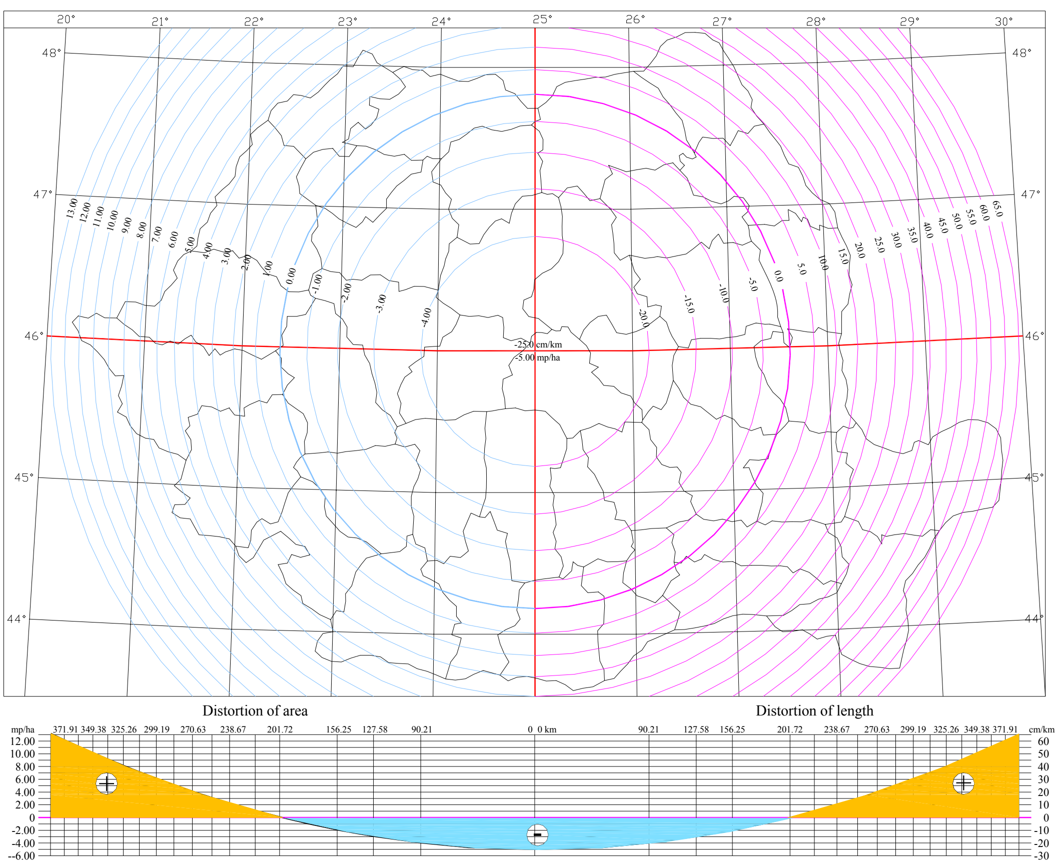

Fig. 1. Distortions of area and distortions of length for Romania

Table 3 Grid coordinates in quasi-stereographic projection
(ellipsoid Krasowski and ellipsoid GRS-80)

φ Krasowski	λ Krasowski	φ_{GRS-80}	λ_{GRS-80}	X Krasowski	Y Krasowski	X φ_{GRS-80}	Y φ_{GRS-80}	Vector length [m]	Vector orientation [º]
44°00’	23°00’	43°59’58”.76951	22°59’54”.51443	279745.946	339584.472	279714.773	339464.008	124.432	283.8791
44°00’	24°00’	43°59’58”.81525	23°59’54”.55501	278260.749	419789.190	278229.488	419668.764	124.418	283.8308
44°00’	25°00’	43°59’58”.85581	24°59’54”.60298	277765.709	500000.000	277734.209	499879.750	124.307	283.6898
44°00’	26°00’	43°59’58”.90506	25°59’54”.66111	278260.749	580210.810	278229.294	580090.937	123.931	283.6630
44°00’	27°00’	43°59’58”.95780	26°59’54”.69328	279745.946	660415.528	279714.647	660295.424	124.116	283.7705
44°00’	28°00’	43°59’59”.04540	27°59’54”.75544	282221.530	740608.056	282191.491	740488.320	123.447	284.3513
45°00’	22°00’	44°59’58”.78945	21°59’54”.37844	393298.359	263525.088	393267.444	263404.580	124.410	284.0129
45°00’	23°00’	44°59’58”.84052	22°59’54.41838	390844.438	342339.555	390813.577	342219.087	124.358	284.0343
45°00’	24°00’	44°59’58”.88207	23°59’54”.46480	389372.230	421166.628	389341.137	421046.308	124.272	283.9004
45°00’	25°00’	44°59’58”.92366	24°59’54”.51007	388881.518	500000.000	388850.206	499879.780	124.230	283.7791
45°00’	26°00’	44°59’58”.96671	25°59’54”.55941	389372.230	578833.372	389340.757	578713.319	124.110	283.6775
45°00’	27°00’	44°59’59”.02067	26°59’54”.59769	390844.438	657660.445	390813.147	657540.285	124.168	283.7817
45°00’	28°00’	44°59’59”.08189	27°59’54”.64550	393298.359	736474.912	393267.488	736354.813	124°003	283.9826
45°00’	29°00’	44°59’59”.13901	28°59’54”.68963	396734.353	815270.450	396703.784	815150°305	123.972	284.1385
46°00’	21°00’	45°59’58”.80389	20°59’54”.27089	507779.948	190288.451	507749.115	190168.602	123.752	283.9695
46°00’	22°00’	45°59’58”.83666	21°59’54”.29382	504375.865	267693.572	504344.522	267573.389	124.203	283.7590
46°00’	23°00’	45°59’58”.88084	22°59’54”.33486	501944.716	345118.213	501913.205	344998.080	124.197	283.6694
46°00’	24°00’	45°59’58”.92704	23°59’54”.36197	500486.162	422555.857	500454.560	422435.452	124.484	283.6594
46°00’	25°00’	45°59’58”.98315	24°59’54”.40368	500000.000	500000.000	499968.613	499879.610	124.414	283.7642
46°00’	26°00’	45°59’59”.03730	25°59’54”.45992	500486.162	577444.143	500454.944	577324.049	124.085	283.8095
46°00’	27°00’	45°59’59”.08616	26°59’54”.49480	501944.716	654881.787	501913.508	654761.501	124.268	283.8389
46°00’	28°00’	45°59’59”.14172	27°59’54”.53673	504375.865	732306.428	504344.883	732186.067	124.285	283.9608
47°00’	22°00’	46°59’58”.88489	21°59’54”.18176	615470.882	271898.022	615439.159	271777.669	124.464	283.5929
47°00’	23°00’	46°59’58”.94189	22°59’54”.21838	613063.625	347920.866	613032.118	347800.470	124.450	283.7055
47°00’	24°00’	46°59’58”.98546	23°59’54”.25344	611619.397	423957.089	611587.700	423836.581	124.607	283.6259
47°00’	25°00’	46°59’59”.03835	24°59’54”.29606	611138.009	500000.000	611106.415	499879.515	124.558	283.6736
47°00’	26°00’	46°59’59”.09421	25°59’54”.33842	611619.397	576042.911	611588.007	575922.412	124.520	283.7762
47°00’	27°00’	46°59’59”.16519	26°59’54”.36685	613063.625	652079.134	613032.906	651958.283	124.695	284.1534
47°00’	28°00’	46°59’59”.21573	27°59’54”.42755	615470.882	728101.978	615440.229	727981.432	124.382	284.1477
48°00’	23°00’	47°59’58”.98148	22°59’54”.12092	724218.009	350747.943	724185.828	350627.773	124.405	283.3423
48°00’	24°00’	47°59’59”.01529	23°59’54”.12974	722788.788	425370.538	722756.112	425249.706	125.173	283.1863
48°00’	25°00’	47°59’59”.10400	24°59’54”.12063	722312.401	500000.000	722280.918	499878.118	125.883	283.9072
48°00’	26°00’	47°59’59”.15890	25°59’54”.18532	722788.788	574629.462	722757.455	574508.015	125.424	283.9259
48°00’	27°00’	47°59’59”.24787	26°59’54”.25192	724218.009	649252.057	724187.900	649131.025	124.721	284.4777

Table 4. Distortions of length and distortions of area in double stereographic projection
Ellipsoid GRS-80

Case 1, α=1.00, K=1.00, R=6378137.000

φ	φ’	φ’- φ	m	m/km	p	m^2/ha
43°00’	42°48’29”.48383	-0°11’30”.51617	1.001553267	1.553	1.003109	31.089
44°00’	43°48’28”.15189	-0°11’31”.84811	1.001611736	1.612	1.003226	32.261
45°00’	44°48’27”.66260	-0°11’32”.33740	1.001670296	1.670	1.003343	33.434
46°00’	45°48’28”.01680	-0°11’31”.98320	1.001728876	1.729	1.003461	34.607
47°00’	46°48’29”.21430	-0°11’30”.78570	1.001787403	1.787	1.003578	35.780
48°00’	47°48’31”.25390	-0°11’28”.74610	1.001845807	1.846	1.003695	36.950
49°00’	48°48’34”.13334	-0°11’25”.86666	1.001904017	1.904	1.003812	38.117

Case 2 α=1.00, K=1.00, R=6367129.025

φ	φ’	φ’- φ	m	m/km	p	m^2/ha
43°00’	42°48’29”.48383	-0°11’30”.51617	0.999824694	-0.175	0.999649	-3.506
44°00’	43°48’28”.15189	-0°11’31”.84811	0.999883063	-0.117	0.999766	-2.339
45°00’	44°48’27”.66260	-0°11’32”.33740	0.999941522	-0.058	0.999883	-1.170
46°00’	45°48’28”.01680	-0°11’31”.98320	1.000000000	0.000	1.000000	0.000
47°00’	46°48’29”.21430	-0°11’30”.78570	1.000058426	0.058	1.000117	1.169
48°00’	47°48’31”.25390	-0°11’28”.74610	1.000116730	0.117	1.000233	2.335
49°00’	48°48’34”.13334	-0°11’25”.86666	1.000174839	0.175	1.000350	3.497

Case 3, α=1.00, K=0.9951904970, R=6389212.733

φ	φ'	φ'- φ	m	m/km	p	m^2/ha
43°00'	43°00'37".83092	0°00'37".83092	1.000004666	0.0047	1.00000933	0.0933
44°00'	44°00'24".59635	0°00'24".59635	1.000002039	0.0020	1.00000408	0.0408
45°00'	45°00'11".98459	0°00'11".98459	1.000000501	0.0005	1.00000100	0.0100
46°00'	46°00'00".00000	0°00'00".00000	1.000000000	-0.0000	1.00000000	-0.0000
47°00'	46°59'48".64602	-0°00'11".35398	1.000000483	0.0005	1.00000097	0.0097
48°00'	47°59'37".92512	-0°00'22".07488	1.000001898	0.0019	1.00000380	0.0380
49°00'	48°59'27".83880	-0°00'32".16120	1.000004190	0.0042	1.00000838	0.0838

Case 4, φ'$_0$=45.571266862, α=1.0007843543, K=0.9970576345, R=6378848.680

φ	φ'	φ'- φ	m	m/km	p	m^2/ha
43°00'	42°57'33".02937	-0°02'26".97063	1.000000316	0.0316	1.00000063	0.0063
44°00'	43°57'25".61139	-0°02'34".38861	1.000000094	0.0094	1.00000019	0.0019
45°00'	44°57'18".82415	-0°02'41".17585	1.000000012	0.0012	1.00000002	0.0002
46°00'	45°57'12".66862	-0°02'47".33138	1.000000000	0.0000	1.00000000	0.0000
47°00'	46°57'07".14471	-0°02'52".85529	0.999999988	-0.0012	0.99999998	-0.0002
48°00'	47°57'02".25133	-0°02'57".74867	0.999999904	-0.0096	0.99999981	-0.0019
49°00'	48°56'57".98635	-0°03'02".01365	0.999999676	-0.0324	0.99999935	-0.0065

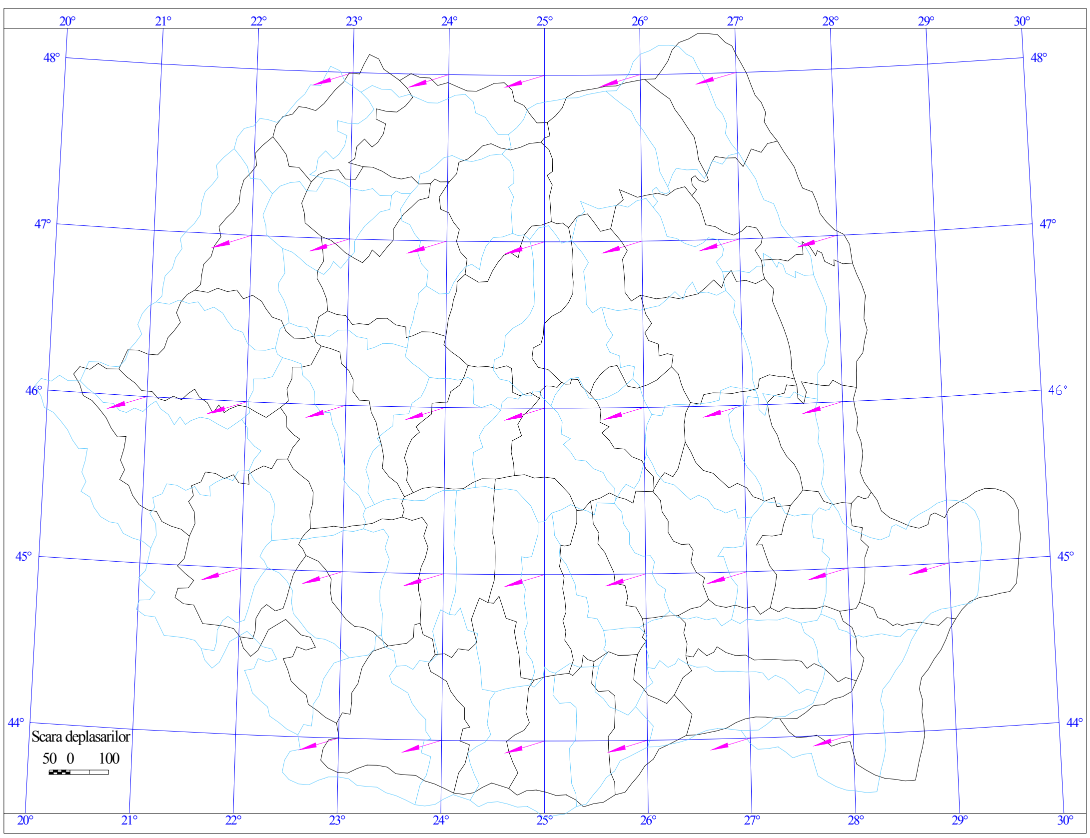

Fig. 2. The vectors determined by the images of the points in quasi-stereographic projection – ellipsoid GRS-80 and ellipsoid Krasovski 1940

3. Conclusions

Analyzing the results from the case studies we can see that the distortions obtained in case 3 (the sphere tangent to the ellipsoid on parallel of latitude 46^o) are comparable to those obtained by means of quasi-stereographic projection. Furthermore, the rectangular coordinates calculated in case 3 lead to values comparable to those calculated through quasi-stereographical projection.

4. References

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