THE DETERMINATION OF TROPOSPHERIC REFRACTION CORRECTIONS FOR GPS MEASUREMENTS

Dumitru ILIOI, Lecturer professor Ph.D., "Gh.Asachi" Technical University Iaşi, dilioi@yahoo.com
Cristian ONU, Assistant professor, Ph.D., "Gh.Asachi" Technical University Iaşi, cristi_onu@yahoo.com
Horațiu HOGAŞ, Lecturer professor Ph.D., "Gh.Asachi" Technical University Iaşi, hhogas@yahoo.com
Gabriel SĂNDULACHE, Lecturer professor Ph.D., "Gh.Asachi" Technical University Iaşi, sandulacheg@yahoo.com

***Abstract**: This paper analyses the tropospheric effects on GPS signals and how tropospheric refraction varies depending on the atmospheric parameters, season and elevation angle. There will be presented the main mathematical formulas for the tropospheric refraction and will be determined how the elevation angle of the observations influences the measurements. There were determined also, as a case study, the size of the corrections that would have to be applied to GPS measurements if the measurements were not to be done by differencing in the relative observation mode.*

***Keywords:** GPS, Geodesy, troposphere, refraction.*

1. Introduction

The propagation of radio waves through the troposphere, is subject to the laws of physics. The signal path bents from the straight geometrical connection between the observer and the satellite, and as a result of tropospheric refraction, the optical distance measured is longer than the direct geometrical range.

The troposphere represents the lowest, gaseous part of the atmosphere situated between the surface of the Earth to about 40-50km. The propagation delay of the GPS signals through the troposphere depends on the water vapor content and on temperature. Hence, tropospheric refraction varies with geographic location and season.

2. The tropospheric refraction models

The troposphere can be defined as the gaseous part of the lower atmosphere where the weather takes place. Given the fact that charged particles are virtually absent and the uncharged atoms and molecules are well mixed, the troposphere can practically be considered a neutral gas. Within the troposphere the temperature decreases with height by *6.5 °C/km* and varies horizontally with only a few *degrees/km*. Furthermore, because nearly 90% of the atmospheric mass is below 16 km altitude and nearly 99% below 30km (Lutgens, Tarbuck 1998), the index of refraction, which is slightly greater than 1 to begin with, decreases with increasing height and becomes nearly 1 at the upper limit of the troposphere, corresponding to the continuously decreasing density of the medium.

The index of refraction does not depend on the frequency of the signal; it depends on air pressure, temperature and water vapor pressure of the atmosphere. Because these three parameters vary so much and are so dynamic within the troposphere, it is very difficult to predict and/or model the index of refraction.

A direct measurement of the refractivity along the signal propagation path is then not feasible. Therefore various models for a description of the height-dependent behavior of the refractivity have been developed. Best results were obtained by Hopfield (1969), who has done the basic research in the field. Input parameters are mostly the meteorological surface data near the observation site.

According with the Helen Hopfield, the impact of the state of the troposphere on the propagation of waves [∂dt], can be characterized by the following algorithm:

$$\partial dt = \partial d_d + \partial d_w = \frac{k_d}{\sin\sqrt{(E^2+6.25)}} + \frac{k_w}{\sin\sqrt{(E^2+6.25)}}, \tag{1}$$

Where [k_d] and [k_w] describe the total effect of the tropospheric refraction in the direction to the zenith, corresponding to the dry term and to the wet term, respectively, and [E] represents the elevation angle of the satellite the way it is seen by the observer, as illustrated in Fig.1.

The dry and the wet items, [∂d_d] and [∂d_w] , are determined separately because their parameters are formulated as distinct functions of height:

$$k_d = 155.2 \cdot 10^{-7} \frac{P}{T} H_d ; k_w = 155.2 \cdot 10^{-7} \frac{4810e}{T} H_w, \tag{2}$$

with [P], the air pressure in Hectopascal [HPa], [e], the partial pressure of the water vapor [HPa], and [T] representing the temperature in Kelvin. [H_d] and [H_w] are the effective altitudes of the dry and the wet terms respectively.

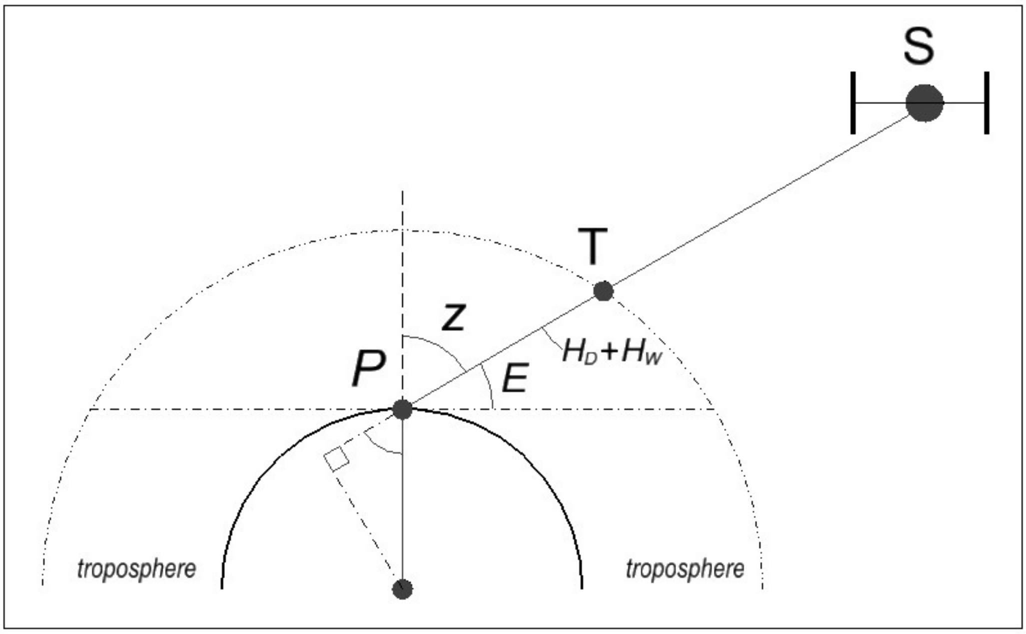

Fig. 1. The tropospheric model

Generally, for [H_w] a mean value is accepted $H_w = 11000m$, but the parameter [H_d] was determined by Helen Hopfield empirically from globally distributed balloon data:

$$H_d = 40136 + 148.72(T - 273.16). \tag{3}$$

For elevations [E > 30°], Harold Black (1978) proposes the simple correction formulas:

$$\partial d_{trop} = (2.31 \cdot P + Q_w) \cos ecE, \tag{4}$$

where [P] is the air pressure in atmospheres [1atm = 1013.25HPa], and [Q_w] is a regional empirical constant with values ranging for Romania from 0.06 during the winter to 0.28 for summer, as showed in Table 1.

The two expressions depending of [P] and [Q_w], to which equation (4) could expand, correspond to the two layers of the troposphere, dry and wet, respectively:

$$\partial d_{trop} = 2.31 \cdot P \cos ecE + Q_w \cos ecE. \tag{5}$$

The global variation by region and season of the regional empiric constant that integrates the atmospheric conditions [Q_w], determined according with the simplified Harold Black model, is presented in Table 1.

Table 1. The global variation of the regional empiric constant

Region and Season	$[Q_w]$
Summer in tropical areas or mean latitudes	0.28
Spring or autumn in mean latitudes	0.20
Winter in maritime latitudes	0.12
Winter in continental mean latitudes	0.06
Polar regions	0.05

3. The determination of tropospheric refraction corrections

Based on the models presented above, there were determined the values of the errors due to the tropospheric refraction, associated with a set of GPS observations made in the summer of 2009 and in the winter of 2010 for a series of cadastre works carried out in Suceava County area.

The atmospheric conditions corresponding to the two sessions of observations were recorded in Table 2.

Table 2. The atmospheric parameters associated with the two sessions

Atmospheric parameters	Summer	Winter
Temperature [T]	24°C	-22ºC
Atmospheric pressure [P]	1004.35 HPa	1030.42 HPa
Partial water vapor pressure [e]	32.40 HPa	5.82 HPa

In Table 3 there were also presented the actual dry altitudes, along with the dry and the wet parameters $[k_d]$ and $[k_w]$ that characterize the total effect of the tropospheric refraction in the direction to the zenith, determined with the equations (2) and (3), for both sessions of the GPS measurements.

Table 3. The parameters calculated based on the Hopfield model

Determined parameters		Summer	Winter
Dry altitude		43705.3 m	36864.2 m
The total tropospheric refraction effect to the zenith	Dry term	2.2938 m	2.3487 m
	Wet term	0.3016 m	0.0759 m

The errors due to the tropospheric refraction, associated with the GPS observations made in the summer of 2009 and in the winter of 2010 in Suceava County area, for the given atmospheric conditions, determined with the Hopfield algorithm were tabulated in Table 4.

Table 4. The errors due to the tropospheric refraction at GPS measurements in Suceava County area [Hopfield model]

[E]		90°	60°	45°	30°	15°	10°	5°
[m]	Summer	2.29	2.65	3.24	4.57	8.74	12.82	23.55
	Winter	2.35	2.71	3.32	4.68	8.95	13.13	24.11
[m]	Summer	0.30	0.35	0.43	0.60	1.16	1.72	3.32
	Winter	0.08	0.09	0.11	0.15	0.29	0.43	0.83
[m]	Summer	2.60	3.00	3.67	5.18	9.90	14.54	26.86
	Winter	2.42	2.80	3.42	4.83	9.25	13.56	24.95

The errors due to the tropospheric refraction, associated with the GPS observations made in the summer of 2009 and in the winter of 2010 in Suceava County area, determined with the simplified Black algorithm were tabulated in Table 5.

Table 5. The errors due to the tropospheric refraction at GPS

measurements in Suceava County area [Black model]

[E]		90°	60°	45°	30°	15°	10°	5°
[m]	Summer	2.29	2.65	3.24	4.59	8.86	13.21	26.32
	Winter	2.35	2.71	3.32	4.70	9.07	13.53	26.95
[m]	Summer	0.28	0.32	0.40	0.56	1.08	1.61	3.21
	Winter	0.06	0.07	0.08	0.12	0.23	0.35	0.69
[m]	Summer	2.57	2.97	3.64	5.15	9.94	14.82	29.53
	Winter	2.41	2.78	3.41	4.82	9.31	13.87	27.64

From the analysis of the values from Tables 4 and 5, one can observe that the effect of tropospheric refraction increases with increasing zenith angle [Z] or, in other words, with decreasing the elevation angle [E]. For elevations [E < 5°] the errors at GPS observations due to the tropospheric refraction and hence the corrections that need to be applied to the measured ranges, easily exceed 20 m. For our case studied, the resulting range error for GPS signals because of the tropospheric refraction vary from less than 3 m to more than 26 m on Hopfield model and from less than 3 m to more than 29 m on Black model, depending on the elevation angle.

The differences between the errors due to the tropospheric refraction at GPS measurements calculated based on the Hopfield model versus those determined on the simplified Black model, especially for angles of elevation of the satellites observed greater than 30° are virtually negligible, as seen in Table 6.

Table 6. Differences between the errors determined with Hopfield and Black model respectively

[E]		90°	60°	45°	30°	15°	10°	5°
[m]	Summer [Hopfield]	2.60	3.00	3.67	5.18	9.90	14.54	26.86
	Summer [Black]	2.57	2.97	3.64	5.15	9.94	14.82	29.53
Differences		0.03	0.03	0.03	0.03	-0.04	-0.28	-2.67
[m]	Winter [Hopfield]	2.42	2.80	3.42	4.83	9.25	13.56	24.95
	Winter [Black]	2.41	2.78	3.41	4.82	9.31	13.87	27.64
Differences		0.01	0.02	0.01	0.01	-0.06	-0.31	-2.69

On the other hand, the portion of the wet term, which depends on the distribution of water vapor in the atmosphere and is therefore harder to model, represents only a small fraction of the total influence. For this paper case studied, the error that occurs at the GPS signal propagation thru the wet layer of the troposphere, reaches only around 10% of the total error due to tropospheric refraction at GPS measurements.

Moreover, the values in Tables 4 and 5, show that the differences from summer to winter conditions between the total errors due to the tropospheric refraction in determining the pseudo-range at GPS signal propagation are also within 8-10% from one another. Same thing, when station distances are smaller [< 50km] and when the height differences are small (in non mountainous regions, as those in the Suceava county area), the atmospheric conditions are sufficiently correlated with one another which means that the water vapor content of the air is almost identical horizontally.

4. Conclusions

The effect of tropospheric refraction increases severely for GPS observations at low elevation angles. It is hence advisable not to make observations for the satellites that are seen by the observer under 10-15° above the horizon.

The differences from summer to winter conditions between the total errors due to the tropospheric refraction in determining the pseudo-range at GPS signal propagation are limited in size.

For a small network it is not advisable to introduce the observed meteorological data into the adjustment of the determinations separately for each station. If the stations are close together, the tropospheric residual error disappears almost completely by differencing in the relative observation mode.

5. References

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