CONTRIBUTIONS TO THE IMPROVEMENT OF THE MEASURING TECHNOLOGY AND PRECISION OF THE CYCLIC DETERMINATION OF THE VERTICAL DEFORMATION OF CONSTRUCTIONS USING HIGH PRECISION GEOMETRIC LEVELING MEASUREMENTS

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Abstract: *The paper presents a new field method for conducting high precision geometrical leveling measurements used to determine the vertical displacements and deformations of construction by using intermediate bench marks positioned on the studies constructions. The newly proposed method ensures a superior accuracy, comparable to the one using the connection marks in the line of levels/traverses.*

Using the Gheorghe Nistor algorithm, which is used to determine the vertical displacements and deformations of construction based on the cyclic variations of the difference in elevation, one can eliminate the use of the parallactic angle between the line sight and the axis of the level tube, increasing thus the efficiency of the field and office calculations. The mathematical model allows a complete estimation of the precision of determination of the vertical displacements and deformations of the intermediate marks from the studied construction, in connection with determining the accuracy from each connection marks.

The study of the precision confirms entirely that by using the measurement technique from two conjugated station point, one can obtain a precision which is quite close to the one obtained for the connection marks.

Keywords: *Behaviour, Building, Geometric leveling, Vertical deformation, Elevation, Level, intermediate mark.*

1. Introduction

Within the large research and analyse process of the behaviour of the buildings a distinct importance have the vertical displacements and deformations, that appear both during researches on models and during their existence.

Measuring the vertical deformations consist in periodical determinations of the elevation of the mobile marks (settlement marks), from the building with reference to some immobile/fixed marks which are settled on stabile soils out of the influence radius of the building. From the difference in elevations of the current and initial periods of observations we obtain the values of the vertical deformations. The determinations of the vertical deformations are made using the high precision geometrical leveling or the method of closed circuit or supported traverse using the method of the networks restrained on fixed marks of the reference network. The high precision geometrical leveling measurements are made using special levels and rods with invartape.

As a general rule, we use geometrical leveling from the midpoint, with two elevations of the instrument, the readings being made on both of the rod.

This paper is to present a new method of determination of the vertical deformations of the intermediate marks in the line of levels/traverses, which ensures a sure control of the results.

2. The Measuring Technology and the Calculation of the Vertical Deformations

The operation of studying the *in situ* behaviour of an industrial building it is necessary the measuring of the vertical deformations. In Fig. 1, R_1 and R_2 represent the fixed marks which are setting the horizontal reference plan; 1, 2,…, *j*,…, (*n* –1) – the connection marks of supported leveling traverse; 7, 8,…, *k*,…,14 – intermediate marks; S_1 , S_2,…, S_i,…, S_n – the station midpoints which have a stable position in all periods of observations; h_1 , h_2 ,…, h_i,…, h_n – the differences in elevation between the connection marks; h'_{jk} and h''_{jk} – the differences in elevation between the connection mark *j* and the intermediate one *k*, measured from the conjugated stations S_i and S_{i+1}; D_i – the length of the aims between the stations S_i and connection mark, *j*.

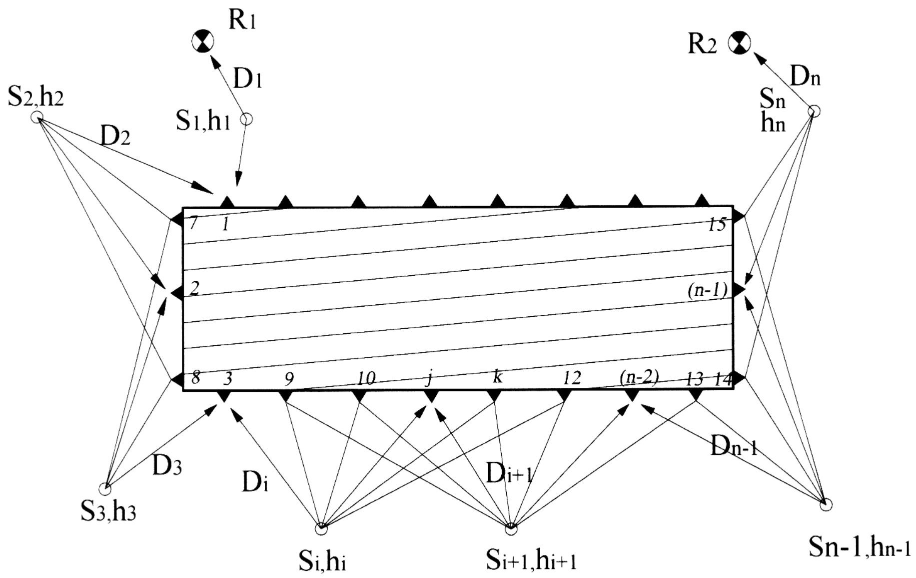

Fig. 1. Location of the fixed, connection and intermediate marks – local reference network. Plan view

Within the usual way of measuring the vertical deformations, the readings on the rod into the intermediate marks are made from a single station, after there have been previously made the readings into the connection marks.

On the terms of the existence of the residual systematic error of the from not absolute parallelism between the axis of the level tube and the line of sight, represented by the parallactic angle ε (Fig. 2), at the determination of the difference in elevation between the connection marks. The influence of this systematic error is eliminated by practicing geometrical leveling from the midpoint.

At the determination of the difference in elevation between the connection marks and the intermediate ones, due to the different length of the aims, the linear deviations from the rod, which correspond to the ε angle, will be different, implying thus measurement errors. For example, considering the upward inclined aim, the value of the difference in elevation between the connection j and the intermediate one k, measured from station S_i , will be given as (Figs. 1 and 2):

$$h_{jk}^{i} = L_0^j - L_0^k = \left(L_{01}^j + e \right) - \left(L_{01}^k + e + \delta h_{jk}^{'} \right) = \left(L_{01}^j - L_{01}^k \right) - \delta h_{jk}^{'} = h_{jk}^{'} - \delta h_{jk}^{'}, \tag{1}$$

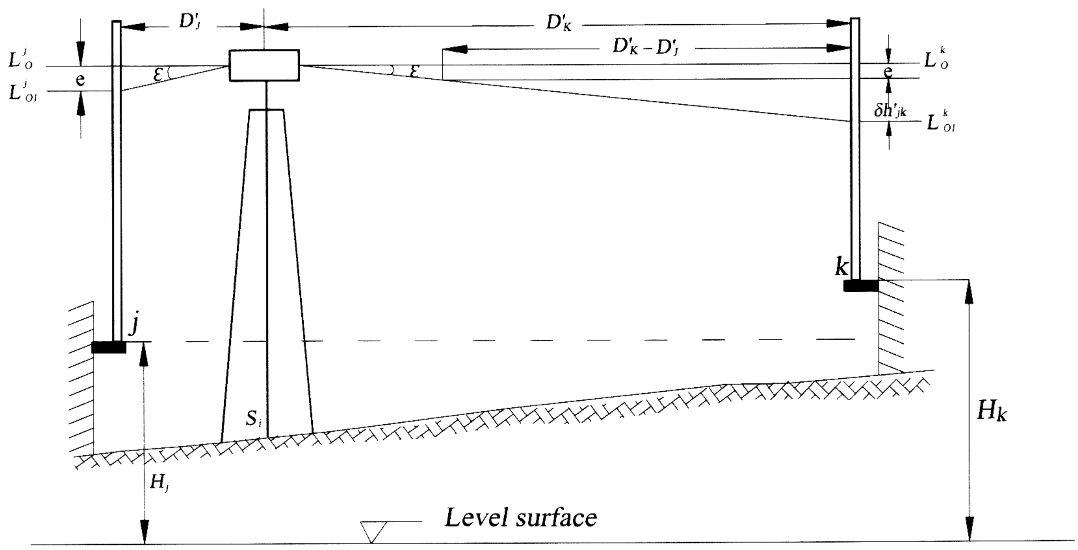

Fig. 2. Schema of the station point for geometrical leveling and the unequal lengths of the aims from the station point to the connaction and intermediate marks. Elevation

where the value of the deviation is given by the following formula:

$$\delta h_{jk}^{'} = \left(D_k^{'} - D_j^{'} \right) \tan \varepsilon. \tag{2}$$

In consequence, using the presented method, all the intermediate marks will be measured from two conjugated stations, for both the possibility of determination of the angle ε or only for control of the measurements and the enlargement of the precision of measuring the vertical deformations.

The determination of the same difference in elevation is done from the conjugated station S_{i+1} (Fig. 1), resulting thus:

$$h_{jk}^{i+1} = L_0^j - L_0^k = \left(L_{01}^j + e + \delta h_{jk}^{''} \right) - \left(L_{01}^k + e \right) = \left(L_{01}^j - L_{01}^k \right) + \delta h_{jk}^{''} = h_{jk}^{''} + \delta h_{jk}^{''}, \tag{3}$$

where the value of the linear deviation will be:

$$\delta h_{jk}^{''} = \left(D_j^{''} - D_k^{''} \right) \tan \varepsilon. \tag{4}$$

In relations (2) and (4), $D_j^{'}$ and $D_k^{'}$ are the lengths of the aims from station S_i, while $D_j^{"}$ and $D_k^{"}$ – the lengths of the aims from station S_{i+1} up to the marks. It is obvious that for each station $D_j^{'} \neq D_k^{'}$ and $D_j^{"} \neq D_k^{"}$.

When marking the leveling from the midpoint with two different lines of sight of the instrument, each provisional difference in elevation will result as an arithmetic mean of the values emerged from using two difference in elevation of the line of sight:

a) for the differences in elevation between the connection marks of the supported traverse:

$$h_i = \frac{1}{2}\left(h_i^{\mathrm{I}} + h_i^{\mathrm{II}} \right); \tag{5}$$

b) the difference in elevation between the connection mark, *j*, and the intermediate one, *k*:

$$h_{jk}^{'} = \frac{1}{2}\left(h_{jk}^{'\mathrm{I}} + h_{jk}^{'\mathrm{II}} \right), \text{ from station } S_i; \tag{6}$$

$$h_{jk}^{"} = \frac{1}{2}\left(h_{jk}^{"\mathrm{I}} + h_{jk}^{"\mathrm{II}} \right), \text{ from station } S_{i+1}. \tag{7}$$

For the case of the classical method, when the vertical deformations are obtained from the difference of the elevations, first we shall calculate and compensate the elevations of the connection marks that from the line of levels/leveling traverse, using the formula [1]:

$$H_j = H_{R_1} + \sum_{i=1}^{j}\left(h_i + K_h \cdot D_i \right),\ j = \overline{1, n-1},\ i = 1, n. \tag{8}$$

The unitary correction for measuring cycle can be further calculated using the previously determined closing error

$$f_h = \sum_{i=1}^{n} h_i - \left(H_{R_2} - H_{R_1} \right),\ K_h = -\frac{f_h}{\sum_{i=1}^{n} D_i}. \tag{9}$$

Depending on the aforementioned elevations we can calculate the elevations of the intermediate marks. Thus, mark *k* will have two elevations, resulted from the elevations of the connection mark *j*, and the differences in elevation measured from the stations points, S_i and S_{i+1}:

$$H_k^{'} = H_j + h_{jk}^{i} = H_j + h_{jk}^{'} - \delta h_{jk}^{'}, \tag{10}$$

$$H_k^{"} = H_j + h_{jk}^{i+1} = H_j + h_{jk}^{"} + \delta h_{jk}^{"}, \tag{11}$$

after which, the final elevation will be:

$$H_k = \frac{1}{2}\left(H_k^{'} + H_k^{''} \right). \tag{12}$$

The linear deviations $\delta h_{jk}^{'}$ and $\delta h_{jk}^{''}$ are calculated using relations (2) and (4) after we determined beforehand the value of the parallactic angle *ε*, as indicated in [3]. Relying on the elevation of the marks, which have been cyclically determined, we can calculated the values of the vertical deformations, using the formulas:

$$\pm\Delta H_j^t = H_j^t - H_j^0, \tag{13}$$

$$\pm\Delta H_k^t = H_k^t - H_k^0, \tag{14}$$

where: H_j^t and H_k^t are the elevations of the marks from the present period *t*, $(t = \overline{1, N})$ and H_j^0 and H_k^0 – the elevations from the initial period (zero).

The problem of determining the vertical deformations using the presented method is very much simplified by using the so-called Gh. Nistor algorithm which allows the direct calculation of the differences in elevation that have been periodically measured. By using this algorithm [1], it is no longer necessary the calculation of the parallactic angle *ε* and neither the calculation of the linear deviations, $\delta h_{jk}^{'}$ and $\delta h_{jk}^{''}$.

Since the differences in elevation that have been periodically/cyclically measured include both the influence of the vertical deformations and the inherent measuring errors, we are calculating the compensated errors of the vertical deformations into the connection marks using the first formula [1]:

$$\Delta H_j^t = \sum_{i=1}^{j} \left[\left(h_i^t - h_i^0 \right) - K^t \cdot D_i \right], \ \left(i = \overline{1, n};\ j = \overline{1, n-1} \right), \tag{15}$$

where: h_i^0 and h_i^t represent the provisional differences in elevation measured in the initial and present cycles; D_i – the length of the aim; K^t – the unitary correction, given by

$$K^t = \frac{\sum_{i=1}^{n} \left(h_i^t - h_i^0 \right)}{\sum_{i=1}^{n} D_i}. \tag{16}$$

Since at the measuring of the vertical deformations of the constructions the station points are materialized, remaining the same during in all the observation periods, that means that length of the aims remain unchanged. Therefore it disappears also the operation of calculating the parallactic angle *ε*, and also of the deviations $\delta h_{jk}^{'}$ and $\delta h_{jk}^{''}$ for each intermediate mark. Thus for the intermediate mark, *k*, the value of the vertical deformation of the construction, between two cycles of observations, will be:

$$\Delta H_k^t = H_k^t - H_k^0 = \frac{1}{2}\left[\left(H_j^t + h_{jk}^{',t} - \delta h_{jk}^{',t}\right) + \left(H_j^t + h_{jk}^{",t} + \delta h_{jk}^{",t}\right)\right] - \\ - \frac{1}{2}\left[\left(H_j^0 + h_{jk}^{',0} - \delta h_{jk}^{',0}\right) + \left(H_j^0 + h_{jk}^{",0} + \delta h_{jk}^{",0}\right)\right], \tag{17}$$

or finally

$$\Delta H_k^t = H_j^t - H_j^0 + \frac{1}{2}\left[\left(h_{jk}^{',t} - h_{jk}^{',0}\right) + \left(h_{jk}^{",t} - h_{jk}^{",0}\right) + \left(\delta h_{jk}^{',0} - \delta h_{jk}^{',t}\right) + \left(\delta h_{jk}^{",t} - \delta h_{jk}^{",0}\right)\right]. \tag{18}$$

In the above written relation, the difference between the first two terms $\left(H_j^t - H_j^0\right)$ was calculated using formula (15) and represents the vertical deformation of the connection mark *j*. Also, by keeping unchanged the position of the station points, the deviations $\delta h_{jk}^{'}$ and $\delta h_{jk}^{"}$ from each period will remain the same, their influence being canceled by subtraction

$$\delta h_{jk}^{',0} - \delta h_{\mathrm{jk}}^{',\mathrm{t}} \approx 0;\ \delta h_{jk}^{",t} - \delta h_{jk}^{",0} \approx 0. \tag{19}$$

The value of the vertical displacement and deformation, measured into the intermediate mark, will have the following expression:

$$\Delta H_k^t = \Delta H_j^t + \frac{1}{2}\left[\left(h_{jk}^{',t} - h_{jk}^{',0}\right) + \left(h_{jk}^{",t} - h_{jk}^{",0}\right)\right], \tag{20}$$

where: ΔH_j^t is the vertical deformation of the connection mark from the *t* period; $h_{jk}^{',t}$ and $h_{jk}^{',0}$ – the differences in elevation measured in station S_i for the present and initial period; $h_{jk}^{",t}$ and $h_{jk}^{",0}$ – the differences in elevation measured in the station S_{i+1}. Making the notations:

$$\Delta h_k^{'} = h_{jk}^{',t} - h_{jk}^{',0},\ \ \Delta h_k^{"} = h_{jk}^{",t} - h_{jk}^{",0}, \tag{21}$$

we can write the final calculation formula of the vertical deformation of the intermediate mark, *k*, for the *t* period:

$$\Delta H_k^t = \Delta H_j^t + \frac{1}{2}\left(\Delta h_k^{'} + \Delta h_k^{"}\right). \tag{22}$$

3. The Mathematical Model for Estimating the Precision of the Vertical Deformations

For studying the behaviour of the buildings *in situ* is represented by the geodetic methods with whose aid one can establish the absolute values of the vertical and horizontal displacements and deformations depending on the fixed points and marks of a reference network. The geodetic methods for determining the vertical deformations, the one the high precision geometrical leveling ensures the highest accuracy.

In what follows the mathematical model for estimating the precision of the vertical deformations (settlements and erections) determined in the intermediate marks is studied. The estimation of the precision as exactly as possible is made using then mean square errors, as follows.

For each station, the difference in elevation between marks *j* and *k* is made relying on the readings made on the base and supplementary scales of the invar rod tape. For instance, for station S_i, it will result:

$$h_{jk}^{',b} = L_{0,j}^{b} - L_{0,k}^{b}\,;\; h_{jk}^{',s} = L_{0,j}^{s} - L_{0,k}^{s}\,. \tag{23}$$

The mean square errors of the difference in elevation will be calculated using the formula of function of directly measured independent value [1], [4], [5] resulting thus:

$$s_{h_{jk}^{',b}} \approx s_{h_{jk}^{',s}} = \pm s_L\sqrt{2}, \tag{24}$$

where s_L is the mean square error a reading on the rod.

In each observation period, the difference in elevation is obtained as a mean of the values resulted from (23). In station S_i, for the first horizon of the instrument it will result:

$$h_{jk}^{'',\mathrm{I}} = \frac{1}{2}\left(h_{jk}^{',b} + h_{jk}^{',s} \right). \tag{25}$$

The mean square error of the difference in elevation (25), can be expressed as:

$$s_{h_{jk}^{',\mathrm{I}}}^{2} = \frac{1}{4}s_{h_{jk}^{',b}}^{2} + \frac{1}{4}s_{h_{jk}^{',s}}^{2} \approx \frac{1}{2}s_{h_{jk}^{',b}}^{2} \Rightarrow s_{h_{jk}^{',\mathrm{I}}} = \frac{s_L\sqrt{2}}{\sqrt{2}} = s_L\,. \tag{26}$$

During each observation period, the provisional difference in elevation is obtained as a mean of the values obtained with two horizons of the instrument

$$h_{jk}^{'} = \frac{1}{2}\left(h_{jk}^{',\mathrm{I}} + h_{jk}^{',\mathrm{II}} \right). \tag{27}$$

The mean square error of the difference in elevation (27) is:

$$s_{h_{jk}^{'}} = \pm\frac{s_L}{\sqrt{2}}. \tag{28}$$

The partial vertical deformation, measuring in one station, is obtained as a difference between the difference in elevation measured in the two cyclical observation with (21). For example, for station S_i, will result:

$$\Delta h_{k}^{'} = h_{jk}^{',t} - h_{jk}^{',0}. \tag{29}$$

The mean square error of the vertical deformation will be given as:

$$s_{\Delta h_k^{'}}^2 = s_{h_{jk}^{',t}}^2 + s_{h_{jk}^{',0}}^2 \approx 2s_{h_{jk}^{',0}}^2 \approx 2\frac{s_L^2}{2} \Rightarrow s_{\Delta h_k^{'}} = \pm s_L. \tag{30}$$

The average partial vertical deformation is obtained with reference to the values determined from the conjugated station, S_i and S_{i+1}, will result:

$$\Delta h_k^t = \frac{1}{2}\left(\Delta h_k^{'} + \Delta h_k^{''}\right). \tag{31}$$

The mean square error of the average partial deformation will be:

$$s_{\Delta h_k^t}^2 = \frac{1}{4}s_{\Delta h_k^{'}}^2 + \frac{1}{4}s_{\Delta h_k^{''}}^2 \approx \frac{1}{2}s_{\Delta h_k^{'}}^2 \Rightarrow s_{\Delta h_k^t} = \pm\frac{s_L}{\sqrt{2}}. \tag{32}$$

The mean square errors of the vertical deformation, determined in the intermediate mark *k*, with reference to the connection mark *j*, and computed with relation (22), will be expressed as:

$$s_{\Delta H_k^t}^2 = s_{\Delta H_j^t}^2 + s_{\Delta h_k^t}^2 \Rightarrow s_{\Delta H_k^t} = \sqrt{s_{\Delta H_j^t}^2 + \frac{s_L^2}{2}}. \tag{33}$$

For the above written formula, the mean square error of the deformation for connection mark *j*, is calculated using the expression proposed by Gh. Nistor [1], [7]:

$$s_{\Delta H_j^t} = \pm\frac{1}{2n}\sqrt{n_j(n-n_j)\sum_{i=1}^{n}\left(d_i^0 + d_i^t\right)} \tag{34}$$

where: *n* is the total number of station points; n_j – the number of station points between the initial support mark R_1 and the connection one, *j*; d_i^0 and d_i^t – the deviations between the values of the difference in elevation which have been determined with the two horizons of the instrument for each station point.

For example, during a supported traverse with a number of eigh connection marks [1], the mean square error of the vertical deformation for the most exposed mark – at the middle of the traverse – was, $s_{\Delta H_j} = \pm 0.19$ mm. Being given the mean square error of a reading on the rod as $s_L = \pm 0.12$ mm, there it resulted the mean square error of the vertical deformation as $s_{\Delta H_k} = \pm 0.208$ mm which is quite close to the one of the connection mark. When the connection marks are measuring with two horizons of instrument with the intermediate marks using only one horizon, the mean square error of the vertical deformation from the intermediate mark *k* will be obtained as:

$$s_{\Delta H_k^t} = \pm\sqrt{s_{\Delta H_j^t}^2 + s_L^2}. \tag{35}$$

Finally, the confidence interval within whose limits the true magnitude of the vertical displacements and deformation from the intermediate mark *k*, will be found can be expressed as [2],

$$\Delta H_k^t - s_{\Delta H_k^t} \leq \overline{\Delta H_k^t} \leq \Delta H_k^t + s_{\Delta H_k^t}. \quad (36)$$

For the same example the average error will be $s_{\Delta H_k} = \pm 0.225$ mm. The difference between them is very small, one being thus able to renounce the measuring of the intermediate marks with two horizons, so that, the enlargement of the precision is obtained only by measuring from two conjugated station points and using only one horizon.

4. Conclusions

From the effectuated analyse there are emerging the following conclusions:

1. Using the presented method one can reduce the number of the leveling station points as opposite to the case when all the settlement marks would be considered as connection marks.

2. One can be certain ones the correctness of measuring the difference in elevation the connection mark and the intermediate ones from the two conjugated station points.

3. By using the Gh. Nistor algorithm one can eliminate the necessity of determining the parallactic angle ε which is a consequence of the from not absolute parallelism between the axis of the level tube and line of sight and, implicitly, the linear deviations $\delta h'$ and $\delta h''$ for each mark, increasing thus the efficiency of the working operations.

4. The analysis of the precision of determination of the vertical displacements and deformations measured in the intermediate marks, relying on formulas (15) and (17), makes clear the idea according to which, using the technology described in [1], one will obtain a considerably equal precision.

5. The study of the precision, as described in (15) and (17), confirms entirely that, by using the measurement technology from two conjugated station points, one will obtain a precision which is quite close to the one obtained for the connection marks.

6. Also, within common practice of the field measurements, for each leveling station point, the readings on the rod into the connection marks are made with two horizons of the instrument while for the intermediate marks, with only one horizon. This is why, a great volume of work and time is saved since the number of the intermediate marks is the biggest.

7. According to the proposed estimating model one can predict which will be the values of the mean errors in the marks of the buildings which are mostly affected by deformations and will be able to compare them to the necessities individually required by each construction.

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