GEODESY AND ALGORITHM METHOD FOR DETERMINING THE GEOMETRICAL AXES INCLINATION OF TALL BUILDINGS, WITH CIRCULAR SECTIONS

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***Abstract**: Determination of tall buildings inclination is needed to study the character of deformations and explain the causes that they may induce, as well as to identify the measures to be taken to ensure the construction's stability and operation under safe conditions of. The paper presents a geodetic method for determining and monitoring in time the inclination of a chimney on an industrial site of a metallurgical trade company. The paper presents the measurement technology, and the procedure to determine the inclination vector, its orientation in plane, the mathematical model to assess the reliability of results obtained and indication of the optimal working conditions.*

***Keywords**: in situ behaviour of constructions, determination of tall buildings inclination, the inclination vector, deformations.*

1. Introduction

During the selection of the optimum solutions for the design and the rational, safe operation of buildings, a major role is played by a thorough-going study, which must be performed at the beginning of the process, both in the drafting and the designing phases, and at the end, in the phases of building verification, execution and operation. Experimental analysis of constructions behavior, performed by using models or on the stand, as well as tests on real buildings, aim at improving the calculation methods, monitoring them during execution and operation. Besides experimental data, the study and the analysis of the *in situ* behavior, under execution and, particularly, under operation, can lead to more information.

The changes in the position and shape of the ensemble or its components, as well as the observation of the occurrence of evolutive phenomena which might affect the operation safety are established through the *in situ* monitoring. In the study of constructions, geodetic methods are used with remarkable results in all stages.

In time, tall tower buildings, including tall buildings with circular cross sections (chimneys, cooling towers etc.), are affected by inclinations against the vertical axis, due to complex causes, namely the differentiated settlings of foundations and soil, the non-uniform heating of the construction by the sun rays, the action of strong winds, etc.

Inclinations occur in different directions and with different magnitudes. Therefore, during execution and after the release to service, it is essential to perform cyclic geodetic observations on the verticality of geometric axes, in order to verify the execution accuracy, as well as to monitor the inclination of the geometric axis against the vertical axis.

2. Cyclic determination of the chimney inclination by linear and angular geodetic measurements, as against a basis of fixed length

This paper presents a method for the cyclic determination of the chimney inclination by means of linear and angular geodetic measurements as against a basis of fixed length, placed at distances of 50 to 200 m from the building, in accordance with the actual conditions in the area [3]. The ends of the basis, represented by the points A and B (Figure 1 - vertical plane), are placed on solid grounds and outside the influence area of the building. These points have been materialized by metallic bolts (lathe processed bolt lags, at the upper part); if monitored for a longer period of time, the materialization can be also realized with small reinforced concrete marks, ensuring thus their protection between the cycles of measurements.

Since after the execution it is virtually impossible to materialize the control points, for example points P and Q, on the chimney generator at its lower part and especially at its upper part, these points are identified on the chimney surface. As against these points, the visas will be given as tangents to the chimney, to the right and to the left (Figure 1 - horizontal plane), from the station points A and B (P_s, P_d, Q_s, Q_d etc.).

After the materialization of the end points A and B, the basis length, $\overline{AB} = b$, was measured electronically several times in both directions, by means of a Leica TCR 405 high performance total station, considering the mean of the determinations as the final value. It must be mentioned that the basis length is chosen so that the angle between the visas from the station points to the points on the chimney should be approximately a right angle.

The angular values of the azimuthal directions were measured from each station point by means of a high performance total station, using the simple method in both positions of the field glass, considering the average values of the positions (Tables 1 and 4). The reference visa was given towards the opposite point of the basis, then the measurements to the points tangential to the chimney to the left and to the right, in the lower and upper parts, were performed.

After the compensation of the azimuthal measurements in the station points, the horizontal angles α_P, α_Q *and* β_P, β_Q, were measured from the basis and from the upper part (Tables 2 and 5), angles which materialize the directions/straight lines at whose intersection there are the centres of the circular sections of the chimney, using the following formulas:

$$\alpha_P = \frac{1}{2}\left(P_s + P_d\right),\ \alpha_Q = \frac{1}{2}\left(Q_s + Q_d\right) \tag{1}$$

$$\beta_P = \frac{1}{2}\left(P_s^{'} + P_d^{'}\right),\ \beta_Q = \frac{1}{2}\left(Q_s^{'} + Q_d^{'}\right), \tag{2}$$

If the geometrical axis of the chimney is vertical, points P and Q will be situated on the same vertical; therefore the following equations of the horizontal angles between the fixed basis and the direction to the centres of the circular sections from the basis and from the top can be established:

$$\alpha_P = \alpha_Q\ ,\ \beta_P = \beta_Q\ . \tag{3}$$

If the horizontal angles between the fixed basis $\overline{AB}$ and the directions to the points from the basis and from the upper part of the chimney, $\overline{AP}$, $\overline{AQ}$, $\overline{BP}$ and $\overline{BQ}$, are not equal, $\alpha_P \neq \alpha_Q$ and $\beta_P \neq \beta_Q$, it means that the geometrical axis of the chimney is tilted against the vertical axis. The may be caused due to (or during) the construction process, when it is determined at the end of the construction works and the structure's release to service, or it may occur after a certain period of time, owing to the interaction between the chimney and the foundation ground, to the regime of the groundwater and to other causes.

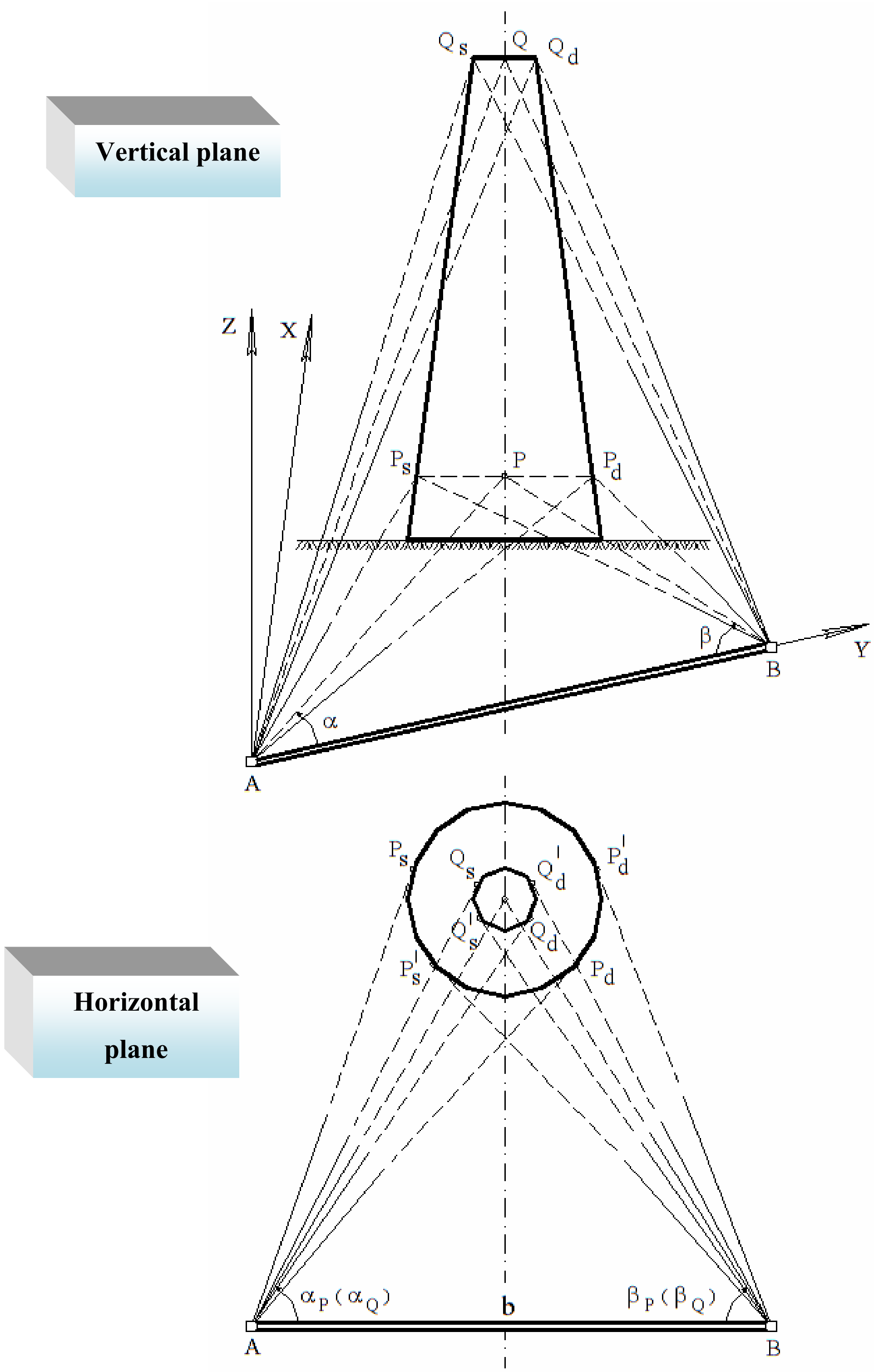

Fig. 1. Determination of the chimney inclination vector

The determination of the components of the chimney inclination vector using the Gh. Nistor method [1] has been performed in direct accordance with the differences between the horizontal angles, measured from the station points

$$\Delta\alpha^{cc} = \alpha_Q - \alpha_P \ , \quad \Delta\beta^{cc} = \beta_Q - \beta_P \ , \tag{4}$$

in a system of rectangular in plane axes, with the origin in point A, in which Y-axis coincides with the basis of fixed length, using the following relations:

$$\Delta X = K_2 \cdot \Delta\alpha^{cc} + L_2 \cdot \Delta\beta^{cc} \tag{5}$$

$$\Delta Y = -K_1 \cdot \Delta\alpha^{cc} + L_1 \cdot \Delta\beta^{cc} \ . \tag{6}$$

In the above relations the following physical quantities were calculated:

$$G = b / \rho^{cc} \sin^2(\alpha_P + \beta_P) \tag{7}$$

$$K_2 = G \sin^2 \beta_P, \quad L_2 = G \sin^2 \alpha_P \tag{8}$$

$$K_1 = G \sin \beta_P \cos \beta_P, \quad L_1 = G \sin \alpha_P \cos \alpha_P \ . \tag{9}$$

The physical quantities K_1, L_1, K_2 and L_2 are calculated on the basis of measured horizontal angles between the fixed basis and the directions to the points from the building basis, using the relations (8) and (9), the results being listed in Tables 2 and 5, and, in Tables 3 and 6 respectively, with a changed sign. When monitoring the inclination for a longer period of time, these physical quantities remain constant for all the cycles of measurements, while the only variables being the angular differences, $\Delta\alpha^{cc}$ *and* $\Delta\beta^{cc}$. It is worth mentioning that, in relations (5) and (6), the angular differences are expressed directly in centesimal / hexadecimal seconds, because the modulus of transformation in radians was introduced in the dimensions of the constants ($\rho^{cc} = 636620\ ^{cc} / \rho^{"} = 206265^{"}$).

The vector size is calculated by means of the inclination vector components, by the following relation:

$$L = \sqrt{\Delta X^2 + \Delta Y^2} \tag{10}$$

and its in plane orientation, in keeping with X-axis, resulting thus:

$$\theta_L = arc\,tg\left(\frac{\Delta Y}{\Delta X}\right), \text{ where } \theta \in [0, 2\pi] \ . \tag{11}$$

Based on the angular differences between the directions to the sections centres, $\Delta\alpha^{cc}$ *and* $\Delta\beta^{cc}$, and on the constants sizes, in Tables 3 and 6 the magnitudes of components ΔX and ΔY were calculated using the relations (5) and (6). Finally, the linear size of the chimney inclination vector and its in plane orientation (Tables 3 and 6) were calculated using the relations (10) and (11), obtaining thus the following values:

- For the initial cycle, 05.09.2007: $L_1 = 117.55mm$, $\theta_{L_1} = 28^g, 02$.
- For the current cycle, 12.10.2007: $L_2 = 119.81mm$, $\theta_{L_2} = 28^g, 01$.

When monitoring the chimney inclination for a longer period of time, by means of formulae (5), (6), (10) and (11), one can either calculate the total inclination vectors, when the angular differences are between the current and the initial cycles, or the partial vectors, resulting between two conjugated cycles, the previous one and the current one.

Table 1. The cycle on 05.09.2007 - azimuthal directions

Station point	Target point	Azimuthal directions Position I	Azimuthal directions Position II	Mean of the positions	Station point	Target point	Azimuthal directions Position I	Azimuthal directions Position II	Mean of the positions
A	B	23.1301	223.1295	23.1298	B	A	356.9675	156.9663	356.9669
	S down	344.4451	144.4435	344.4443		S down	4.4559	204.4600	4.4580
	S up	345.1867	145.1883	345.1875		S up	5.1836	205.1863	5.1850
	D down	352.5367	152.5354	352.5361		D down	10.7134	210.7113	10.7124
	D up	351.8059	151.8115	351.8087		D up	10.2598	210.2628	10.2613

Table 2. The determination of the constants

Station point	Control Point	Azimuthal directions	α β α+β	sin α sin β sin(α+β)	cos α cos β cos(α+β)	b (mm) ρ^{cc} G (mm)	Constants sizes K_1	L_1	K_2	L_2
A	P	348.4902	74.6396	0.921698	0.387907					
	Q	348.4981	50.6183	0.713940	0.700207	50888.5				
	Sum		125.2579	0.922322	-0.386423	636620	0.0470	0.0336	0.0479	0.0798
B	P	7.5852	74.6317	0.921650	0.388022	0.0940				
	Q	7.7231	50.7562	0.715456	0.698658					
	Sum		125.3879	0.921531	-0.388306					

Table 3. The determination of the inclination vector and its orientation

Control point	α α^1	Δα ^{cc}	β^0 β^1	Δβ ^{cc}	Constants sizes (-K1)	L1	K2	L2	Vector components ΔX	ΔY	Value of the horizontal displacement	
P	74.6396		50.6183									
Q	74.6317		50.7562		-0.0470	0.0336	0.0479	0.0798	106.35	50.08	**117.55**	**28.02**
		-79.25		1379.75								
											L (mm)	θ$_L$(g)

Table 4. The cycle on 12. 10. 2007 - azimuthal directions

Station point	Target point	Azimuthal directions Position I	Azimuthal directions Position II	Mean of the positions	Station point	Target point	Azimuthal directions Position I	Azimuthal directions Position II	Mean of the positions
A	B	16.0662	216.0644	16.0653	B	A	337.8894	137.8904	337.8899
	S down	337.3683	137.3649	337.3666		S down	385.3787	185.3762	385.3775
	S up	338.1180	138.1103	338.1142		S up	386.1080	186.1090	386.1085
	D down	345.4784	145.4803	345.4794		D down	391.6329	191.6339	391.6334
	D up	344.7463	144.7496	344.7480		D up	391.1849	191.1823	391.1836

Table 5. The determination of the constants

Station point	Control Point	Azimuthal directions	α β α+β	sin α sin β sin(α+β)	cos α cos β cos(α+β)	b (mm) ρ^{cc} G (mm)	Constants sizes K_1	L_1	K_2	L_2
A	P	341.4230	74.6423	0.921715	0.387868					
	Q	341.4311	50.6155	0.713910	0.700237	50888.5				
	Sum		125.2579	0.922322	-0.386422	636620	0.0470	0.0336	0.0479	0.0798
B	P	388.5054	74.6343	0.921666	0.387985	0.0940				
	Q	388.6461	50.7562	0.715455	0.698658					
	Sum		125.3904	0.921515	-0.388342					

Table 6. The determination of the inclination vector and its orientation

Control point	α α^1	Δα ^{cc}	β^0 β^1	Δβ ^{cc}	Constants sizes (-K1)	L1	K2	L2	Vector components ΔX	ΔY	Value of the horizontal displacement	
P	74.6423		50.6155									
Q	74.6343		50.7562		-0.0470	0.0336	0.0479	0.0798	108.39	51.03	**119.81**	**28.01**
		-80.75		1406.25								
											L (mm)	θ$_L$(g)

Figure 2 shows the chimney inclination vector and the orientation of the inclination direction against the X-axis of the chosen system of axes.

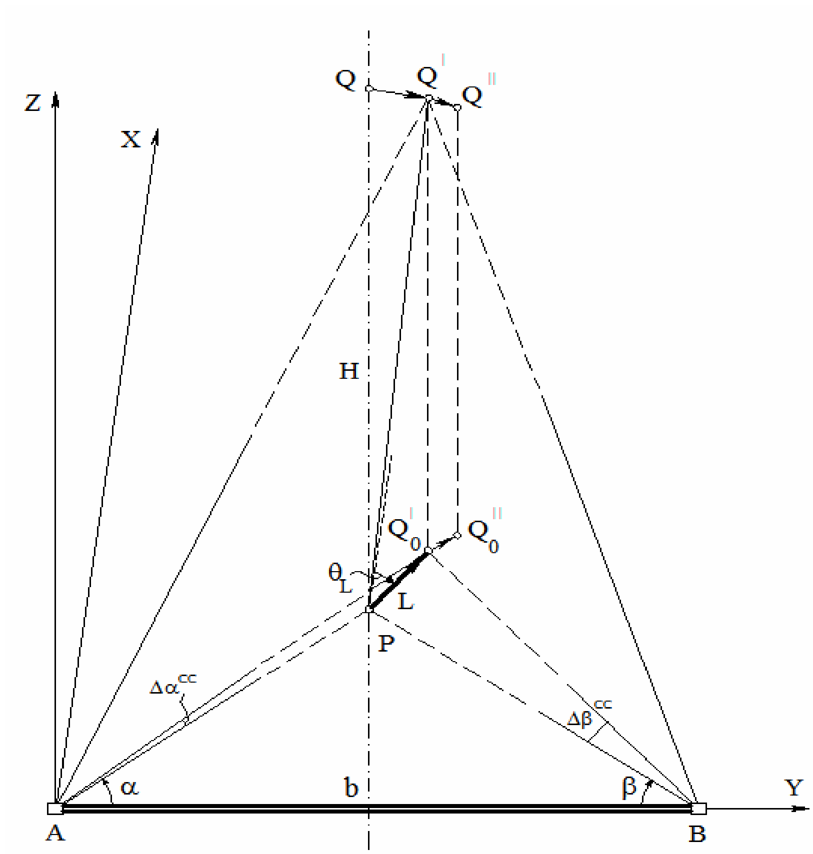

Fig. 2. The inclination vector and its orientation

3. Assessment of the results accuracy

In order to assess the results accuracy, it is premised that the sizes K_1, L_1, K_2 and L_2 are constant for all the cycles of measurements, and the measurement errors of the angular differences have a fundamental influence on the results. The squares of the mean errors of the inclination vector components are expressed using simplified formulae [1], calculated in accordance with the variable sizes of the angular differences, $\Delta\alpha^{cc}$ *and* $\Delta\beta^{cc}$. Therefore:

$$s_{\Delta X}^2 = K_2^2\, s_{\Delta\alpha}^2 + L_2^2\, s_{\Delta\beta}^2 \tag{12}$$

$$s_{\Delta Y}^2 = K_1^2\, s_{\Delta\alpha}^2 + L_1^2\, s_{\Delta\beta}^2 \quad , \tag{13}$$

where the squares of the angular differences errors depend on the errors of the horizontal angles measured from the station points:

$$s_{\Delta\alpha}^2 = s_{\alpha_P}^2 + s_{\alpha_Q}^2\,, \quad s_{\Delta\beta}^2 = s_{\beta_P}^2 + s_{\beta_Q}^2 \quad . \tag{14}$$

The square mean error of the inclination vector will be calculated by the relation

$$s_L = \pm\sqrt{s_{\Delta X}^2 + s_{\Delta Y}^2} \tag{15}$$

The angles measured in the station points being obtained with the same accuracy, the angular differences will have the errors:

$$s_{\Delta\alpha} \approx s_{\Delta\beta} \approx \pm s_{\alpha}\sqrt{2} \quad . \tag{16}$$

In this case, the mean square error of the chimney inclination vector will be calculated by the relation:

$$s_L = \pm s_{\Delta\alpha}\sqrt{K_1^2 + K_2^2 + L_1^2 + L_2^2} \quad , \tag{17}$$

with a value $s_L = \pm 0,68\,mm$.

The confidence interval, within whose limits the real inclination vector size will be situated, with a probability of P = 68.3%, will be expressed by the following double inequality:

$$L - s_L \le \overline{L}_{68.3\%} \le L + s_L \quad . \tag{18}$$

4. Determination of the chimney height

The unknown chimney height, necessary for the rehabilitation works, can be determined either from one or both station points, in order to control the determination and to obtain a superior accuracy of the determined value. The determination of the chimney height from one station point, for example A, is performed as follows [3] (Figure 3):

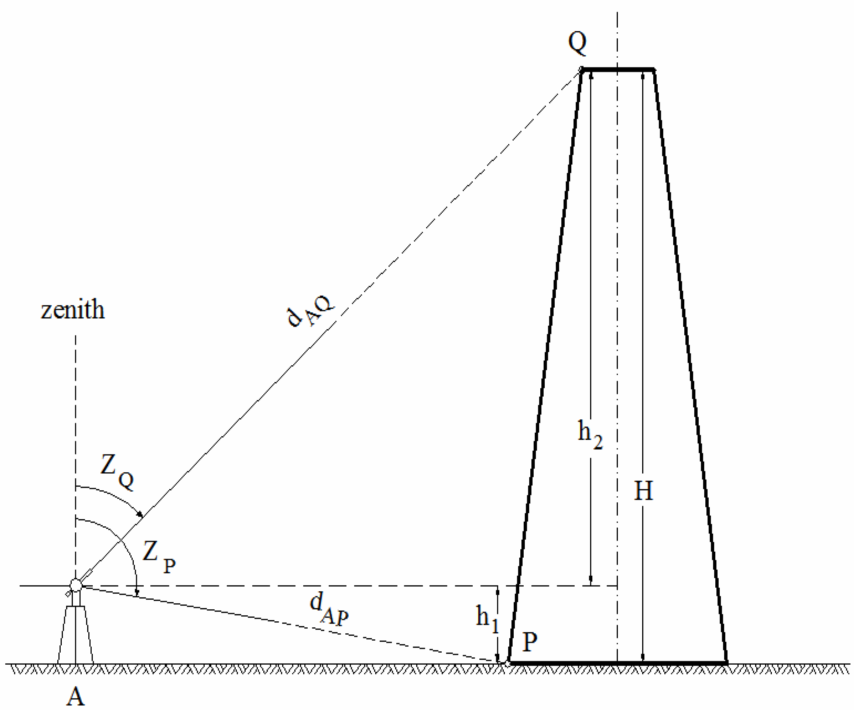

Fig. 3. Determination of the chimney height

The total station is installed in the point A and centred while aiming at the middle of chimney, to the upper part Q, and reading the zenith angle, Z_Q. The field glass is tilted, aiming at point P, from the base of the chimney, while reading the zenith angle Z_P. Each time, the tilted distances measured by laser, d_{AQ} and d_{AP}, will be read simultaneously with the sighting. Based on these measurements, the heights are calculated:

$$h_1 = d_{AP}\cos(200^g - Z_P), \quad h_2 = d_{AQ}\cos(200^g - Z_Q) \quad , \tag{19}$$

after which the chimney height will then be:

$$H = h_1 + h_2 \quad . \tag{20}$$

For control purposes, the height can be determined similarly from the station point B. It is worth mentioning that the tilted distances and the zenith angles have also been measured with the second position of the field glass, using average values for the calculus, the resulting chimney height being H = 32.31m.

5. Conclusions

It is possible for the relative small inclination, not raising operation safety problems, to have existed since the end of the execution, including the rehabilitation works.

The results accuracy matched the aimed objective, the square mean error of the chimney inclination vector being $s_L = \pm 0,68\,mm$.

Determination of the chimney axis inclination vector from the vertical, with an accurate method, such as the trigonometrical method, was absolutely necessary, in order to find the inclination amplitude after the shortening and the reinforcement of the chimney, so that it could be monitored in time as to its size and direction.

According to the data from the determinations performed during these last two cycles of measurements, it resulted that the magnitude of the chimney inclination vector is L = 119.81 mm, with an orientation (in plane) of 28^g against the X-axis of the rectangular system, the inclination being directed towards the two buildings in close vicinity.

It is worth mentioning that an inclination of approximately 12 cm is acceptable and does not imply major risks. In case of a powerful earthquake, the behaviour of the chimney will depend preponderantly on its resistance structure and on the quality of the rehabilitationworks.

Because when determining the chimney inclination it cannot be foreseen in which direction the inclination against the vertical axis will occur, it is recommended the fixed basis to be chosen such that the lengths of the determination straight lines be almost equal and the angle between the two determination straight lines be approximately a right one. In this case, the pedal curve degenerates into an error circle, so that the determination errors of the inclination vector will be identical, irrespective of the inclination direction.

6. References

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