THE DESCRIPTION OF THE MAIN TECHNIQUES OF RESOLVING THE PHASE AMBIGUITIES, THE ADVANTAGES, DISADVANTAGES AND THEIR CHARACTERISTICS

Florin Gabriel PAIS, Eng. - National Centre of Geodesy, Cartography, Photogrammetry and Remote Sensing, florinpais@yahoo.com

Abstract: *The paper presents the main techniques of resolving the phase ambiguities as well as the advantages, disadvantages and their characteristics. The characteristics are presented in terms of the used positioning method, conditions and periods of observations, the used method for calculating the initial solution, the type of observations used for finding the initial solution, etc.*

Keywords: *GPS, phase ambiguity, techniques.*

1. Introduction

The phase ambiguity is the initial unknown integer number of wavelengths, from the distance satellite-receiver, which is not measured by the receiver. In general, the resolution of phase ambiguity is necessary for the fast and high precision relative positioning. The process of fixing the ambiguities is to transform the estimations of the real value into integer value ambiguities.

In the course of time we tried to assess and resolve these ambiguities developing certain techniques or algorithms. There are several techniques for resolving phase ambiguity based on the used method: static, rapid static or kinematic. The development of different techniques of resolution of phase ambiguity began twenty years ago. In this article we are presenting certain techniques such as:

- ⇨ Fast ambiguity resolution approach (FARA) (Frei, 1991)
- ⇨ Fast ambiguity search filter (FASF) (Chen and Lachapelle, 1994)
- ⇨ Least squares ambiguity decorrelation adjustment method (LAMBDA) (Teunissen, 1996)
- ⇨ Least squares ambiguity search technique (LSAST) (Hatch, 1990; 1991)

All the techniques presented in the tables of advantages, disadvantages and characteristics are described below. These techniques have approximately the same characteristics.

2. Methods of resolving phase ambiguities

2.1 Fast Ambiguity Resolution Approach (FARA) Method

The steps to resolve phase ambiguities by this method are: the calculation of float solution of the carrier phase, the choice of sets of ambiguities to be tested, the calculation of the fixed solution for each set of ambiguity and finally the test for the fixed solution with the least square method. This technique is based on the results of the initial solution: the vector solution, the matrix of variance-covariance and the posterior variance factor. To generate series of combinations of integer values ambiguities to be tested, we are using a search algorithm which is based on statistical concepts. For example, a part of the vector solution, containing the real values of phase ambiguities is x(xi,……,xu)T

Thus, the standard deviation ei of the parameter of ambiguity xi, can be calculated with the following formula:

$$e_i=\sigma_0 \cdot \sqrt{Q_{ii}} \quad (1)$$

where σ_0^2 – represents the posteriori variance factor

Q_{ii} – represents the matrix of variance-covariance

According to a confidence level α, the algorithm computes the superior and inferior limits of the search space of integer phase ambiguities x_{Ai} using the following formula:

$$x_i - \varepsilon \cdot e_i \leq x_{Ai} \leq x_i + \varepsilon \cdot e_i \quad (2)$$

In this case there are calculated all possible combinations of integer values that satisfy the conditions of equation (2).

Thus, several vectors of ambiguities are formed. The final solution consists in choosing the vector of ambiguities with the smallest standard deviation. This vector must have a single standard deviation (there are two combinations with the same standard deviation). In this case, the standard deviation is close to σ_0. The FARA method needs phase carrier data. It is using double difference code observations to find an initial solution.

2.2 Least Square Ambiguity Search Technique (LSAST) Method

The steps to resolve phase ambiguities by this method are: the estimation of initial coordinates of unknown point, then sets of ambiguities around the initial point are defined and finally we apply the ambiguity search algorithm by means of least squares method. This method is one of the phase ambiguity resolution techniques called On-the-Fly. The technique consists in fixing the ambiguities when the receiver antenna is in motion.

This technique is based on the residual to research and evaluate the right combination of ambiguities that will be used for calculating the final solution [Hatch 1990]. In this case, an initial solution is calculated, using the pseudorange observations. Thus, we are establishing a search space associated with this position in which different combinations of integer ambiguities are tested. This solution is calculated by simple difference between stations.

Also, to reduce the search space and validate phase ambiguities, the phase measurements are divided into a set of primary measures and a set of secondary measures. For the first set corresponds a primary group of satellites and for the second set corresponds a secondary set of satellites. We can make this division only if the number of observed satellites is greater than four. The float solution is computed using only four satellites (the primary satellites). Then, we are going to solve the ambiguities for the secondary satellites using the computed position of the primary satellites.

Then we will calculate the residuals for each combination and we will make a discrimination and validation test. The phase measurements of the first set define the search space while the phase measurements of the second set are used to validate the ambiguities. LSAST test the set of ambiguities at successive epochs. So, in each epoch, the sets of ambiguities that are tested remain the same, but the determinations of position and residuals are changing. The set of ambiguities with the smallest variation factor is the right set.

2.3 Modified Cholesky Decomposition Method

This technique was developed by Landau and Euler (1992). It is slow for real time applications when the speed of data collection is larger than 1Hz. Also, the time to look for the ambiguities during the initial epochs is much greater. This method is based on the decomposition of matrices. The total number of possible combinations of phase ambiguities is 21^5=4084101 if there are six satellites. The five ambiguities of double-difference have an uncertainty of ± 10 cycle.

2.4 On-The-Fly (OTF) Method

The On-The-Fly phase ambiguity resolution is affected by many factors such as the satellite geometry and the distance between the fix and mobile receiver. In general, these effects are sought and used to reduce the time needed to resolve ambiguities. In this technique of resolution of phase ambiguity in motion OTF, we calculate an approximate value of phase ambiguities starting from the coordinates obtained by pseudorange processing.

Then, we are performing tests with different integer values and we choose the combination of ambiguities that meets a series of statistical tests and validation criteria. There are some differences between the methods « on-the-fly » but they have the same target: to establish the right procedure of finding the right combination of ambiguities. These differences consist in: the way to validate or reject combinations of ambiguities, the number or sequence of applied criteria and finally the criteria used in the process to stop the searches of ambiguity combinations. If we are using dual frequency receivers, the resolution of phase ambiguity is very fast, in a few epochs.

Also, if the real solution of the ambiguity is close to an integer, the chance to properly assess this amount is large. The correct solution in the search space is identified using methods of search by least squares. Abidin has developed the OTF algorithm following the same procedures as the LSAST method. This method uses a greater number of validation tests.

2.5 Fast Ambiguity Search Filter (FASF) Method

This method is based on a technique called Kalman filter and a process of research at each epoch. An important feature to select the search method is the number of points that should be sought. Also, this method is affected by cycle slip. The only method that is not affected by cycle slips is AFM (Ambiguity function method). The method has an important feature: the ambiguities research areas are determined recursively and they are connected to each other.

Thus, the ambiguities are related to each other and considered as a sequence. If we are using dual frequency receivers, the method will work very well thanks to a small research area. Generally, to avoid the ambiguities that are not necessary and to have a bigger chance to fix them we must use a maximum number of possible solutions as a threshold. For example, if we have a great number of sets of phase ambiguities, the chance to have a failure of ambiguity resolution is great.

2.6 Least squares AMBiguity Decorrelation Adjustment (LAMBDA) Method

It’s a method developed for solving on-the-fly phase ambiguities, based on the estimation of integers. The efficiency of calculation is obtained by decorrelating integers. It is important for applications that rely on short observation time and very large ambiguity research areas. This method aims to divide the states of ambiguities and covariance matrix. The target of division consists in transforming an area of research in another area more efficient.

This algorithm reduces the search space of integer values using the correlation structure of float solutions. The main effect of this method is to transform the states of ambiguity so that the correlation between them (remain) be minimum. Thus, the number of ambiguities is kept but they can be found more efficient. In this case, the elongated ellipsoid is transformed into a spheroid. A float solution is calculated with the method of least squares or Kalman filter.

Then we define an area to search all possible combinations. To find the right set of ambiguities we must reduce the number of combinations. In this case, the computation time to search the integer ambiguity is inversely proportional to the number of analyzed combinations.

Then we can obtain the estimated integer ambiguity from the float solution and its estimated covariance matrix. Finally, we can calculate the fixed solution.

The LAMBDA method creates combinations of ambiguities which are based on accurate measurements. The main advantages of this method are: the reduction of the number of possible ambiguities combinations and the ability to fix the ambiguities (of finding a solution) in only one epoch using code and phase measurements, but treated separately. This method is considered the best because it offers the highest probability of finding (produce) the correct integer values for ambiguities. Therefore, the LAMBDA method is the right method for resolving the phase ambiguities and very efficient for applications in real time.

Table 1: Advantages and disadvantages and of the phase ambiguity resolution

Method	***Advantages***	***Disadvantages***
FARA Method Frei et Beutler (1990)	*Correlation between the values of ambiguities to minimize the number of epochs required*	*This technique is affected by cycle slips compared to AFM and the grid of possibilities is not as dense as AFM (Ambiguity function method)*
LSAST Method Hatch (1991)	*Potential solutions of the primary group that does not satisfy the observations of the secondary group are immediately rejected. This method is more efficient than any other method that looks for all possible ambiguities for all satellites. In this case only the combinations of primary satellites are recorded. Minimize the number of combinations of ambiguities that is tested. In this case the speed of processing the observations is increasing. The increase of the number of satellites decreases the number of viable solutions and the time of processing the observations.*	*The method is based on the processing of observations in simple difference and do not establish mathematical correlations between the phase observations just like in dual difference [Hofmann-Wellenhof et al. 2001]. This method is very sensitive to errors. Thus, if an error happens to one of the primary satellites, the calculated positions are all wrong. If the error is very large, the estimated position may be far from the true position. In this case the correct ambiguities are not sought.*
M. Cholesky Décomposition Method Euler et Landau (1992)	*The method provides significant improvement in computational speed of the process of looking for ambiguities, but not in performance.*	*Resolution of ambiguity starting from combination of code and phase data from dual frequency is possible with only a few epochs of data.*
OTF Method Abidin (1993)	*The advantage of this technique is that it is not necessary to place the receiver over a known point to fix the ambiguities.*	*The OTF technique can have only poorly improvements.*
FASF Method Chen et Lachapelle	*Correlation between the values of ambiguities to minimize the number of*	*The method does not work in some*

(1994)	*epochs required* *Small number of sets of ambiguities which must be tested with a test of discrimination. Thus, many sets of possible ambiguities shouldn't be looked for completely.* *The residual are already calculated when a complete set of integer ambiguity is found.* *The ambiguities can be searched in real time, so the GPS data can be collected until the ambiguities are fixed.*	*cases because of calculation of standard deviation for fractionally fixed ambiguities. This calculation is done starting from least squares estimation.* *The application of least squares for FASF works well when some epochs of observations are used for ambiguity resolution. In this case, the distortion of standard deviation is great when there are a lot of observations.*
LAMBDA Method Teunissen (1996)	*Uses a combination of measures to minimize the correlation between the values of ambiguity.* *It is very efficient for applications in real time because of the reduced number of sets of possible ambiguities.*	*To fix the ambiguity in only one epoch by combining the phase and code measurements* *The large number of epochs to resolve the ambiguity (7 epochs) compared to other methods (for example Null method - 2 or 3 epochs).*

Table 2**:** The technical characteristics of the phase ambiguity resolution

Characteristics	***FARA Method***	***LSAST Method***	***M. Cholesky Décomposition Method***	***OTF Method***	***FASF Method***	***LAMBDA Method***
Authors	*Frei and Beutler (1990)*	*Hatch (1991)*	*Euler and Landau (1992)*	*Abidin (1993)*	*Chen and Lachapelle (1994)*	*Teunissen (1996)*
Observations conditions	*Satellites >15°* *Base line <10 km* *The more the better*	*Satellites >15°* *Base line <10 km* *The more the better*	*Satellites >15°* *Base line <10 km* *The more the better*	*Satellites >15°* *Base line <10 km* *The more the better*	*Satellites >10°* *Base line = some km* *The more the better*	*Satellites >15°* *Base line <10 km* *It's not important the number of satellites*
The period of observation to solve the ambiguity	*1-5 minutes*	*Instantaneous Solution*	*Some seconds*	*Some seconds*	*5 minutes*	*Some seconds*
The technique of measurement	*Fast-Static*	*Fast-Static*	*Kinematics*	*Kinematics*	*Fast-Static*	*Kinematics*
The effect of multipath on phase measurements (magnitude)	*Des résiduelles et le facteur de variance plus grandes* *The residuals and variance factor are large*	*Potential rejection of good solution* *The residuals and variance factor are large*	*Potential rejection of good solution (level of cm)*	*Potential rejection of good solution*	*The periodic variations of residuals*	*Potential rejection of good solution (level of cm)*
The effect of multipath on code measurements (magnitude)	*No*	*The low estimate of initial coordinates*	*Rejection of good solution (level of cm)*	*The low estimate of initial coordinates*	*The low estimate of initial coordinates*	*Rejection of good solution (level of cm)*
The ambiguity search method (on code and phase / phase)	*Both* *(To provide a solution from one epoch, combine code and phase measurements)*	*Independent* *(To provide one solution at each epoch, it's using phase measurement)*	*Both* *(To provide a solution from one epoch, combine code and phase measurements)*	*Both* *(To provide a solution from one epoch, combine code and phase measurements)*	*Both* *(To provide a solution from one epoch, combine code and phase measurements)*	*Both* *(To provide a solution from one epoch, combine code and phase measurements)*
The data processing	*Multi epochs (Provided a*	*Only one epoch (Provided a*	*Multi epochs (Provided a solution*	*Multi epochs (Provided a*	*Multi epochs (Provided a*	*Multi époques (Provided a*

method (of finding a solution)	*solution at several epochs)*	*solution at each epoch)*	*at several epochs)*	*solution at several epochs)*	*solution at several epochs)*	*solution at several epochs)*
The method used for the calculation of initial solution	*The solution by least squares*	*The solution by least squares*	*The solution by least squares*	*The solution by least squares*	*Kalman filtre*	*The solution by least squares*
The type of observation used to find an initial solution	*Carrier phase solution*	*Code Solution*	*Carrier phase solution*	*Code Solution*	*Carrier phase solution*	*Carrier phase solution*
The field of search	*The area of ambiguity*	*The area of ambiguity*	*The area of ambiguity*	*The area of positioning*	*The area of ambiguity*	*The area of ambiguity (sets of ambiguïties)*
The search space for ambiguity	$k\sigma_N$ *of carrier phase solution*	$K\sigma_x$ $K\sigma_y$ $K\sigma_z$ *of code solution*	$k\sigma_N$ *of carrier phase solution*	$K\sigma_x$ $K\sigma_y$ $K\sigma_z$ *of code solution* $k\sigma_N$	$k\sigma_N$ *of carrier phase solution*	$k\sigma_N$ *of carrier phase solution*
The type of receivers used	*Dual frequency receiver*	*Single frequency and dual frequency receiver*	*Single frequency and dual frequency receiver*	*Single frequency and dual frequency receiver*	*Single frequency and dual frequency receiver*	*Single frequency and dual frequency receiver*
The criterion of selection of fixed solution	*The minimum variation of* σ_0^2	*The minimum variation of* σ_0^2	*The minimum variation of* σ_0^2	*The minimum variation of* σ_0^2	*Nr of sets of possible ambiguities <threshold The minimum variation of* σ_0^2	*The minimum variation of* σ_0^2
The criterion of acceptance of fixed solution	*The Fischer test*	*The Fischer test*	*The Fischer test*	*The Fischer test Number of validation tests is great*	*The Fischer test Nr of sets of possible ambiguities =1 (one solution)*	*The Fischer test*

k – is a constant which is obtained statistical from Student distribution
σ_N *- represents the standard deviation of ambiguity N*
The advantage of instantaneous solution is that the incorrect ambiguities or cycle slips will not affect the solutions of the next epoch.

3. References

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