Abstract: With the development of the International GNSS Service, formerly the International GPS Service, whose primary object is to provide highest quality data and products for research, education and multidisciplinary application, the concept of Precise Point Positioning began to receive more and more interest on the problem called “positioning”. Nowadays because of this development, the PPP technique it started to grow on the detriment of the relative GPS positioning. PPP it is able to offer point determination by processing un-differenced dual-frequency receiver, combine with precise orbit and clock corrections offered by IGS to obtain centimeter accuracy.

The aim of this article is to present the results obtained by both type of processing technique.

Keywords: IGS, Precise Point Positioning, relative positioning, accuracy

1. Introduction

The vast majority of commercially available software utilizes the principles of relative positioning. However, in the late 1990s, the Jet Propulsion Laboratory (NASA) pioneered a new technique that did not require differencing to obtain precise positions. They labeled it Precise Point Positioning (PPP) and implemented it in their GIPSY/OASISII GPS processing software.

Precise GPS point positioning (PPP), as an alternative to differential GPS Surveying that let us use only one GPS receiver – in our case we use dual frequencies receiver. However, the positioning accuracy is affected from global disturbances in addition to other unmodelled errors and biases. This is not the only type of source of errors.

The first question concerning processing GPS data is:

What are the sources of the errors?

The primary limitation of the GPS point positioning are unmodelled errors and biases. These errors are: ionospheric and tropospheric delays, multipath error, ephemeris errors, satellite altitude, atmospheric loading, ocean loading and residual satellite clock errors.

In table 1 we present different types of errors that affect the GPS measurements. But we have to pay attention to this type of errors because we have to know exactly what type of errors is affecting the processing technique. Rigorous estimate of uncertainties requires full knowledge of the error spectrum—both temporal and spatial correlations (never possible). We need to understand and account for the dominant errors at the frequencies we’re most interested in.
| Table 1 |
|-----------------|-----------------|-----------------|
| Signal propagation effects | Unmodeled motions of the station | Unmodeled motions of the satellites |
| Receiver noise | Monument instability | |
| Ionospheric and tropospheric effects | Loading of the crust by atmosphere, oceans, and surface water | |
| Signal scattering (antenna phase center/multipath) | | |
| Atmospheric delay (mainly water vapor) | | |

The ionospheric effect, which is the dominant error source in point positioning after the application of precise GPS orbit and clock products, can be mitigated effectively by using dual-frequency measurements or using the ionospheric model offered by IGS or Berne University.

2. Relative positioning

There are essentially two ways in which measurements from two receivers are used to account for biases, and hence improve accuracy:

1. Each set of measurements at a receiver are independently used to derive a position which is in error by more or less the same amount. This is the DGPS procedure implemented for precise navigation applications using pseudo-range data.

2. Differencing measurements between receivers leads to an observable that is essentially free of biases (or at least substantially reduced if the receivers are not too far apart). This is the GPS surveying mode of differential positioning using carrier phase data.

Differential or relative positioning requires at least two receivers set up at two stations (usually one is known) to collect satellite data simultaneously in order to determine coordinate differences. This method will position the two stations relative to each other (hence the term “relative positioning”) and can provide the accuracies required for land surveying. There are two type of relative positioning:

- Differential Positioning (Code Pseudo-Range Tracking)
- Differential Positioning (Carrier Phase Tracking)

The term differential positioning is sometimes used interchangeably with relative positioning. However since differential positioning is more often associated with the specific type of relative positioning which applies correction measured at a “known” site to measurements at an “unknown” site, relative positioning will be the term used to describe the general concept as shown in Figure 1.
We assume we can write the actual observation to be the sum of a modeled observation, plus an error term:

$$P_{observed} = P_{model} + \text{noise}$$

Next we apply Taylor’s theorem, where we expand the computed model using provisional parameter values and ignore second and higher order terms:

$$P (x, y, z, \tau) = P(x_0, y_0, z_0, \tau_0) + (x - x_0) \frac{\partial P}{\partial x} + (y - y_0) \frac{\partial P}{\partial y} + (z - z_0) \frac{\partial P}{\partial z} + (\tau - \tau_0) \frac{\partial P}{\partial \tau}$$

$$= P_{computed} + \frac{\partial P}{\partial x}\Delta x + \frac{\partial P}{\partial y}\Delta y + \frac{\partial P}{\partial z}\Delta z + \frac{\partial P}{\partial \tau}\Delta \tau$$

So, the equation for each satellite in view – for \( m \) satellites we can write this system of \( m \) equation in matrix form:

### 3. Precise point positioning

NASA’s Jet Propulsion Laboratory scientists developed PPP which is a new high precision mode of GPS positioning. It provides around 1 cm accuracy with single receiver and without any ground control. PPP should not be confused with average point positioning which is performed in real time using pseudo ranges, and gives about 5-10 m precision (Sanli, 1999).
The main idea basing PPP idea is that we have precise orbits and clock information from some other source we can position ourselves (along with receiver clock and tropospheric bias) very accurately.

The major difference between this to type of processing DGPS and PPP is that the PPP technique it is free from the reference receiver, its measurements and also its corrections – figure 2.

Unlike in relative positioning, common mode errors do not cancel in PPP. The station that we want to compute the coordinates by using the PPP technique contain also errors which need to be removed. These are satellite dependent errors including GDOP, clock error, orbit error, propagation dependent errors including ionosphere, troposphere, multipath, and receiver dependent errors including receiver clock error, antenna phase centre variation and measurement uncertainty.

Both the carrier phase and pseudorange observables are important to PPP. The relevant expressions are in units of distance:

\[
\phi_{pk} = \rho_k^P - c dt_k + c dt^P + \frac{c}{f_i} N^P_k + I^P_{k,i,p} + T^P_k + d_{k,i,c} + d_{k,i,p} + d_{k,i,p} + e_{k,i,p}^P
\]

\[
\rho_k = \rho_k^P - c dt_k + c dt^P + I^P_{k,i,p} + T^P_k + d_{k,i,c} + d_{k,i,p} + d_{k,i,p} + e_{k,i,p}^P
\]

Where:
- \( i \) = subscript identifying L1 or L2
- \( f_i \) = frequency
- \( k \) = receiver station identifier
- \( p \) = satellite identifier
- \( \phi_{pk} \) = measured carrier phase scaled to distance (meters)
- \( \rho_k^P \) = measured pseudorange
- \( \rho_k \) = geometric topocentric distance
- \( N^P_k \) = integer ambiguity
- \( d_{k,i,c} \) = receiver clock error
- \( d_{k,i,p} \) = satellite clock error
- \( I^P_{k,i,p} \), \( I^P_{k,i,p} \) = ionosphere for phase and pseudorange and frequency \( f_i \)
- \( T^P_k \) = troposphere
- \( d_{k,i,c}, d_{k,i,p} \) = multipath for phase or range respectively
\(d_{R}^{P}, d_{P}^{P}\) = satellite hardware delay for phase or range respectively
\(e_{r.m.s.}^{P}, e_{r.m.s.}^{R}\) = random measurement noise for phase or range respectively

By using dual frequency receiver we are able to eliminate the ionospheric errors by using the narrow-lane combination. This is defined by the metric difference L1-L2 which isolates the ionospheric component, hence its name:

\[L_i \equiv L_1 - L_2 = d_{1\text{ON}} + \lambda_1 b_1 - \lambda_2 b_2 = d_{1\text{ON}} + \lambda_1 (b_1 + b_2) + (\lambda_1 - \lambda_2) b_2 = d_{1\text{ON}} + \lambda_1 b_\Delta - \lambda_1 b_2\]

The largest difference between relative processing and PPP is the way that the satellite and receiver clock errors are handled. Instead of between-receiver differencing to remove the satellite clock errors, PPP uses highly precise satellite clock estimates. These satellite clock estimates are derived from a solution using data from a globally distributed network of GPS receivers.

We can use different software to process precise point positioning but we need this typical features included in the software:

- Orbit integration with appropriate force models;
- Accurate observation model (Earth model, media delay...) with rigorous treatment of celestial and terrestrial reference systems;
- Reliable data editing (cycle-slips, outliers);
- Estimation of all coordinates, orbits, tropospheric bias, receiver clock bias, polar Motion, and Earth spin rate
- Ambiguity resolution algorithms applicable to long baselines;

### 4. Case study

The case study has been carried out in Oradea. The data was obtain from the permanent station on the Faculty of Construction and Architecture. The data are from different hours and different intervals. The vertical value is the correction clock given in nanoseconds, and the horizontal value is time series.

In figure 3 we have the first hour.
In figure 4 we have the charts after 4 hours. We can observe that the improvement of time is inessential.
The figure 3 and 4 are related to PPP technique. We will not describe the relative positioning charts because we are more interested in the PPP technique.

After processing data using relative positioning we were able to obtain the final coordinates. In table 2 it is presented the results, from both types of processing, deltas and rms.

<table>
<thead>
<tr>
<th>CARTESIAN</th>
<th>ESTIMATED</th>
<th>A-PRIORI</th>
<th>Delta(m)</th>
<th>RMS(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X (m)</td>
<td>4037694.6293</td>
<td>4037695.0331</td>
<td>-0.4038</td>
<td>0.4014</td>
</tr>
<tr>
<td>Y (m)</td>
<td>1626553.3843</td>
<td>1626553.0278</td>
<td>0.3565</td>
<td>0.3580</td>
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<td>Z (m)</td>
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<td>4646396.2833</td>
<td>0.3460</td>
<td>0.3496</td>
</tr>
</tbody>
</table>

The A-Priori coordinates are the coordinates obtain by using relative positioning and estimated are the coordinates obtain by using Precise Point Positioning.

5. Conclusions

When we are using GPS technology we are able to obtain the position of a point by two different methods: precise point positioning - when we employ only one GPS receiver, and the second method is using differential (relative) positioning – when we use two or more GPS receiver simultaneously tracking the same satellites. We show that by using only one GPS receiver and suitable software for processing we are able to obtain positioning accuracy comparable to the differential positioning.

Also as a remark we can observe that the accuracy on both methods depends mostly on the duration session. Our recommendation is that if it is used the precise point positioning technique you need at least 6 hours of data to obtain results sufficiently accurate for the surveying application.
It is possible this due to poor satellite receiver geometry, the second suspect being the ionospheric effect and also multipath error. Obviously to have a pertinent observation on this technique it is necessary to due detailed investigation in the field and also in the office.

6. References

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