GEOID MODELLING BY ASTRO-GEODETIC MEASUREMENTS

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***Abstract**: Regionally or globally geoid determination is a problem of equal scientific and practice interest. To do this, there are several possibilities, each of them with certain advantages. Terrestrial methods are able to detect short wavelength structures of the geoid and the astro–geodetic method is one of them beside gravimetric, GPS and levelling. New developments in the fields of CCD cameras, time dissemination and measurement or geodetic instruments reconsider the astro-geodetic method as accuracy and efficiency one.*

***Keywords:** astro-geodetic, geoid, CCD, vertical deviation.*

1. Introduction

Geoid determination as physical figure of the Earth and reference equipotential surface represents one of the major task of geodesy, integrated in the present efforts of understanding the Earth as a system. Development of satellite geodesy, lunar laser ranging (LLR), satellite laser ranging (SLR) and radio-interferometry with very long baseline (VLBI) represent a major change of geodetically concepts as follows: static $\rightarrow$ dynamic, Earth $\rightarrow$ Earth system, Newtonian $\rightarrow$ Einstein's theory of gravity. Earth system means that its constituent parts of different nature, solid, liquid, gaseous, interact between them in a complex manner, showing large variations in space and time and influencing each other. Earth's gravitational potential and Earth's magnetic field represent the basement of research in geodesy and geophysics, which together with complementary data (seismic, global atmosphere, volcanology, solar and planetary astronomy) represent the most valuable sources of information about the structure, composition, evolutionary processes and the past-present-future behaviour of our planet.

The geoid represents the surface which in land surveying is usual named *sea level*. This is the surface that closely approximates sea level in the absence of disturbing forces as winds, ocean currents and tides. Practically, the geoid cannot be expressed into a simple mathematical form. We can see the geoid as the equilibrium and rest state of the ocean's surface extended under the continents. In the every point on the Earth’s surface we have a reference direction, tangent to the force line of the terrestrial gravity field. This direction is named *local vertical* or *plumb line* direction and it is normal to the geoid, not to the Earth’s topographical surface. In contrast, the ellipsoid is a man–made surface, with an easy mathematical description and which, theoretically, approximates as good as possible the geoid. Therefore, through the same point on the Earth we have the second reference direction, normal to the ellipsoid. Angular difference between normal to the geoid and normal to the ellipsoid, in the same point on the Earth's topographical surface, represent so called vertical deviation (Helmert definition). In a similar way is defined the vertical deviation at the geoid surface (Pizzetti definition), but this cannot be directly observed because of terrestrial relief. Practically, these angles indicate the relative position ellipsoid–geoid. *The astro–geodetic method of geoid determination is the only one can directly provides vertical deviation at the Earth's surface, by comparisons between astronomical and geodetic coordinates* [1], [2]. Another quantity implied in the geoid modelling is the geoid–ellipsoid separation (sometime named geoid undulation) which represents the distance, along the vertical (or plumb line), between geoid and ellipsoid. The separation can be obtained by several techniques such as gravimetric measurements, combined GPS–levelling or from gravity potential coefficients determined by satellite measurements. Knowing ellipsoid-geoid separations it is possible to derive vertical deviations but at the geoid surface, which is relative useless because the almost all terrestrial measurements (except spatial distances) are realized towards local vertical at the Earth's surface. The configuration of the vertical and the shape of the geoid are in reciprocally correlation and the astronomical coordinates are determined respect to the local vertical from observation's point.

As we seen, the geoid reflects the anomalies in the masses distribution and the density inhomogeneous in the terrestrial crust over a certain zone or global, for the whole Earth. Satellite missions as CHAMP (CHAllenging Minisatellite Payload), GOCE (Gravity field and steady-state Ocean Circulation Explorer) or GRACE (Gravity Recover and Climate Experiment) provide global long–wave geoid models (smooth surfaces) which mainly serve as reference for a better global or zonal geoid model. *Thus, a combination between satellite derived models and regional data from terrestrial measurements is the suitable solution for obtaining a high resolution geoid* [3]. Terrestrial methods are able to detect short wavelength structures of the geoid and the astro–geodetic method is one of them beside gravimetric, GPS and levelling. *All this methods have both advantages and disadvantages, general rules being using them in various combination* [3], [4], [5].

Regarding the geoid determination in our country we can affirm that Romania is really a black spot on the actual geoid European map. After 1990, although the modern geodetic instruments entered in use, both in public and private sector, many of them are not suitable for very precise measurements such as implied in geoid determination. Considerably GPS and levelling measurements were made over national territory but, most of them, are disparate and inferior qualitative. What is more, until now, only few singular experiments were made to obtain a geoid model from GPS–levelling combination, principally at TUCE–FG (Technical University of Civil Engineering – Faculty of Geodesy) [6]. Also, some gravimetric measurements were achieved last years but over limited areas and unincorporated at national level. In these circumstances the national geoid model is an old one, previous 1990, obtained by combination of astro–geodetic and gravimetric measurements. For this aim, military experts in collaboration with experts from former Geological and Geophysical Institute (actual "Sabba S. Stefanescu" Institute of Geodynamics of the Romanian Academy) designed and realized a national gravimetric network, structured on two orders, with over 440 points uniformly distributed over whole national territory. Also, military experts from Military Topographic Direction (former D.T.M.) and Military Astronomical Observatory (former O.A.M.), especially in the 1965–1975 decade, measure 146 Laplace points with complete determination of astronomical latitude, longitude and azimuth and 118 astro–gravimetric points. It is important to retain that the observations time were usually of 24 nights long for one Laplace point and 15 nights long for one astro–gravimetric point, for a team formed by one geodesist, one technician and 15 soldiers. The precision obtained for latitude determination was ±0".3, ±0".45 for longitude and ±0".5 for azimuth [7], [8]. It is strange that until now, the army does not disseminate for civil use any data regarding national geoid (quasi–geoid) model, especially after 1990. *The calculation of vertical's deviation by* *comparison of the old astronomical determination with the present GPS determination, cannot lead than at less conclusive results*. This, because the old astronomical determinations were based on the old less precise stars catalogue (FK3, FK4) and reduced at the old mean pole CIO (Conventional International Origin). CIO was defined with less accuracy than present mean pole CTS (Conventional Terrestrial Pole), at which the GPS technology is reported. In addition, the reductions of the old astronomic determinations at the CIO mean pole were effectuated on the bases of the Earth's pole instantaneous coordinates, which it was determined by BIH (Bureau International de l'Heure) through classical methods with more less accuracy in comparison with the present accuracy provided by IERS (International Earth Rotation Service). *These reasons with somehow theoretical feature and more practical reasons, in this moment make that the old astronomical determination does not present than historical importance for Romanian astronomic geodesy, as well as, for all world's country that passed at GPS technology*.

In many European countries as Austria, Switzerland, Croatia, Germany, Portugal and Holland was started campaigns for astronomical measurements with the task of geoid modelling. These measurements, more or less extended, implied a few technical solutions and algorithms, depending in the main of the allocated funds.

2. Main steps of an astro-geodetic modelling of the geoid

Astro-geodetic measurements in contrast with classical geodetic measurements are not quite standardized. In the last decade, depending on various factors as chosen instruments, duration of observations, timing method or adjustment solution for measurements, was developed some technical solution (observations platforms). On principle, at implementing of a method for astro-geodetic modelling of the geoid, we identified 6 major steps, as follows.

1) Developing an rigorous mathematical algorithm for astro–geodetic determinations of vertical deviation:

– The algorithm is based on the azimuthal and zenithal angular observations for a big number of stars, uniformly distributed on the celestial sphere, without the utilization of the ephemerides of the observations.

2) Designing and performing a mobile observing platform for astro–geodetic determinations of vertical deviation:

– The system can consists in a high accuracy electronic theodolites or electronic total station equipped with servo motors, a GPS time receiver, a micro CCD camera and portable computer for platform–system management.

3) Automating of vertical deviation determination by astro–geodetic measurements:

– The observing platform have to provide real time vertical deviation at a satisfactory precision by astronomic and geodetic measurements. This step consists in implementing the mathematical algorithm into dedicated software for astronomical observations guidance, analyzing and controlling.

4) High density astro–geodetic, GPS and levelling observations:

– Performing high density measurements for an accuracy geoid modelling and for extracting certain data regarding precision, accuracy and standardization. Measurements will consist in: i) vertical deviation determinations by astro–geodetic method using developed observing platform, ii) GPS measurements of ellipsoidal heights and geometric levelling measurements of orthometric (normal) heights from which are extracted separation (undulation) between geoid and reference ellipsoid.

5) High accuracy geoid modelling by astro–geodetic, GPS and leveling measurements and refining the global satellite geoid models:

– Generating geoid models based on i) astro–geodetic technique, ii) GPS–levelling combination, iii) astro–geodetic and GPS–levelling combination, iv) refined global satellite models (CHAMP/GRACE/GOCE/EGM) by astro–geodetic and/or GPS–levelling combination;

– Precision an accuracy evaluation of geoid models, comparisons between models, suitable grid geoid surface for users by an adequate choice of the step grid and interpolation method, data validation and interpretations, model adjusting.

6) Standardization of procedure for geoid determination by astro–geodetic, GPS and levelling measurements, results dissemination.

3. Astro-geodetic measurements

Firstly, astro-geodetic geoid modelling supposes an algorithm/procedure for the astro–geodetic determination of vertical deviation and a proper measurement device. Using an electronically total station (ETS) of new generation represents a satisfactory and less expensive solution instead designing and making a new one instruments (in Germany, for astro-geodetic modelling of the geoid was developed a *Transportable Zenith Camera* at Institut für Erdmessung, Hannover [10].) ETS is a high accurate geodetic instrument made for precise angular and distance measurements. Generally such instruments are used for complex geodetic works as high-speed railway alignment; montage of great bridges' components or running bridges, various industrial equipments positioning as turbine engine, or for tunnels layout. The instrument is endowed by a very sensitive (biaxial) compensator which measure how big is the tilt of the instrument vertical axis related to local vertical. Based on this continuously measurement, all azimuthal and zenithal measurements are corrected for this tilt. Supplementary, such instruments are gifted by servo-motors which permit a fast passage between first and second position of the telescope. Accuracy angular measurement is about 0".5, standard deviation is 0".1 and the dual axis compensator has an accuracy of about 0".3. Moreover, for night or underground measurements, these kinds of devices have graphic LCD colour touch screen and illuminated reticular wire. The instruments play a crucial role, firstly because establish the spatial reference for astronomical positioning and secondly, because all angular measurements are made at highest possible precision for a geodetic instrument. We have to remind that one arc second at the Earth's centre subtend about 30 meters on the Earth's surface. 0".1 precision, how the instrument is capable to measure an angular quantity, is even a limit value for a geoid accuracy of mm level. However, taking into account that vertical deviation is a quantity with a generally slow variation over the Earth's surface, and a GPS time receiver can ensure minimum 0.01 time second accuracy for time measurements is feasible to attain a cm level accuracy for geoid determination. Then, more measurements (which mean more CCD images acquisition for a single star), more observed stars, eliminating time and operator errors, coupling the astro–geodetic determinations with GPS-levelling measurements are so many solutions to obtain satisfactory proposed results. ETS endowed by a CCD micro camera and GPS timing become a very efficient platform for high accuracy geoid determination.

Measurements consist in azimuthal and zenithal angular observations for a big number of stars, uniformly distributed on the celestial sphere, without the utilization of the ephemerides of the observations. Applying the theory of conditional measurements with unknowns, we get up at one right rigorous algorithm for the determination with maximum efficiency of all three fundamental elements of astronomical geodesy. Supplementary, the introduction of an adequate matrix of weights will be taking in account the unequal weights of all direct measurements: angular measurements and corresponding times. The stars are looked like simple bright points on the celestial sphere. For these stars, in the initial step, do not matter the constellation, the catalogue number, the name or the magnitude and not even the equatorial coordinates (the right ascension and declination).

Every star will be precise observed in both position of the telescope, azimuthal and zenithal too, of course each angular measurement being accompanied of the corresponding value of the time. These observations over a single star will prove itself of great efficiency concerning productivity and accuracy mainly due to a micro CCD camera which will be mounted instead of the total station eyepiece. It is the typical case of different accuracy measurements (depending of the star position on the sky) in which the weights will provide uniformity in the final accuracy of the results and the coherence of these. CCD camera will record, for a single star, in both positions of the total station, a lot of images. From these images will be calculated the star trajectory in the CCD array frame and next, automate, the moment of intersection between star trajectory and reticular wire (azimuthal and zenithal too) of the telescope's total station. The time value together with the angular reading (separate for azimuthal and zenithal observation) at the total station and equatorial coordinates at the time moment of observation will generate one equation. Thus, separate for azimuthal and zenithal observation, for a single star will obtain two equations. Finally, for a n number of observing stars will obtain $2n$ equations with 3 unknowns, that will ensure a satisfactory redundancy. Of course, equatorial coordinates at the observation's epoch must be reduced from catalogue epoch (usually J2000.0) to the observation epoch that implies calculus of proper motion, precession, nutation, annual parallax and aberration, diurnal parallax and aberration and not in the last refraction from zenithal measurements [11], [12].

Modelling implies measurements of different type, such as GPS and levelling together with astro–geodetic. From GPS measurements of ellipsoidal heights and geometric levelling measurements of orthometric (or normal) heights are extracted separations (undulations) between geoid (quasi-geoid) and reference ellipsoid. In this way we will obtain the second quantity necessary for an adequate geoid modeling.

We have to establish an adequate density of observations' points over the interest area through a feed-back process: measurements – geoid modelling – accuracy and precision estimating – densification – re-measurement. Of course, the quality of the measurements plays a crucial role in this process besides the density of the observations' points. All measurements have to validate before introduction them into a geoid model. It is important to remark that do not exists strictly rules for geoid modelling, only general rules corresponding to the chosen method.

4. Some remarks

The astro-geodetic modelling of the geoid have a scientific character and applicative too, as we already pointed out in the previous sections. The results of old astronomical and geodetically determinations were always expensive, as a result of low productivity, frequently ambiguous accuracy through the ignorance both of the weights’ observations and rigorous method of processing, which to allow a scientific estimation of the accuracy of the determinations. At these added the missing of the coherence of the determinations as a result of a long duration of stars' observations, often one month or even two months for a single astronomical point. We intend to remove the previous difficulties, beginning at the both existence of the high productivity computing resources and the present electronic devices for the high accuracy angular measurements. It is important to remark that the determination of the three fundamentals elements of the astronomical geodesy, with accuracy and rapidity, is a present problem in the context of GPS technology, as a method of detail studying of both the geoid and the deviation of the geoid from the adopted reference ellipsoid.

An important advantage of the astro–geodetic method is the fact that a local geoid can be determined directly from pointy (un-relative) determination, within the computation area. This is in contrast with the gravimetric method, where the application of the well known Stokes formula theoretically requires coverage over the whole Earth. This gravimetric method presents other disadvantages: it is an indirect method then it is not immediate and a gravimeter (relative or absolute) is very expensive. However, due to the enhancement of the astro–geodetic technique within the last years, the method is now significantly less time-consuming and expensive than before. It has to be considered that the astro–geodetic method only provides relative geoid undulations (not quasi-geoid). In order to get absolute information, the geoid height has to be known at one station in minimum. That implicates an error accumulation with increasing distance to the reference station that may be a slightly disadvantage. Some systems for astro-geodetic vertical deviation measurement are based on zenithal cameras [9], [10]. Obviously, it is easier to work with these devices because it is not necessary to orient the instrument in the horizontal plane and measurements are limited to zenithal distances ones. At the zenith or near to the zenith the refraction is zero or very small, which is an advantage. On the other hand, observing stars into a small zenithal area it is a strong limitation. Instead of, an alt-azimuthal system will be capable to observe stars in almost any position on the celestial sphere. Every star will be observed both azimuthal and zenithal, so that every observed star will provide two equations, which means more information (redundancy).

Regionally or globally geoid determination is a problem of equal scientific and practice interest. Geoid knowing is important not only for geodesy (almost all measuring data on the Earth surface have to corrected by vertical deviation), but for geology, geological prospection, geophysical studies, earthquakes studies (seismically zoning of the national territory) and not at least for civil engineering (especially for constructions as water storage dams, subway, railway or road tunnels, nuclear plants, particle accelerators, long bridges and so on). The determination of a precise and high-resolution geoid is a subject of many research projects, the combination of different geodetic networks and observation types being of great importance on a national and on a regional/European level. The compatibility of spatial reference frames, the height systems and the geoid are main objectives for all European national authorities in the field of geodesy. In the last years, some theoretical and practical methods for geoid determination were developed and an accuracy of a few centimeters and even millimeters is feasible, depending on time and effort.

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