Evaluation of surfaces calculation precision from the digital base of the orthophotographs and from field measurements using control areas of geodetic trapezium

Valeriu MOCA, Professor -"Ion Ionescu de la Brad" University of Agricultural Sciences and Veterinary Medicine, Iaşi, valmoca@uaiasi.ro
Mihaela CARDEI, PhD – "Gheorghe Asachi" Technical University, Iaşi, cardei_mihaela@yahoo.com
Oprea RADU, Lecturer - "Ion Ionescu de la Brad" University of Agricultural Sciences and Veterinary Medicine, Iaşi, opricaradu@yahoo.com
Cristian HUŢANU, PhD - "Ion Ionescu de la Brad" University of Agricultural Sciences and Veterinary Medicine, Iaşi, hutanucrst@yahoo.com
Constantin SAVU, Eng. – S.C. „Casa Verde" S.R.L., Suceava, costyca81@yahoo.com

Abstract: *For evaluating the accuracy in determining the surfaces from the general cadastre informational system the primary data from the analog and digital archive of the basic cadastral plans were analyzed. Therefore, the following documents were used: basic cadastral plan, on a scale of 1:10 000 (aerophotography 2006); direct topographic measurements made with GPS receivers of geodetic precision in 2011.*

The verification and testing the results achieved was performed using the commune coordinates of ground control points (buildings' corners, crossroads, parcels' limits etc.). Depending of the coordinates determined in the Stereographic projection 1970 the surfaces of the agricultural physical blocks were calculated. The accuracy of the georeferentiation procedure has been made evident by evaluating the errors on the coordinates of the control points and on the digitize / measured surfaces in the field, in the case of a geodetic trapezium, at a scale of 1:5 000, nomenclature L-35-4-B-a-3-III and the control area of 540.9816 ha.

Keywords: *digital orthophotographs, Global Position System – GPS, Ground Control Point – GCP, control areas of geodetic trapezium.*

1. Introduction

The basic elements of the general cadastre –the parcel, the construction and the owner – are identified and recorded in the cadastral documents after performing the proper cadastral measurements in each territorial – administrative unit (10). Among the types of measurements used at introducing the general cadastre, photogrammetry made itself noticed; with the passage of time, this method experienced three different stages of evolution: analog, analytic and digital. Nowadays, LiDAR technology represents a modern measuring technique, complementary to the traditional aerial cartographic technique and capable of replacing it, that provides 3D photogrammetric products (9). The basic topographic plan made between 1951 and 1990 was created using the photogrammetric technology. The plan was made at scales of 1:2 000, 1:5 000 and 1:10 000 and it included almost 85% of Romania's territory. Between 2001 and 2010, in order to create the digital and analog format of the orthophotoplan aerophotogrammetric elevations were performed; the resolution of the cartographic representation was at a scale of 1:10 000 (4).

The access and the administration of the cadastral data base from the informational system of the digital ortophotoplan, at the level of the territorial - administrative units are performed by the Agency for Payments and Interventions in Agriculture in Romania.

The global positioning technologies based on satellite navigation systems started being used in Romania after 1990. The A class GPS National Geodetic Network that was created until the beginning of 2008 included, according to the official data: 28 permanent stations uniformly distributed that were determined using the three spatial coordinates (3D), the precision level being of less than ± 1 cm (1). Referring to the positioning methods used, the **kinematic method** was most appreciated. It relies on the use of two frequencies GNSS receivers and a short observation period thus providing centimetric accuracy (3).

The kinematic method of absolute positioning is also known as ***Real Time Kinematic*** (RTK) as it allows the determination of spatial coordinates (3D), in real time. In the case of Romania’s territory, the ***Romanian Position Determination System (ROMPOS***) was adopted. This method permanently provides corrections for point positioning from the support geodetic network as well as for the elevation points of topographic details (11).

Starting with 2009, Romania adopted the ***European Terrestrial Reference System 1989 (RO_ETRS 89)*** along with a series of standard software applications, the most important of which being the ***TransDatRO*** programme (12).

2. Method and Results

The first phase in realizing the present case study consisted in calculating the mathematical base of the cadastral sheets that, cartographically speaking, included the limits of the administrative territory from Bilca commune, Suceava County. The calculation was made at a scale of 1:5 000. The second phase consisted in the vectorization of the basic cadastral plan, at a scale of 1:5 000, including the graphic data corresponding to the field’s limits and respectively, to the parcels and the categories of land-use at the time of 1978. The third and last phase consisted in examining the data regarding the informational system of the agricultural physical blocks present on the digital orthophotoplan at a scale of 1:10 000 from the administrative territory of Bilca commune, based on the aerophotogrammetrical elevations of 2006. The comparative analysis of the data regarding the surfaces that resulted after the vectorization of the basic cadastral plan, at a scale of 1:5 000 and respectively after the analysis of the digital orthophotoplan, at a scale of 1:10 000, was made using field measurements. The measurements were made using the **GNSS South S 82 T** receiver, a dual-frequency GPS receiver of geodetic precision.

2.1. Geographic location and cartographic presentation of the territory from Bilca commune

The geographic position of the cadastral - administrative unit of Bilca commune is situated in the northern part of Suceava County, having in the north part the state boundary with Ukraine. The surface covered by Bilca territorial – administrative unit is mostly situated in the Dragomirnei Plateau (geomorphologic subunit of Suceava Plateau) and a small part of it in Radauti Depression. The territory and the geographic position of Bilca commune – a rural settlement from the Valley of Suceava River – were first attested in the official documents issued somewhere around 1810. At the same time, a first cartographic representation of the territory of Bilca commune was included in a fragment of a military Austrian map published around 1900 (7). The cartographic documentation of the territory of the Bilca commune, with its two basic components (incorporated and unincorporated area) was aerophotographed in 1978 and published in 1982 using the Stereographic projection system of coordinates – 1970.

Depending of the Stereographic projection pole – 1970, latitude (**φ_0 = 46° North)** and longitude (**λ_0 = 25° East)**, the central point of the Bilca commune was situated in the outer area of the null deformation circle, at a distance **of 221.985 km** (Figure 1).

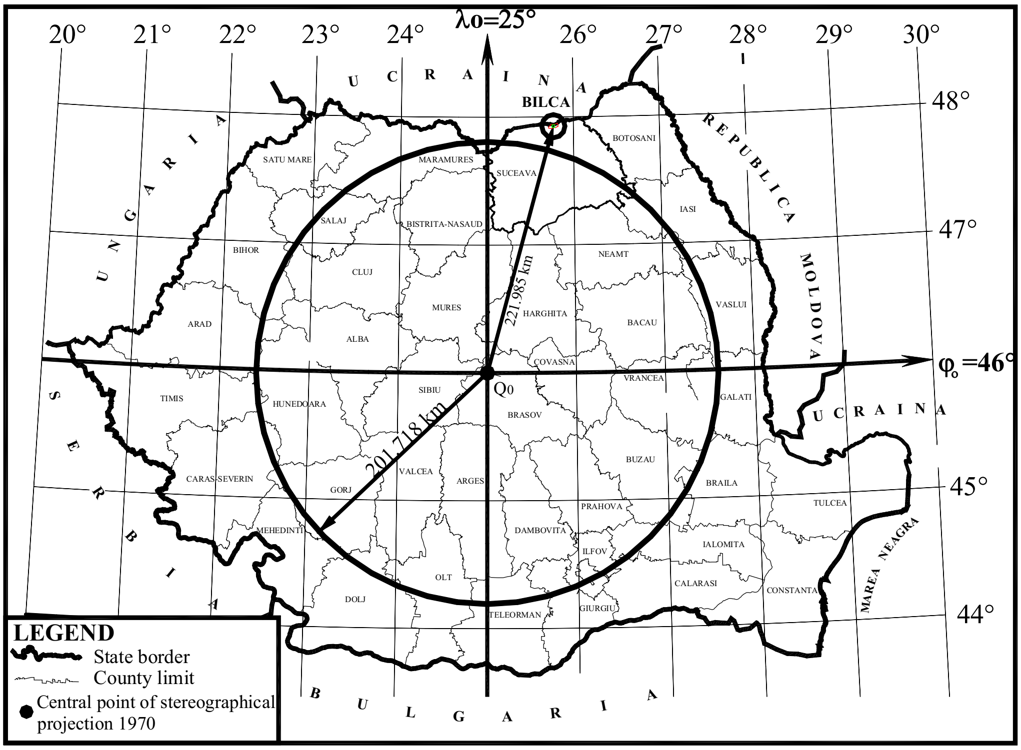


Figure 1 – The geographical position of Bilca commune related to the Stereographic projection pole –1970

From the cadastral activity between 1978 and 1982 resulted the basic cadastral plan sheets at a scale of 1:5 000 pointing out the existent graphic entities.

The cadastral delimitation of the territory belonging to the Bilca commune was made using: the Romanian state border with Ukraine in the **North**; the territorial limit of Fratautii Noi commune to the **East**; the territorial limit of the communes Gălăneşti and Vicovu de Jos to the **South**; the territorial limit of Vicovu de Sus commune to the **West** (Figure 2).

From the general scheme of the cartographic framing of the territorial administrative unit resulted the following map and plan sheets (geodetic trapezia), at standard scales, with the official nomenclature of the Stereographic projection – 1970:

- **cadastral map, scale 1:50 000**, with the following nomenclature of the 2 geodetic trapezia: L-35-4-A (Vicovu de Sus); L-35-4-B (Bilca);
- **topographic map, scale 1:25 000,** with the following nomenclature of the 4 geodetic trapezia: L-35-4-A-b; L-35-4-A-d; L-35-4-B-a; L-35-4-B-c;
- **basic cadastral plan, scale 1:10 000**, with the following nomenclature of the 5 geodetic trapezia: L-35-4-A-b-4; L-35-4-A-d-2; L-35-4-B-a-3; L-35-4-B-a-4; L-35-4-B-c-1;
- **basic cadastral plan, scale 1:5 000**, with **13 plan sheets** (geodetic trapezia): **12 plan sheets full/empty**, with the surfaces of the neighboring territories and **1 plan sheet full/empty**, with the nomenclature **L-35-4-B-a-3-III** (Figure 2).

The case study was performed on the accuracy of the georeferentiation process of the graphic fund of the **basic cadastral plan**, at a scale of 1:5 000, with the nomenclature **L-35-4-B-a-3-III** that included 100% of the Bilca territory.

This cadastral plan sheet represented the main support of the graphic entities measured and georeferentiated directly in the field and analyzed according to the errors resulted from the coordinates and the surfaces.

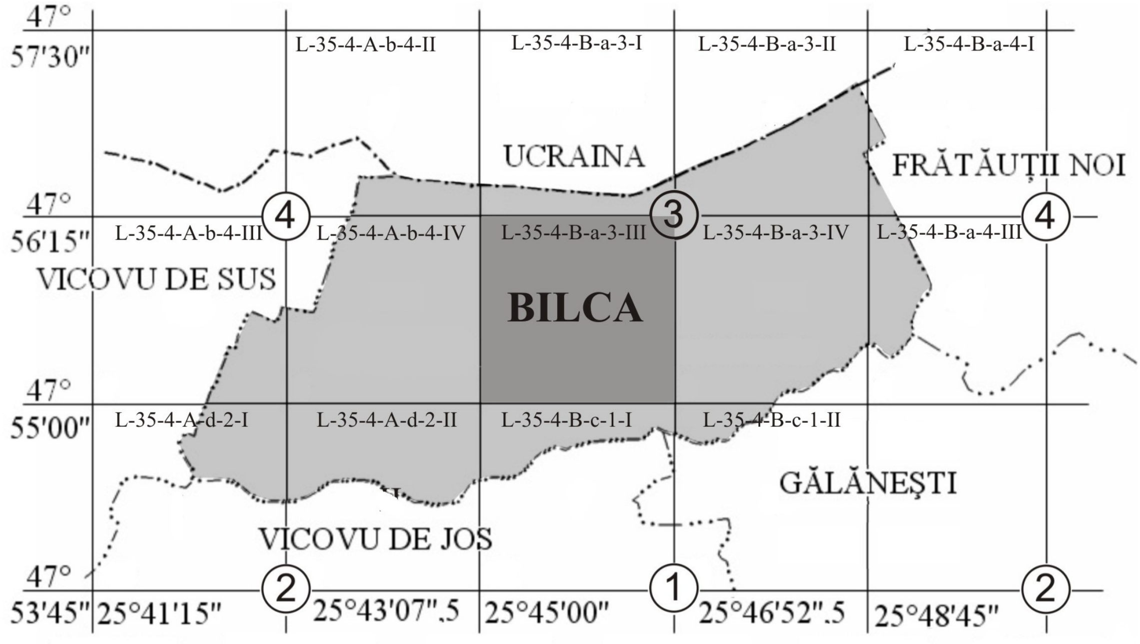


Figure 2 – Cadastral delimitation and cartographic presentation of the territory of Bilca commune

2.2 Calculation of Stereographic plane coordinates – 1970, depending of the geographic coordinates from the reference ellipsoid Krasovski – 1940

Depending of the geographic coordinates of the corners of the geodetic trapezia from the surface of the reference ellipsoid Krasovski – 1940 the calculation of the Stereographic coordinates – 1970 was made using the formulas with constant coefficients (5, 6).

The algorithm for the up mentioned calculation included first of all, the determination of the stereographic coordinates from the “*tangent plan”*, parallel to the unique secant plan – 1970, using the relations:

$$X_{tg} = (a_{00} + a_{10}f + a_{20}f^2 + a_{30}f^3 + a_{40}f^4 + a_{50}f^5 + a_{60}f^6)\ 1.000 + \\ + (a_{02} + a_{12}f + a_{22}f^2 + a_{32}f^3 + a_{42}f^4)\ l^2 + \\ + (a_{04} + a_{14}f + a_{24}f^2)\ l^4 + \\ + (a_{06} + \ldots)\ l^6 + \qquad [m]$$

$$Y_{tg} = (b_{01} + b_{11}f + b_{21}f^2 + b_{31}f^3 + b_{41}f^4 + b_{51}f^5)\ l + \\ + (b_{03} + b_{13}f + b_{23}f^2 + b_{33}f^3)\ l^3 + \\ + (b_{05} + b_{15}f + \ldots)\ l^5 + \qquad [m]$$

where: $\mathbf{f = 10^{-4}(\varphi_i - \varphi_0)''}$ - the latitude difference between the given point $\mathbf{P_i(\varphi_i, \lambda_i)}$ and the projection pole $\mathbf{Q_0\ (\varphi_0, \lambda_0)}$, that reaches values higher than 10 000″;

$\mathbf{l = 10^{-4}(\lambda_i - \lambda_0)''}$ - the longitude difference between the given point $\mathbf{P_i(\varphi_i, \lambda_i)}$ and the projection pole $\mathbf{Q_0\ (\varphi_0, \lambda_0)}$.

The coordinates from the secant plan of the Stereographic projection – 1970 resulted from the modification of the scale of the tangent plane, using the **C=0,999 750 000** coefficient.

$$\mathbf{X_{sec} = X_{tg}\ x\ 0,999\ 750\ 000} \qquad \mathbf{[m]}$$

$$\mathbf{Y_{sec} = Y_{tg}\ x\ 0,999\ 750\ 000} \qquad \mathbf{[m]}$$

The plane rectangular coordinates from the corners of the geodetic trapezia, at a scale of 1:5 000, having the nomenclature **L-35-4-B-a-3-III (Bilca)** have been expressed in the official system of coordinates of the Stereographic projection – 1970, having their origin translated 500 000 m to the South and 500 000 m to the West in relation to the origin's axes in real system. Referring to the origin and the axes of the conventional system, the stereographic coordinates – 1970 have been achieved using the following relations:

$$\mathbf{X\langle 70\rangle = X_{sec} + 500\,000\ \ m}$$
$$\mathbf{Y\langle 70\rangle = Y_{sec} + 500\,000\ \ m}$$

The numeric values of the geographic coordinates from the ellipsoid Krasovski – 1940 and the Stereographic coordinates – 1970 of the corners of the trapezium taken into consideration are presented in Table 1.

Table 1. Geographic coordinates on the ellipsoid Krasovski – 1940 and Stereographic coordinates –1970 of the L-35-4-B-a-3-III trapezium

Point no.	Point position	Geographic coordinates		Stereographic coordinates	
		φ(° ′ ″)	λ(° ′ ″)	X<70> (m)	Y<70> (m)
1	North - West	47 56 15	25 45 00.0	715 630.758	556 038.999
2	North - East	47 56 15	25 46 52.2	715 653.569	558 373.914
3	South - West	47 55 00	25 45 00.0	713 314.268	556 061.166
4	South - East	47 55 00	25 46 52.5	713 337.085	558 397.005
Trapezium's area		**540.9816 ha**		**541.0387 ha**	

2.3. The calculation of the areas of the geodetic trapezia from the ellipsoid Krasovski – 1940 and the Stereographic projection plan – 1970

Given its representation, the Stereographic projection -1970 is a conformal projection that keeps the angles unaltered but distorts the lengths and the areas from the ellipsoid.

The ellipsoid area (T) between two latitude parallels (φ_i) and (φ_j), respectively between two meridians of longitude (λ_m) and (λ_n) was determined using the relation:

$$\mathbf{T = [\Delta T\,(\varphi_j)_{\Delta\lambda=1'} - \Delta T\,(\varphi_i)_{\Delta\lambda=1'}]\,(\lambda_n - \lambda_m)'} \qquad [km^2] \text{ or } [ha]$$

where: $\Delta T\,(\varphi_j)_{\Delta\lambda=1'}$ and $\Delta T\,(\varphi_i)_{\Delta\lambda=1}$ are elements of ellipsoid area comprised between the Ecuador and the latitude parallel (φ_j) and respectively (φ_i), for a longitude difference of **one minute** ($\Delta\lambda = 1'$) that were calculated using formulas with constant coefficients (A, B, C, D)

$$\mathbf{\Delta T\,(\varphi_i)_{\Delta\lambda=1'} = A \sin\varphi_i - B \sin 3\varphi_i + C \sin 5\varphi_i - D \sin 7\varphi_i} \qquad [km^2] \text{ or } [ha]$$

$$\mathbf{\Delta T\,(\varphi_j)_{\Delta\lambda=1'} = A \sin\varphi_j - B \sin 3\varphi_j + C \sin 5\varphi_j - D \sin 7\varphi_j} \qquad [km^2] \text{ or } [ha]$$

$(\lambda_n - \lambda_m)'$ is the longitude difference expressed in minutes and parts of minutes, which, in the case of the geodetic trapezium, at a scale of 1:5 000, is of **1′.875**.

The trapezium's area (S) represented in the Stereographic projection plane – 1970 was determined according to the plane rectangular coordinates (X<70>, Y<70>) of the trapezium's corners, at a scale of 1:5 000, with the formulas:

$$\pm 2S = \sum_{i=1}^{n} x_i\,(y_{i+1} - y_{i-1}) = \sum_{i=1}^{n} y_i\,(x_{i+1} - x_{i-1}) \qquad [ha]$$

The values determined for the **ellipsoidal area (T)** and **the area from the projection plane (S)** depending of the geographical ellipsoidal coordinates and respectively, of the Stereographic plane coordinates 1970, in the case of the L-35-4-B-a-3-III trapezium (Bilca), at a scale of 1:5 000 are presented in Table 1.

2.4. The deformation calculation from the secant plane of the Stereographic projection -1970

For evaluating the deformations from the central point of the L-35-4-B-a-3-III trapezium (Bilca) untranslated coordinates (X_{sec}, Y_{sec}) were used with the origin of the system of coordinates in the projection pole situated in the centre of Romania's teritory (Figure 1).

Depending of the position of the point taken into consideration from the middle of the L-35-4-B-a-3-III trapezium, at a scale of 1:5 000, situated at **221.985 km** from **the projection pole Q_0 (φ_0, λ_0)** the following length and surface deformations were calculated.

- Relative linear deformation from the unique secant plane – 1970 (D_{sec})

$$D_{sec} = D_o + \frac{L^2}{4R_0^2} = 0.000\ 052\ 753\ km/km = 5.27 cm/km$$

where: **D_0 = 0.000 250 km/km** is the relative linear deformation in the projection's pole Q_0 (φ_0, λ_0), from the unique secant plane - 1970;

$L^2 = (x_{sec}^2 + y_{sec}^2)$ is the distance between the plane image of the projection's pole $O(X_0, Y_0)$ and the central point of the considered geodetic trapezium $M(X_{sec}, Y_{sec})$;

R_0 = 6378.956 681 km is the average curvature ray of the ellipsoid Krasovski 1940 for the latitude φ_0= 46° N of the Stereographic projection pole – 1970.

- Linear deformation module from the unique secant plane -1970 (μ)

$\mu = 1 + D_{sec}$ = 1.000 052 753

- Areolar deformation module from the unique secant plane -1970 (p)

$p = \mu^2$ = 1.000 105 509

- Relative areolar deformation from the unique secant plane -1970 (P)

P = (p – 1) =0.000 105 509

- Total areolar deformation from the unique secant plane -1970 (ΔT)

ΔT = P x T = 0.0571 ha = 571 m^2

ΔT = (S – T) = 0.0571 ha = 571 m^2

where: **P** – relative areolar deformation from the unique secant plane -1970;

T – the trapesium's area from the reference ellipsoid Krasovski 1940;

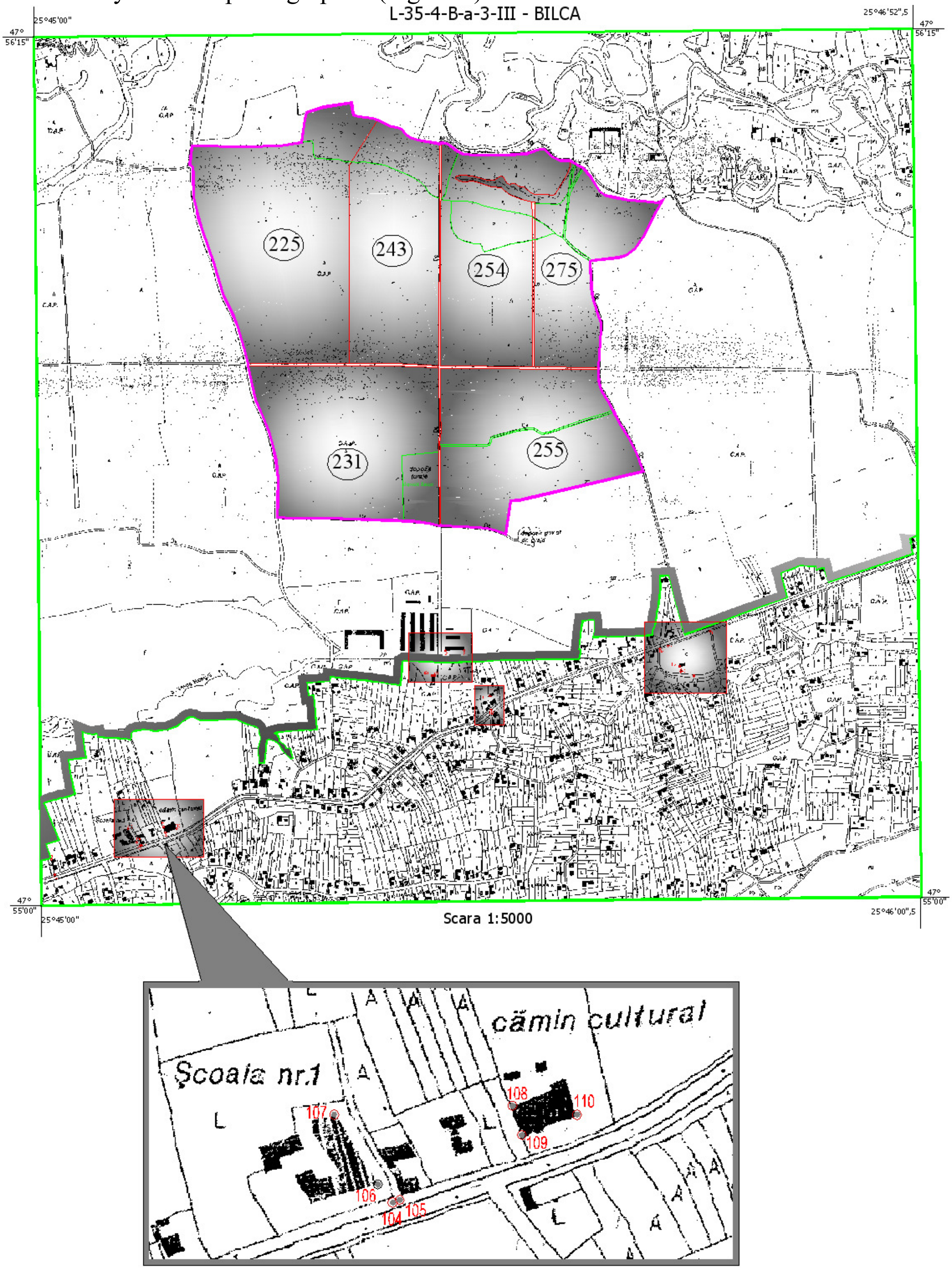
S – the trapesium's area from the unique secant plane - 1970.

The regional deformations determined in the case of the geodetic trapezium taken into consideration were made evident by a relative linear deformation of **+5.27 cm/km** and total areolar deformation of **+571 m^2.** From the point of view of the cartographic representation, the length and surface deformations answer to the accuracy characteristics of drawing up the basic cadastral plan at large and very large scales.

2.5. Georeferentiation of the basic cadastral plan, scale 1:5 000

For the integration of raster data in the Stereographic projection system – 1970 the **affine transformation method** was used. This method is currently used in the georeferentiation process of scanned topographic plans and maps (2).

For the accurate georeferentiation of raster data the graphic fund of the basic cadastral plan was used, at a scale of 1:5 000, with the nomenclature L-35-4-B-a-3-III that included the limits of cadastral sectors and cadastral parcels that existed in 1978, the year the Bilca territory was aerophotographed (Figure 3).



LEGEND: ○ **106 and 107** – ground control points, the School building Bilca;
○ **108, 109 and 110** – ground control points, Community centre's building Bilca;
225, 231, 243, 254, 255, 275 – agricultural physical blocks from the unincorporated Bilca area.

Picture 3 – Georeferentiation of the basic cadastral plan, scale 1:5 000
and the position of ground control points

In the case of affine transformation in a bidimentional space, separate corrections are introduced for each of the two directions of the axes of coordinates. The affine transformation operation includes translation, rotation and the scale factor but the deformations are different on the two axes of coordinates (X, Y). The most often used transformation formulas are those of the affine transformation, like: **Xc = AX + BY + C**

Yc = DX + EY + F

The correlation/rectification of the raster image in CAD software was performed according to the 24 control points, of which: 4 points are represented by the geodetic trapezium's corners and 20 intersection points of the axes of the kilometric grid with the side of 0.5 km. The distribution and the selection of the control points of the mathematical transformation of the raster image took into consideration the uniform distribution principle of the images on the graphic support of the cadastral plan, thus assuring the accuracy.

The values of the coordinates of the control points calculated through georeferentiation (**x_r and y_r**) and the values of the measured control points (**x_i and y_i**) that are represented by the trapezium's corners and by the points of the kilometric network are presented in Table 2.

Table 2. Stereographic plane coordinates – 1970, calculated and measured, of the trapezium's corners and of the points of the kilometric network

Point no.	Point position	Calculated stereographic coordinates (m)		Measured stereographic coordinates (m)		Coordinated differences (m)		
						(x_r-x_i)	(y_r-y_i)	RMSE
		Xr < 70 >	Yr < 70 >	Xi < 70 >	Yi < 70 >	XR_i	YR_i	R_i
		L-35-4-B-a-3-III Trapezium						
1	North West	715629.342	556039.690	715630.758	556038.999	-1.416	+0.691	1.576
2	North East	715653.385	558376.010	715653.569	558373.914	-0.184	+2.096	2.104
3	South West	713312.343	556058.641	713314.268	556061.166	**-1.925**	**-2.525**	**3.175**
4	South – East	713336.653	558397.586	713337.085	558397.005	-0.432	+0.581	0.724
1.1	Intersection of kilometric grid axes	715500.738	556500.988	715500.000	556500.000	+0.738	+0.988	1.233
1.2		715500.765	557000.963	715500.000	557000.000	+0.765	+0.963	1.229
1.3		715500.791	557501.045	715500.000	557500.000	+0.791	+1.045	1.310
1.4		715500.818	558000.979	715500.000	558000.000	+0.818	+0.979	1.275
2.1		715000.711	556500.666	715000.000	556500.000	+0.711	+0.666	0.974
2.2		715000.764	557000.802	715000.000	557000.000	+0.764	+0.802	1.107
2.3		715000.817	557501.069	715000.000	557500.000	+0.817	+1.069	1.345
2.4		715000.869	558000.943	715000.000	558000.000	+0.869	+0.943	1.282
3.1		714500.331	556500.344	714500.000	556500.000	+0.331	+0.344	0.477
3.2		714500.438	557000.641	714500.000	557000.000	+0.438	+0.641	0.776
3.3		714500.544	557501.094	714500.000	557500.000	+0.544	+1.094	1.221
3.4		714500.651	558000.916	714500.000	558000.000	+0.651	+0.916	1.123
4.1		713999.969	556500.022	714000.000	556500.000	**-0.031**	**+0.022**	**0.038**
4.2		714000.244	557000.480	714000.000	557000.000	+0.244	+0.480	0.538
4.3		714000.518	557501.118	714000.000	557500.000	+0.518	+1.118	1.232
4.4		714000.793	558000.890	714000.000	558000.000	+0.793	+0.890	1.191
5.1		713499.020	556499.700	713500.000	556500.000	-0.980	-0.300	1.025
5.2		713499.334	557000.318	713500.000	557000.000	-0.666	+0.318	0.738
5.3		713499.649	557501.142	713500.000	557500.000	-0.351	+1.142	1.194
5.4		713499.963	558000.863	713500.000	558000.000	-0.037	+0.863	0.863
	Total Root Mean Square Error - RMSE							**1.157**

The accuracy of the georeferentiation process of the basic cadastral plan was evaluated by pointing out and verifying the differences between the values of the plane stereographic coordinates of the 24 control points. Depending of the values of these differences, that are presented in Table 2, between the calculated coordinates and the measured coordinates, the following parameters of transformation precision were determined (2, 8).

RMS error (RMSE) is the distance between the input (map or reference) location of a ***Ground Central Point (GCP)*** and the transformed location for the same GCP.

It is a difference between the desired output coordinate for a GCP and the actual output coordinate for the same point is transformed with the geometric transformation.

Root Mean Square Error (**RMSE**) is calculated with a distance equation:

$$RMS_{error} = \sqrt{(x_r - x_i)^2 + (y_r - y_i)^2} \quad [m]$$

where: x_i **and** y_i are the input source coordinates;
x_r **and** y_r are the retransformed coordinates.

The RMS error of each point is calculated and reported to evaluate the GCPs. This is calculated with a Euclidean distance formula:

$$R_i = \sqrt{XR_i^2 + YR_i^2} \quad [m]$$

$$R_1 = \sqrt{(-1.416^2) + (0.691^2)} = 1.576\ m$$

where: R_i – the RMS error for GCPi;
XR_i – the X residual for GCpi;
YR_i – the Y residual for GCpi.

The values determined for the 24 control points ranged between a minimum value of **0.038 m** and a maximum value of **3.175 m** (Table 2).

For the residuals, the following calculations are made to determine the total RMS error, the X RMS error, and the Y RMS error:

$$R_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n} XR_i^2}\ ; \quad R_y = \sqrt{\frac{1}{n}\sum_{i=1}^{n} YR_i^2}\ ; \quad T = \sqrt{R_x^2 + R_y^2} \quad \text{or} \quad \sqrt{\frac{1}{n}\sum_{i=1}^{n} XR_i^2 + YR_i^2}$$

where: R_x – total X RMS error;
R_y – total Y RMS error;
T – total RMS error;
n – the number of GCPs and **i** – GCP number;
XR_i – the X residual for GCPi;
YR_i – the Y residual for GCPi.

The total root mean square error (total RMS error) calculated for the 24 control points (GCPs) was evaluated using the value of **1.157 m**, that confirms a relatively accurate georeferentiation of raster images.

A normalized value representing each point's RMS error in relation to the total RMS error:

$$E_i = \frac{R_i}{T}$$

$$E_{41} = \frac{0.038}{1.157} = 0.033 \quad m$$

where: E_i – error contribution of GCPi;
R_i – the RMS error for GCPi;
T – total RMS error.

In the case study of the georeferentiation process, the standard errors of each analyzed point ranged between a minimum value of **0.033 m** and a maximum value of **2.744m**.

2.6. Evaluation of the georeferentiation accuracy depending of the field measurements with GPS receivers of geodetic accuracy

In the case of the performed study as ground control points were used a series of punctiform elements represented by buildings' corners, means of communication, enclosures and others. The field measurements were performed using the GNSS South S82T receiver and the real time kinematic positioning method that is also known as RTK (*Real Time Kinematic*).

Using the SurvCE field software and the service provided by the Romanian Position Determination System (ROMPOS-RTK) the plane Stereographic coordinates – 1970 of 24 ground control points were determined; they are presented in Table 3.

Table 3. 1970 Plane Stereographic coordinates – 1970, calculated and measured GPS ground control points from Bilca territory

Point no.	Point position	Calculated stereographic coordinates (m)		Measured stereographic coordinates (m)		Coordinated differences (m)		
						(x_r-x_i)	(y_r-y_i)	RMSE
		Xr < 70 >	Yr < 70 >	Xi < 70 >	Yi < 70 >	XR_i	YR_i	R_i
		L-35-4-B-a-3-III Trapezium						
101	Church fence	713439.983	556082.706	713438.021	556085.165	**1.962**	**-2.459**	**3.146**
102	Church fence	713397.872	556095.493	713395.982	556094.555	1.890	0.938	2.110
103	School fence	713463.269	556282.466	713460.851	556283.366	2.418	-0.900	2.580
104	School fence	713475.962	556320.084	713473.103	556320.277	2.859	-0.193	2.866
105	Property fence	713477.360	556325.031	713474.546	556323.956	2.814	1.075	3.012
106	School no.1	713482.963	556312.402	713482.851	556312.331	0.112	0.071	0.133
107	School no.1	713519.983	556289.165	713520.058	556289.081	**-0.075**	**0.084**	**0.113**
108	**Community center**	713524.656	556383.619	713524.788	556383.820	-0.132	-0.201	0.240
109	**Community center**	713509.459	556389.041	713509.277	556388.895	0.182	0.146	0.233
110	**Community center**	713519.996	556417.971	713520.079	556418.133	-0.083	-0.162	0.182
111	City hall	713872.945	557251.217	713873.140	557251.073	-0.195	0.144	0.242
112	City hall fence	713839.143	557252.735	713836.865	557253.904	2.278	-1.169	2.560
113	Cemetery	713999.203	557702.777	713999.387	557702.932	-0.184	-0.155	0.241
114	Cemetery	714007.219	557724.762	714007.073	557724.727	0.146	0.035	0.150
115	Cemetery	714046.520	557836.283	714046.331	557836.260	0.189	0.023	0.190
116	Cemetery	713952.130	557755.643	713952.064	557755.805	0.066	-0.162	0.175
117	Steeple	713941.953	557758781	713941.237	557758.491	0.716	0.290	0.772
118	Steeple	713938.905	557758.962	713937.877	557758.215	1.028	0.747	1.271
119	Graveyard	713929.644	557792.570	713928.061	557792.964	1.583	-0.394	1.631
120	Building 1 CAP	713995.841	557181.681	713994.351	557182.564	1.490	-0.883	1.732
121	Building 1 CAP	713994.252	557132.870	713994.092	557132.613	0.160	0.257	0.303
122	Building 2 CAP	713926.982	557103.683	713927.193	557103.642	-0.211	0.041	0.215
123	Building 2 CAP	713927.665	557097.383	713927.476	557097.110	0.189	0.273	0.332
124	School fence	713476.287	556320.86	713475.540	55632.65	0.747	-0.790	1.087
	Total Root Mean Square Error - RMSE							**1.063**

The accuracy of the georeferentiation operation was also verified by direct field measurements made with GPS receivers.

The resulted data were used to establish the differences between the calculated coordinates (x_r and y_r) and the measured coordinates (x_i and y_i). Depending of these values the same parameters were established with the up mentioned algorithm.

The RMSE values determined for the 24 control points ranged between a minimum limit of **0.113 m** and maximum of **3.146 m.**

The total root mean square error (Total RMS Error) calculated for the 24 ground control points was estimated using the value **1.063 m** that confirms the inclusion of the field measurements within the accuracy limits for the georeferentiation process of raster images.

The standard errors of each ground control point were calculated depending on the partial errors and respectively, the total mean square error thus resulting values ranging between **0.106 m** (School building Bilca) and **2.960 m** (Table 3).

The RMSE – residual values of the 24 control points (GPS) resulted from the measurements made with GPS equipment presented values relatively close to the ones analyzed previously, for the control points from the vectorized plan.

The digital conversion of the existent analog cartographic material was made with a **graphic error of +/- 0.2 mm** that corresponds to the precision of the numerical scale.

The maximum graphic error admitted in reporting the points from the geodetic and elevation network, for the cadastral plan, **scale 1:5 000** was of **+/- 1.00 m.**

2.7. Calculation of vectorized / digital and field measured surfaces

The calculation of the surfaces of the six agricultural physical blocks required the use of the plane Stereographical coordinates – 1970 of the points from the geometric contour (Figure 3). For this purpose the following elements were used: the plane coordinates extracted from the vectorized support of the cadastral plan, scale 1:5 000, those from the digital orthophotoplan – scale 1:10 000 and from the direct field measurements. In calculating their surfaces the inventory of plane coordinates (x_i, y_i) of the points from the geometric contour of each physical block were used along with the following general relations:

$$2S = \sum_{i=1}^{n} x_i \left(y_{i+1} - y_{i-1}\right) \quad \text{and} \quad -2S = \sum_{i=1}^{n} y_i \left(x_{i+1} - x_{i-1}\right) \qquad [\mathrm{m}^2]$$

The resulting surfaces of the six agricultural physical blocks were determined using the previously mentioned methods are included in Table 4.

Table 4 Surfaces calculated on digital support and GPS measured of the physical agricultural blocks from Bilca territory

Agricultural physical block's code	Agricultural physical blocks' surface			Surface differences	
	Vectorized cadastral plan 1:5 000/1978	Digital on the orthophotoplan 1:10000/2006	Measured in the field, GPS in 2011	$\pm (S_i - S)$	$\pm (S_j - S)$
	S_i (ha)	S_j (ha)	S (ha)	ha	ha
225	22.7634	22.9631	22.9875	-0.2241	-0.0244
231	18.6214	18.7384	18.7485	-0.1271	-0.0101
243	14.4787	14.4989	14.5134	-0.0347	-0.0145
254	14.3317	14.6294	14.6582	-0.3265	-0.0288
255	17.1101	17.0118	16.9222	+0.1879	+0.0896
275	10.6842	10.4438	10.4546	+0.2296	-0.0108
Roads	1.3149	1.3309	1.3001	+0.0148	+0.0308
TOTAL	**99.3044**	**99.6163**	**99.5845**	**-0.2801**	**+0.0318**

For integrating the determined surfaces in the vectorized cadastral plan and, respectively in the digital orthophotoplan, in the case of the six physical agricultural blocks, it was included the condition that their surface were equal to the surface measured in the field, of the entire cadastral sector of **99.5845 ha**, with compulsion on this size. The accepted tolerance was calculated with the formula: $\mathbf{T = 0.0003\ N\sqrt{S}\ (m^2)}$

where: **N** – scale plan's denominator

S – the surface expressed in square meters.

For the plan's scale of 1:5 000 and the cadastral sector's surface of **995.845 $\mathbf{m^2}$** resulted a tolerance of **1.497 $\mathbf{m^2}$**. In the case of the orthophotoplan scale 1:10 000 and the same surface of the cadastral sector resulted the value of **2.294 $\mathbf{m^2}$**.

In the case of the surfaces measured with GPS equipments the following formula for calculating the tolerance was used for closing the physical blocks on the total surface of the cadastral sector: $\mathbf{TS = Et\ \sqrt{S}\ (m^2)}$

where: **Et** – the accepted error in appreciating the points' position in the field and the limits between the properties (0.05–0.10 m for the incorporated areas and 0.10–0.20 m, for the unincorporated areas);

S – the surface expressed in square meters.

After compensating the surfaces of the vectorized / digital physical blocks on the total surface of the cadastral sector of **99.5845 ha**, they were constrained on the **control area of 540.9816 ha**. This surface of the geodetic trapezium, nomenclature L-35-4-B-a-3-III, represents the undistorted measure from the projection plan.

3. Conclusions

The distorsions determined in the case of the L-35-4-B-a-3-III geodetic trapezium from the cadastral territory of Bilca commune were pointed out with a **relative linear deformation of +5.27 cm/km** and a **total areolar deformation of +571 m^2** that respects the precision requirements for drawing up the basic cadastral plan at large and very large scales.

The correlation/rectification of the raster images was made possible with a CAD software that used the cadastral plan at a scale of 1:5 000, depending of the usage of the **24 control points** represented by the corners of the geodetic trapezium and the intersection points of the axes of the kilometric grid with the side of **0.5 km**.

The accuracy of the georeferentiation process of the cadastral plan was made evident depending of the differences between the coordinates of the control points determined from the raster image and the real / measured ones in the field with GPS receivers of geodetic precision; the mean square error was **of 1.157 m** and respectively **of 1.063 m**.

The discrepancy between the vectorized / digital surfaces and the measured surfaces was compensated first of all on the values of the physical blocks measured in the field after which they were constrained to the **control area of 540.9816 ha** of the trapezium.

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