DYNAMIC MODEL FOR DATA PROCESSING OF REPEATED GEODETIC MEASUREMENTS

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***Abstract**: This paper presents a dynamic method for in block data processing of repeated geodetic measurements performed in planimetric networks for direct determination of points movements from one processing epoch to another or from an initial epoch to the actual epoch, and the points movement velocities on the two directions.*

***Keywords:** Adjustment, movement velocity, network*

1. Introduction

By repeated observations in planimetric geodetic networks, complemented by proper measurements data processing, can be computed the horizontal crustal movements displacements in the area covered by geodetic network.

Actually, very often, determination of horizontal crustal movements or of structures on which the points are placed, it is done by separate adjustment of each measurements epoch, and displacements are obtained by comparing the results of adjustment.

By this dynamic method it is outlined that network point positions are affected even inside the measurement epoch.

Data processing results are as good (precision of displacements velocities on the two directions) as more measurements are available in more observation epochs.

2. Ground markers establishment

Geodetic network used for the case study was realized with the goal to determine the recent crustal movements in the area. The markers consisted on deep ground markers including mechanical and thermic isolation by a protective tube and this pipe take as possible only the crustal movements. Forced centering systems included on each ground marker ensure the installation of geodetic instruments and accessories, practically without centering errors.

3. Block data processing of horizontal measurements

Horizontal measurements were performed with a Kern DKM 3 theodolite (can measure $0^{cc}.5$) by complete series method, at least 4 series, on sighting targets. After station data adjustment were obtained standard deviation of $1\text{-}2^{cc}$.

Data processing of horizontal observations was performed by least square adjustment method including as supplementary unknowns, the points velocities expressed in mm/year.

A first step in formulation of functional-stochastic model was the setting of corrections equations. As we consider that the performance of all geodetic observations at one epoch covers usually a longer time interval, we assume that the network point positions are influenced just in this time interval by crustal movements. This hypothesis requires the introduction of supplementary unknowns, abbreviated $\dot{X}_P$ and $\dot{Y}_P$, representing the projections of P site velocity on the correspondent directions:

$$\dot{X}_P = \left(\frac{dX}{dt}\right)_P ; \quad \dot{Y}_P = \left(\frac{dY}{dt}\right)_P \tag{1}$$

The time interval necessary for performing a single geodetic observation it is much short than the time interval necessary for performing of all geodetic observations in the network, and we can assume a measurement moment t_i for each observation included in the processing model.

Therefore, $\tilde{X}_P^i$, $\tilde{Y}_P^i$ coordinates of point P at an observing moment t_i, affected only by his displacement velocity can be written as:

$$\begin{aligned} \tilde{X}_P^i &= X_P'^I + dX_P + \Delta t_{I-i}\dot{X}_P \\ \tilde{Y}_P^i &= Y_P'^I + dY_P + \Delta t_{I-i}\dot{Y}_P \end{aligned} \tag{2}$$

with $X_P'^I, Y_P'^I$ provisional coordinates of P point, at the initial moment t_I and (Fig. 1):

$$\begin{aligned} X_P^I &= X_P'^I + dX_P \\ Y_P^I &= Y_P'^I + dY_P \\ \Delta t_{I-i} &= t_i - t_I \end{aligned} \tag{3}$$

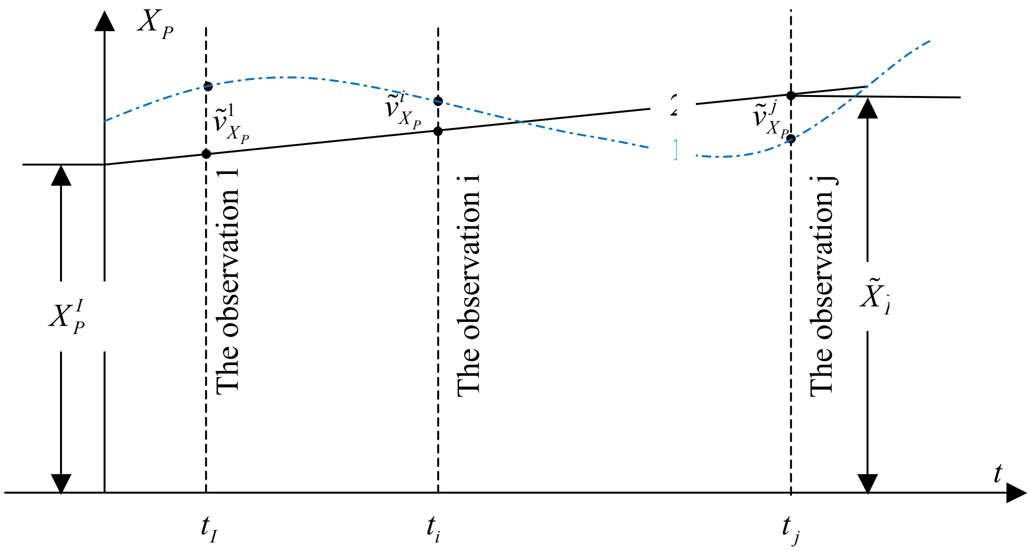

Fig. 1. Time series of X coordinate of point P. 1- real time series , 2-linearized variation (in the sense of least squares), $\tilde{v}$ - random variation

Equations (3) express the real situation of P point movements, only for short time intervals. For a model of such movements for longer time intervals, when the linear movement hypothesis cannot be accepted, will be necessary to introduce for the same point more unknowns as those from equation (2), between the measurement epochs I, II, III etc. It needs to be clear distinguished between real coordinate variation in time and the variation obtained by computation (Fig.1) with the $\tilde{v}$ difference between them. These differences have a random character when a correct functional-stochastic model it is realized and the measurements have a high level of precision.

E. Jacobs developed a functional-stochastic model including constant movements velocities (partial or total) for the hole time interval and neglecting the $\tilde{v}$ corrections:

$$\begin{aligned} \dot{X}_P^{I-II} &= \dot{X}_P^{II-III} = \dot{X}_P^{III-IV} = \dot{X}_P; \quad \tilde{v}_{X_P} = 0 \\ \dot{Y}_P^{I-II} &= \dot{Y}_P^{II-III} = \dot{Y}_P^{III-IV} = \dot{Y}_P; \quad \tilde{v}_{Y_P} = 0 \end{aligned} \tag{4}$$

This functional-stochastic model was accepted in this paper, too.

Correction equation forms

We consider two new points (Fig.2) from the geodetic network with a direct connection (with horizontal directions observed). Provisional position of these points it is expressed by provisional coordinates marked with superscript ^{o} (zero).

The most probable position of these points at actual epoch it is done by adjusted coordinates marked with superscript ^{a}. In the adjustment process will be determined corrections added to the provisional coordinates and will be obtained the adjusted coordinates:

$$\begin{aligned} X_i^a &= X_i^o + dx_i^a + \Delta t_{I-a}\dot{X}_i; \quad X_j^a = X_j^o + dx_j^a + \Delta t_{I-a}\dot{X}_j \\ Y_i^a &= Y_i^o + dy_i^a + \Delta t_{I-a}\dot{X}_i; \quad Y_i^a = Y_i^o + dy_i^a + \Delta t_{I-a}\dot{X}_i \end{aligned} \tag{5}$$

with I – initial epoch, considered as measurements origin; Δt_{I-a} difference in time between the actual epoch and the initial epoch.

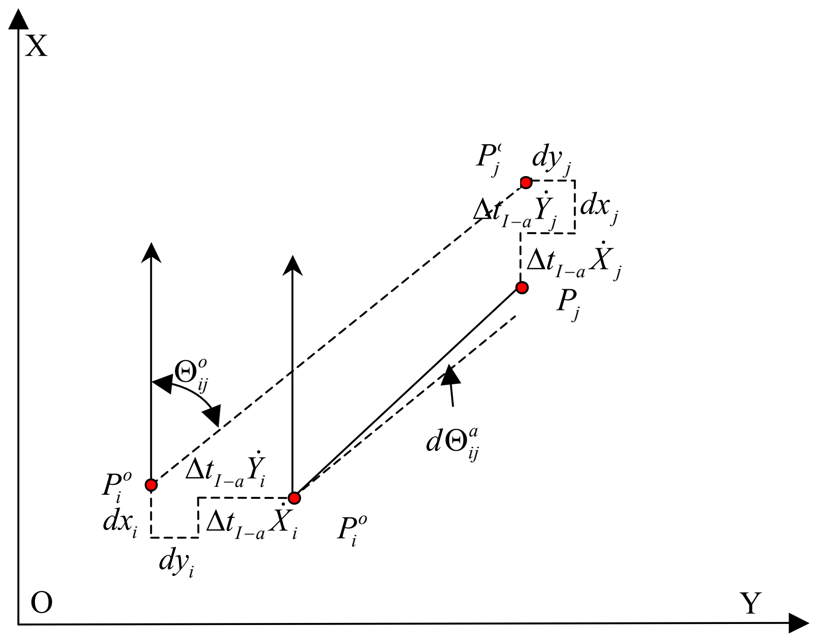

Fig. 2. Orientation variation function of plane coordinates variation

By differentiation of equation who describes the orientation tangent between the two points and by use of following equations:

$$a_{ij} = -\rho^{cc} \frac{\sin \Theta_{ij}^{o}}{D_{ij}^{o}}$$
$$b_{ij} = \rho^{cc} \frac{\cos \Theta_{ij}^{o}}{D_{ij}^{o}} \quad , \tag{6}$$
$$A_{ij} = a_{ij} \Delta t_{I-a}$$
$$B_{ij} = b_{ij} \Delta t_{I-a}$$

it is obtained the following general form of the orientation variation function of plane coordinates variation:

$$\left(d\Theta_{ij}^{a} \right)^{cc} = a_{ij} dx_j^a + b_{ij} dy_j^a - a_{ij} dx_i^a - b_{ij} dy_i^a + A_{ij} \dot{X}_j + B_{ij} \dot{Y}_j - A_{ij} \dot{X}_i - B_{ij} \dot{Y}_i \tag{7}$$

In a point „i" for the observed direction towards „j" point, after adjustment, the following equation should be accomplished:

$$\alpha_{ij}^{l} = -Z_i + \Theta_{ij} \tag{8}$$

with α_{ij} adjusted horizontal direction, Z_i angle of orientation of the station, Θ_{ij} the most probable (adjusted) value of orientation between the two points.

In the previous equation there are considered the known values and the values obtained by adjustment, the general form of the correction equation for a horizontal direction will be:

$$\left(v_{ij}^{a} \right)^{cc} = -dZ_i^a + d\Theta_{ij}^a + l_{ij}^a, \text{ unde } \quad l_{ij}^a = \left(\Theta_{ij}^{o} - \left(\alpha_{ij}^{*} \right)^{a} \right) - \left(Z_i^{o} \right)^{a} \tag{9}$$

In the matrix formulation of the data adjustment by least square method, first the correction equations linear system it is assumed as follows:

$$v = Ax + l \tag{10}$$

The most probable values of unknowns are obtained by least square method as follows:

$$x = -N^{-1} A^{T} Pl \tag{11}$$

For the precisions there are computed the following elements:

- Standard deviation of unit weight:

$$s_o = \pm \sqrt{\frac{[pvv]}{m-n}} \tag{12}$$

with *m* representing the total number of observations from the geodetic network in the measurement epochs, and *n* is the total number of unknowns involved in the model.

- Standard deviation of unknowns for coordinate differences:

$$s_{dx_i} = s_o \sqrt{Q_{XX_i}}$$
$$s_{dy_i} = s_o \sqrt{Q_{YY_i}} \tag{13}$$

- Standard deviation of point velocities:

$$s_{\dot{X}_i} = s_o \sqrt{Q_{\dot{X}\dot{X}_i}}$$
$$s_{\dot{Y}_i} = s_o \sqrt{Q_{\dot{Y}\dot{Y}_i}} \tag{14}$$

- Standard deviation of point positions:

$$s_{X_i} = s_o\sqrt{Q_{FF}^{X_i}}; \quad Q_{FF}^{X_i} = Q_{XX_i} + \Delta t_{I-a}^2 Q_{\dot{X}\dot{X}_i} + 2\Delta t_{I-a} Q_{X\dot{X}_i}$$
$$s_{Y_i} = s_o\sqrt{Q_{FF}^{Y_i}}; \quad Q_{FF}^{Y_i} = Q_{YY_i} + \Delta t_{I-a}^2 Q_{\dot{Y}\dot{Y}_i} + 2\Delta t_{I-a} Q_{Y\dot{Y}_i} \tag{15}$$

- Standard deviation of movements:

$$s_{\Delta X_i} = s_o\sqrt{Q_{\dot{X}\dot{X}_i}} \cdot \Delta t_{I-a}$$
$$s_{\Delta Y_i} = s_o\sqrt{Q_{\dot{Y}\dot{Y}_i}} \cdot \Delta t_{I-a} \tag{16}$$

with $Q = N^{-1}$.

4. Case study

For the case study a geodetic network was considered including 12 points with two of them supposed fixed (Fig.3)

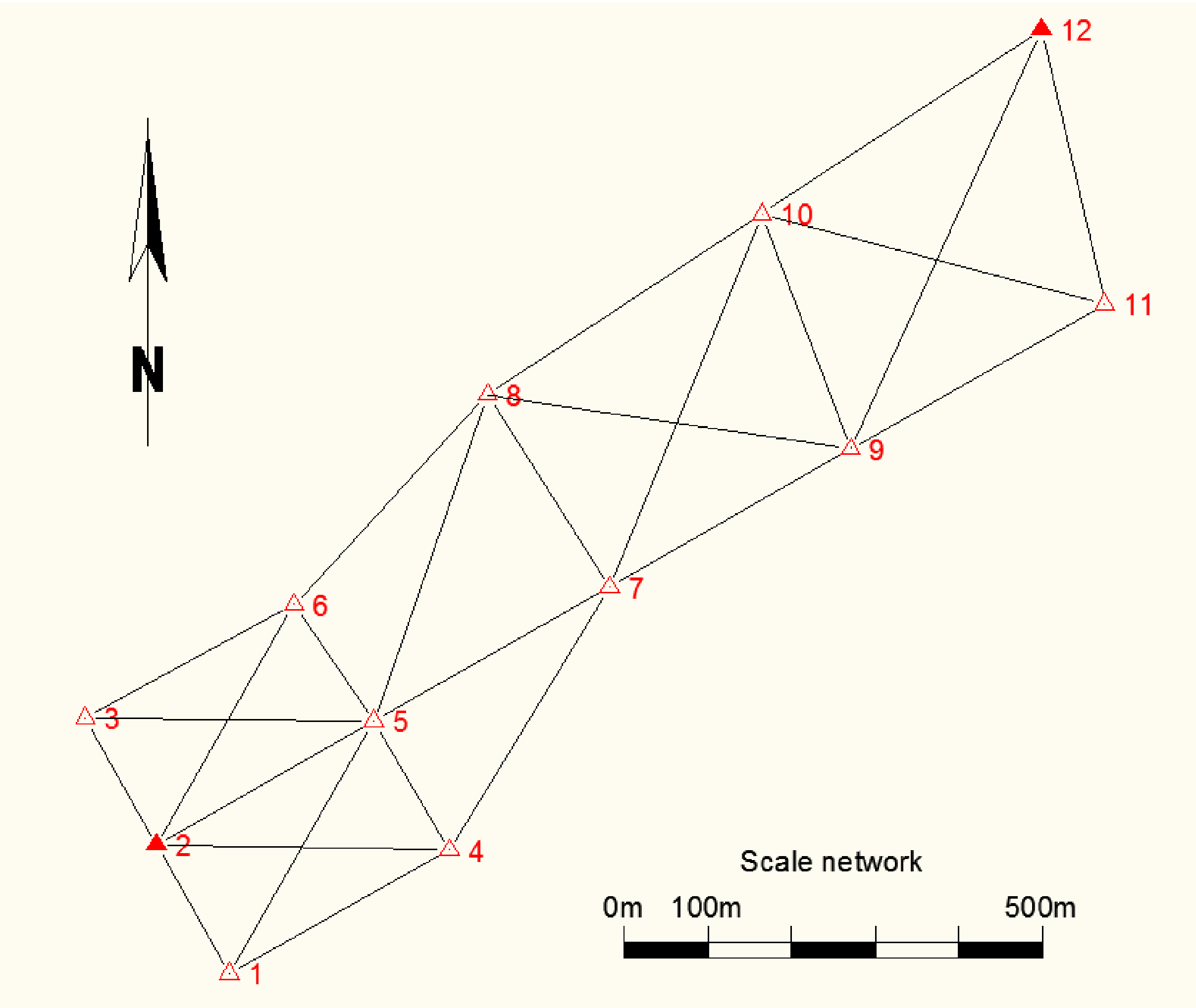

Fig. 3. Geodetic network sketch

In the position of P point situated on the terrestrial surface can be determined by geodetic methods, with „a posteriori" standard deviation s_P, a point displacement ΔD_P between the two measurement epochs, can be considered, only when:

$$|\Delta D_P| > t_s |s_P| \tag{17}$$

with coefficient t_s (2.008 for the case study) dependent of statistic reliability S (usually 95%) and the number of supplementary equations f (52 in the case study) existing on the geodetic network on the two observation epochs.

After data processing, the results are statistically analyzed for an improvement of functional-stochastic model and after, a new data processing can be performed.

For the improvement of model structure, it is analyzed the ratio between the unknown and his standard deviation:

$$t_k = x_i / s_{x_i} \tag{18}$$

If this value it is lower than t_s value from „Student" table, it can be removed in the future from the data processing model with the assumption that the movement can be due to the measurement errors.

After statistical analysis, according to previous steps, the points 4 and 6 were determined with a significant movement on X direction, and point 3 on Y direction. The values obtained are presented in table 1.

Table 1. Results after block data processing

Point	Movements on X		t_k	Movements on Y		t_k
	ΔX [mm]	$s_{\Delta X}$ [mm]		ΔY [mm]	$s_{\Delta Y}$ [mm]	
1	-3.0	2	1.51	-0.9	1	0.52
3	-3.5	2	1.72	**-3.3**	**1**	**3.28**
4	**-2.0**	**1**	**2.02**	-5.1	3	1.70
5	-3.5	2	1.76	-3.1	2	1.56
6	**-4.2**	**2**	**2.09**	-1.9	2	0.97
7	3.7	3	1.23	1.6	3	0.52
8	1.2	3	0.41	1.8	3	0.58
9	4.6	3	1.54	0.4	2	0.18
10	-1.5	2	0.75	-2.4	3	0.79
11	1.9	3	0.64	-0.9	1	0.31

For block data adjustment of observation performed in the two measurement epochs, the velocities are expressed in mm/year. For that, the velocities unknown coefficients were divided to 365, the time interval between the initial epoch and the last analyzed epoch was 154 days and between epochs was of 76 days.

It can be mentioned that was realized and a classical data processing, a separate data processing for each measurement epoch. Adjusted coordinates for each epoch are presented in table 2, and the movements resulted from coordinates difference. Based on these movements point velocities were computed expressed in mm/year. In the same table there are presented the velocities obtained after block data processing.

Table 2. Comparison between velocities obtained by two methods
(Classical – static and dynamic)

CLASSICAL (STATIC) METHOD									DYNAMIC METHOD	
Point	Coord. Epoch 1		Coord. Epoch 2		Movements		Site velocities		Site velocities	
	X [m]	Y [m]	X [m]	Y [m]	[mm]	[mm]	[mm/y]		[mm/y]	
1	8356.193	8714.931	8356.189	8714.930	-4	-1	-9.5	-2.4	-8.1	-2.5
3	8662.716	8542.839	8662.712	8542.836	-4	-3	-9.5	-7.1	-9.3	-8.8
4	8504.098	8978.713	8504.095	8978.712	-3	-1	-7.1	-2.4	-5.4	-13.7
5	8658.547	8887.771	8658.545	8887.777	-2	0	-4.7	0.0	-9.4	-8.3
6	8798.501	8792.312	8798.499	8792.313	-2	1	-4.7	2.4	-11.2	-5.2
7	8819.025	9170.281	8819.028	9170.286	3	5	7.1	11.8	9.9	4.2
8	9049.626	9023.666	9049.627	9023.669	1	3	2.4	7.1	3.3	4.7
9	8984.782	9458.288	8984.788	9458.291	6	3	14.2	7.1	12.4	1.1
10	9266.562	9352.256	9266.561	9352.254	-1	-2	-2.4	-4.7	-4.0	-6.4
11	9158.198	9762.665	9158.205	9762.664	7	-1	16.6	-2.4	5.1	-2.5

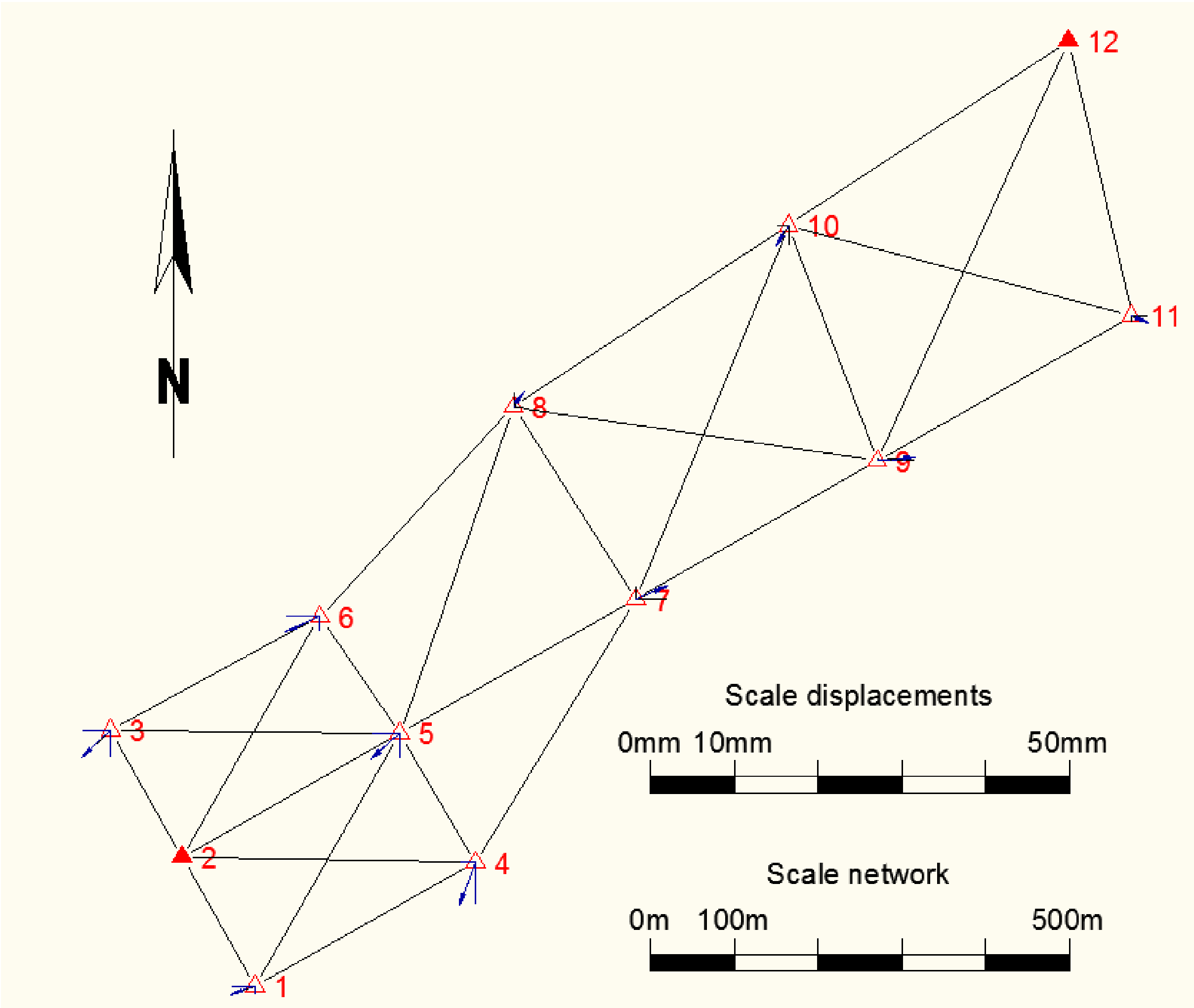

Fig. 3. Geodetic network sketch with movement vectors

5. Conclusions

On the present paper the authors have presented a method for data processing of repeated observations performed in geodetic network with the goal to determine the network points movements. There are also other more complex data processing methods for repeated observations and movements determination involving deeper information on data processing and mathematical statistics. The proposed method it is much more simple and it is accompanied by a statistical analysis of results making it more reliable than conventional methods to determine movements.

From results analysis of the two data processing presented in table 2, it can be observed that in few points there are differences between the velocities obtained. The problem it is that on conventional data processing (separated on measurement epochs) movements can be obtained from coordinate differences, but cannot say anything about these movements, in the sense they are or not significant.

The proposed method of data processing with a dynamic model, the movements and site movement velocities are determined, and the most important is that a statistical analysis of results it is performed which shows that movements are significant or due to measurements errors.

6. References

1. *Jacobs, E.: Vertikale Bodenbewegungen im weiteren Bereich des Niederrheinisch-Westfälischen Steinkohlengebietes;*
2. *Moldoveanu, C.: Geodezie. Noțiuni de geodezie fizică şi elipsoidală, poziționare. Editura MATRIX ROM, Bucureşti 2002;*