MODELLING THE ROMANIAN QUASIGEOID USING EGG97 MODEL, GNSS AND LEVELLING MEASUREMENTS

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***Abstract**: The precise determination of the national quasi-geoid it is one of the most important task of the Romanian geodesy. Mathematical models for generate its surface combine with the available data could reach to multiple solutions. The geometric modelling using Delaunay triangulation with bicubic spline interpolation method for values of the anomaly of the altitude obtained from GNSS and levelling measurements represents a possible solution. In order to determine a precise quasigeoid model, the European model EGG97 can be used as support. In this case the solution is to correct EGG97 model with a surface that was determined from GNSS and levelling measurements. It will be obtain a quasi-gravimetric model with a solid reference to the national normal altitude system. For the determination of this conversion surface it will be use an algorithm developed under Matlab application.*

***Keywords**: geodetic levelling, geoid, quasi-geoid, GNSS, surface generation, geometric modelling, EGG97, Matlab.*

1. Introduction

A surface designated to approximate the figure of the Earth is the quasigeoid – a surface introduced by Molodensky (1960) as a practical solution for the problems that must be resolved in geodesy. The quasigeodetical height known as the anomaly of the altitude could be calculated, theoretical, exactly.

Even the quasigeoid did not be physically defined, as the geoid is, it is used from many countries as surface as reference for the altitudes. The altitude system that uses this surface as reference is the normal altitude system.

2. General considerations

The computations and the determinations of the geoids models enjoy a real interest for the specialist around the world. On international each country tried to determined and modelling with precision a national geoid model that could offer a real support for all the measurements in the terrestrial and spatial geodesy. With a particular database using measurements and geoid models at larger scale (exp: European geoid model EGG97 or Global Geoid model EGM2008), the west European countries and the American Continent have developed such geoid models.

For the determination of the geoid model there are more methods function of available data there for the modelling the quasigeoid on Romanian territory exist a rich mathematical support.

In Romania, the situation of the determination of a quasigeoid is debated in more specialist reunions. Till now, in Romania, there is not developed a quasigeoid model that could assure high precision for the determination made with modern GNSS technology and trans calculate the ellipsoidal heights in national reference altitude system

After a rigorous analysis on the available data for the Romanian territory and that could be used for modelling the quasigeoid surface we have found that the majority of them are represented by GNSS levelling (precise or trigonometric) determination. In this way, a possible solution for the quasigeoid generation is represented by the geometrical method.

3. Modelling the quasigeoid on the Romanian territory

One of the methods for generating surfaces is represented by the Delaunay triangulation method. Delaunay method with cubical interpolation is a known method, being one of the first methods used before the development of the computation technology.

The algorithm creates triangles drawing lines between the scattered data points. The points are connected in such a way that a triangle never is crossed by other triangles. The result of the triangulation is an irregular network form by plan faces that generate a surface. Each triangle defines a plan that forms the nods of a grid with slope and elevation determined by minimum three points that define the triangle. All the grid nodes are defined by the triangle tops. The triangulation with linear interpolation is the right one when the distribution of the scattered date is poor.

The method has optimum solutions for large surfaces, as in the case of the Romanian territory, the points of the study case creating a triangular irregular network that fulfil the conditions of Delaunay triangulation. Besides the linear interpolation used in triangulation the nearest neighbour interpolation method is applied for generating acceptable solutions where data are insufficient and solutions needed to assess the results extrapolated.

The Delaunay triangulation is implemented in Matlab software through "griddata" function. The "griddata" function uses the finite element method for resolving the triangulation. This method is used on large scale in surfaces modelling field.

For the generation of the quasigeoid model using Delaunay triangulation with cubic interpolation it was used a data set of 680 points determined by GNSS satellite methods and levelling measurements (geometric or trigonometric).

The geometrical method for the determination of the anomaly of the altitude assumes the following mathematic relation:

$$\zeta_i = h_i - H_i \quad , \text{ i – considered point}$$

where the difference between the ellipsoidal altitude and normal altitude represents the anomaly of the altitude.

The used data were graphical and analytic filtrated and after was generated a text file under the following structure (Table 1):

Table 1. The coordinates of the points

No. Point	Latitude B (° , „)	Longitude L (° , „)	Ellipsoid height h (m)	Normal height H (m)	Anomaly of the altitude $\zeta_{ETRS89\text{-}Niv}$ (m)
1	46 03 53.70491	23 34 29.58419	282.001	239.854	42.147
2	44 01 57.83694	23 21 08.85045	102.661	61.222	41.439
3	47 06 54.77613	24 28 11.10961	384.642	345.405	39.237
4	47 50 23.05978	22 55 06.62369	166.081	127.137	38.944
…	…	…	…	…	…

The distribution of the scattered data points is represented in the Figure 1. There are uniform distributed on all Romanian territory.

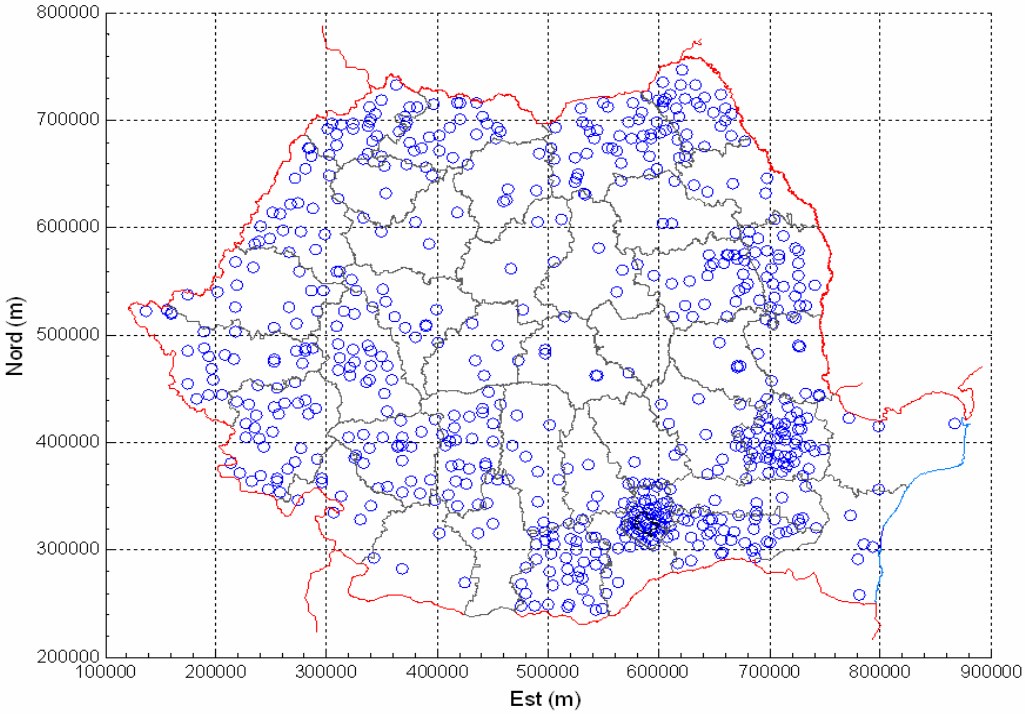

Figure 1. The distribution of the points used to model the quasigeoid on the Romanian territory

A completely new approach regarding the determination of the quasigeoid model on the Romanian territory is represented by the utilisation of the anomalies of the altitude obtained by interpolating the European EGG97 regional model that will be corrected with the values of the anomaly of the altitude determined by GNSS and levelling measurements. The method is not supposed to constrain the European model but to correct it with another surface. In this case, knowing that the European reference for this geoid is the system of normal heights with fundamental zero point Amsterdams Peil Normaal, through this correction with the anomaly of the altitude determined on the normal reference system, practically the reference system of the European model will be change for the Romanian territory. The European quasigeoid model is a gravimetric one, so through its correction with determinations of the anomaly of the altitude from GNSS and levelling measurements it will be obtain a quasigravimetric conversion surface.

The solution it was computed for 3 situation of grid spacing for the quasigeoid model (dN=dE=16000m, dN=dE=8000m şi dN=dE=3000m planimetric coordinates), following multiple steps.

The determination of the anomaly of the altitudes corrections in points of the scattered data set is realised with the relation:

$$v_{i\zeta} = \zeta_{iETRS89\text{-}Niv} - \zeta_{iEGG97\_RO}$$

where: v_ζ – the vector of the differences for the anomaly of the altitude

$\zeta_{ETRS89\text{-}Niv}$ - the vector of the anomaly of the altitude from GNSS and levelling measurements

ζ_{EGG97_RO} – the vector of the anomaly of the altitude from the interpolation into the European EGG97 model