

MODELLING THE ROMANIAN QUASIGEOID USING EGG97 MODEL, GNSS AND LEVELLING MEASUREMENTS

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Abstract: *The precise determination of the national quasi-geoid is one of the most important tasks of the Romanian geodesy. Mathematical models for generating its surface combine with the available data could reach to multiple solutions. The geometric modelling using Delaunay triangulation with bicubic spline interpolation method for values of the anomaly of the altitude obtained from GNSS and levelling measurements represents a possible solution. In order to determine a precise quasigeoid model, the European model EGG97 can be used as support. In this case the solution is to correct EGG97 model with a surface that was determined from GNSS and levelling measurements. It will be obtained a quasi-gravimetric model with a solid reference to the national normal altitude system. For the determination of this conversion surface it will be used an algorithm developed under Matlab application.*

Keywords: *geodetic levelling, geoid, quasi-geoid, GNSS, surface generation, geometric modelling, EGG97, Matlab.*

1. Introduction

A surface designated to approximate the figure of the Earth is the quasigeoid – a surface introduced by Molodensky (1960) as a practical solution for the problems that must be resolved in geodesy. The quasigeoid height known as the anomaly of the altitude could be calculated, theoretical, exactly.

Even the quasigeoid did not be physically defined, as the geoid is, it is used from many countries as surface as reference for the altitudes. The altitude system that uses this surface as reference is the normal altitude system.

2. General considerations

The computations and the determinations of the geoids models enjoy a real interest for the specialist around the world. On international each country tried to determine and modelling with precision a national geoid model that could offer a real support for all the measurements in the terrestrial and spatial geodesy. With a particular database using measurements and geoid models at larger scale (exp: European geoid model EGG97 or Global Geoid model EGM2008), the west European countries and the American Continent have developed such geoid models.

For the determination of the geoid model there are more methods function of available data there for the modelling the quasigeoid on Romanian territory exist a rich mathematical support.

In Romania, the situation of the determination of a quasigeoid is debated in more specialist reunions. Till now, in Romania, there is not developed a quasigeoid model that could assure high precision for the determination made with modern GNSS technology and trans calculate the ellipsoidal heights in national reference altitude system

After a rigorous analysis on the available data for the Romanian territory and that could be used for modelling the quasigeoid surface we have found that the majority of them are represented by GNSS levelling (precise or trigonometric) determination. In this way, a possible solution for the quasigeoid generation is represented by the geometrical method.

3. Modelling the quasigeoid on the Romanian territory

One of the methods for generating surfaces is represented by the Delaunay triangulation method. Delaunay method with cubical interpolation is a known method, being one of the first methods used before the development of the computation technology.

The algorithm creates triangles drawing lines between the scattered data points. The points are connected in such a way that a triangle never is crossed by other triangles. The result of the triangulation is an irregular network form by plan faces that generate a surface. Each triangle defines a plan that forms the nodes of a grid with slope and elevation determined by minimum three points that define the triangle. All the grid nodes are defined by the triangle tops. The triangulation with linear interpolation is the right one when the distribution of the scattered date is poor.

The method has optimum solutions for large surfaces, as in the case of the Romanian territory, the points of the study case creating a triangular irregular network that fulfil the conditions of Delaunay triangulation. Besides the linear interpolation used in triangulation the nearest neighbour interpolation method is applied for generating acceptable solutions where data are insufficient and solutions needed to assess the results extrapolated.

The Delaunay triangulation is implemented in Matlab software through “griddata” function. The “griddata” function uses the finite element method for resolving the triangulation. This method is used on large scale in surfaces modelling field.

For the generation of the quasigeoid model using Delaunay triangulation with cubic interpolation it was used a data set of 680 points determined by GNSS satellite methods and levelling measurements (geometric or trigonometric).

The geometrical method for the determination of the anomaly of the altitude assumes the following mathematic relation:

$$\zeta_i = h_i - H_i, \quad i - \text{considered point}$$

where the difference between the ellipsoidal altitude and normal altitude represents the anomaly of the altitude.

The used data were graphical and analytic filtrated and after was generated a text file under the following structure (Table 1):

Table 1. The coordinates of the points

No. Point	Latitude B (° , ' , ")	Longitude L (° , ' , ")	Ellipsoid height h (m)	Normal height H (m)	Anomaly of the altitude $\zeta_{\text{ETR89-NIV}}$ (m)
1	46 03 53.70491	23 34 29.58419	282.001	239.854	42.147
2	44 01 57.83694	23 21 08.85045	102.661	61.222	41.439
3	47 06 54.77613	24 28 11.10961	384.642	345.405	39.237
4	47 50 23.05978	22 55 06.62369	166.081	127.137	38.944
...

The distribution of the scattered data points is represented in the Figure 1. There are uniform distributed on all Romanian territory.

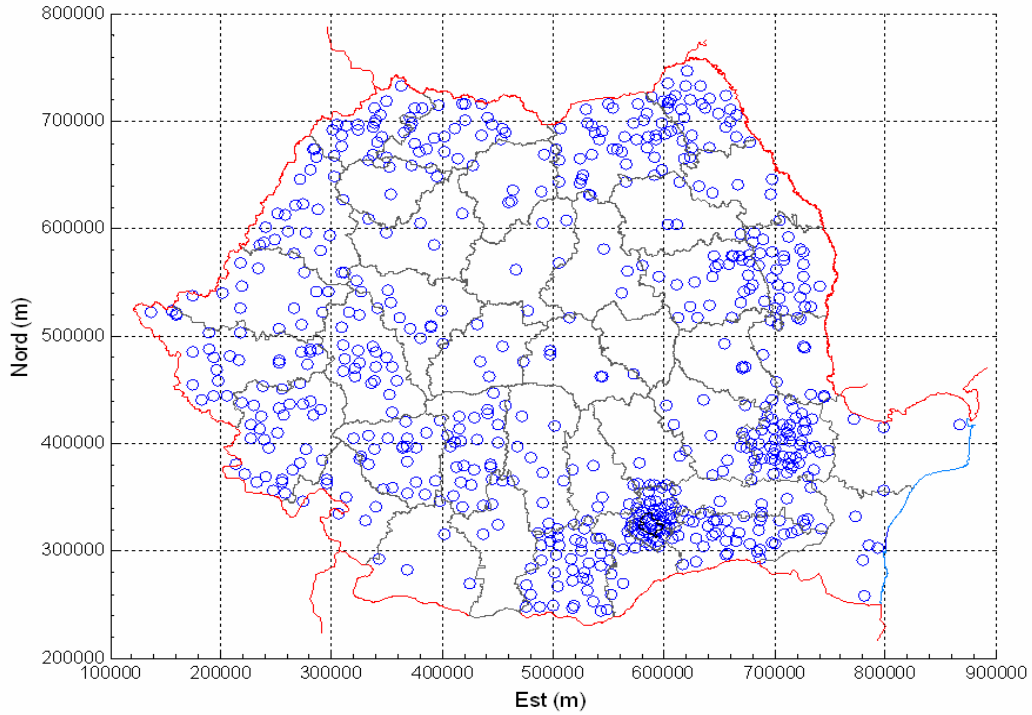


Figure 1. The distribution of the points used to model the quasigeoid on the Romanian territory

A completely new approach regarding the determination of the quasigeoid model on the Romanian territory is represented by the utilisation of the anomalies of the altitude obtained by interpolating the European EGG97 regional model that will be corrected with the values of the anomaly of the altitude determined by GNSS and levelling measurements. The method is not supposed to constrain the European model but to correct it with another surface. In this case, knowing that the European reference for this geoid is the system of normal heights with fundamental zero point Amsterdams Peil Normaal, through this correction with the anomaly of the altitude determined on the normal reference system, practically the reference system of the European model will be change for the Romanian territory. The European quasigeoid model is a gravimetric one, so through its correction with determinations of the anomaly of the altitude from GNSS and levelling measurements it will be obtain a quasigravimetric conversion surface.

The solution it was computed for 3 situation of grid spacing for the quasigeoid model ($dN=dE=16000m$, $dN=dE=8000m$ și $dN=dE=3000m$ planimetric coordinates), following multiple steps.

The determination of the anomaly of the altitudes corrections in points of the scattered data set is realised with the relation:

$$v_{i\zeta} = \zeta_{iETRS89-Niv} - \zeta_{iEGG97_RO}$$

where: $v_{i\zeta}$ – the vector of the differences for the anomaly of the altitude
 $\zeta_{ETRS89-Niv}$ - the vector of the anomaly of the altitude from GNSS and levelling measurements
 ζ_{EGG97_RO} – the vector of the anomaly of the altitude from the interpolation into the European EGG97 model

Then was realised the correction of the European GG97 model with the grid containing the v_{ζ} correction values to obtain the quasigeoid grid on the Romanian territory, ζ_{RO_NOD} .

Figure 1 shows three 5x5 grids representing spatial domains. The first grid is labeled $\zeta_{EGG97NOD}$. The second grid is labeled $v \zeta_{NOD}$. The third grid is labeled ζ_{RO_NOD} . The equation is shown as: $\zeta_{EGG97NOD} + v \zeta_{NOD} = \zeta_{RO_NOD}$. The grids are labeled with $N \max$, $N \min$, $E \min$, and $E \max$.

The grid of the corrections of the anomaly of the altitude $v_{\zeta\text{NOD}}$ has the following representation:

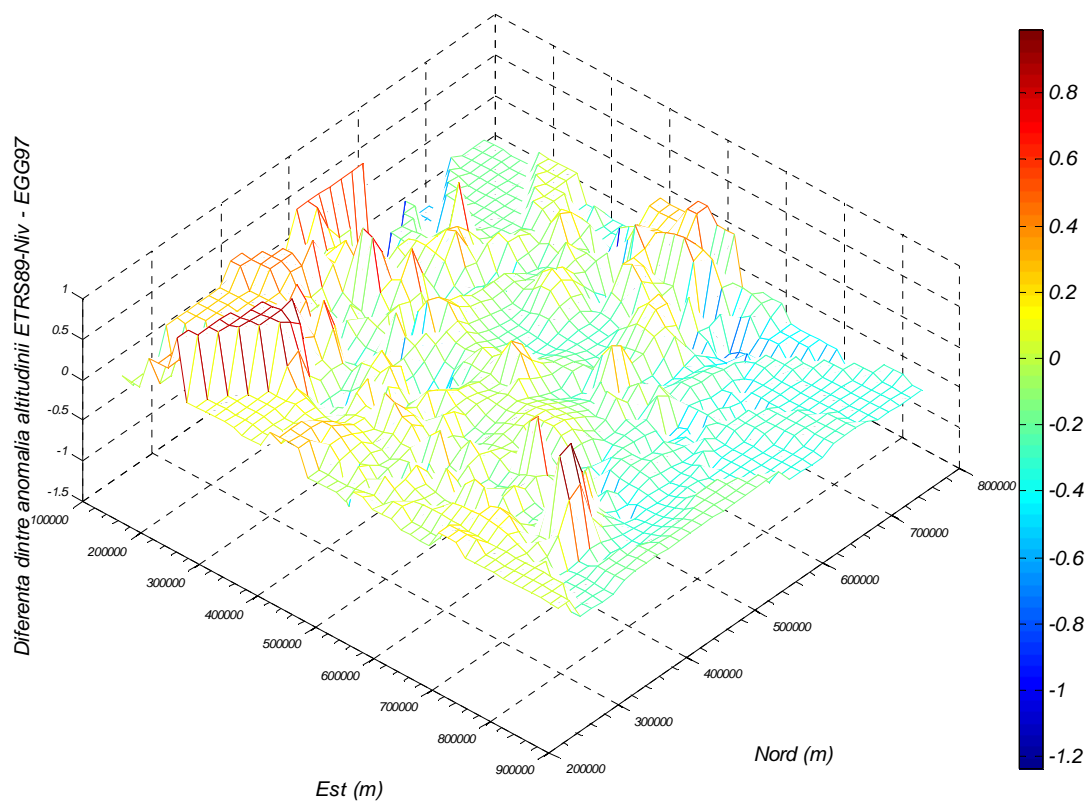


Figure 2. The grid of the correction surface for the EGG97 model, cell spacing $dN=dE=16000m$

In all three spacing situations, for the standard deviation computation, it was utilised the cubic interpolation in regular grid. It was determined the interpolation standard deviation for the correction grid of the anomalies of the altitudes, but for determine the interpolation

error of the final quasigeoid model must be calculated the sum of the standard deviation of the correction grid and the standard deviation of the EGG97 quasigeoid model.

The standard deviations determined in all three situations are presented in Table 2.

Table 2. Standard deviation for the grid interpolation

Spacing	16000 m	8000 m	3000 m
Standard deviation S_0 (m)	0.148	0.084	0.029

The corrections of the differences of the anomaly of the altitude for the points used to determine the grid of the quasigeoid model and the anomaly of the altitude interpolated in the EGG97 model are represented in the following image:

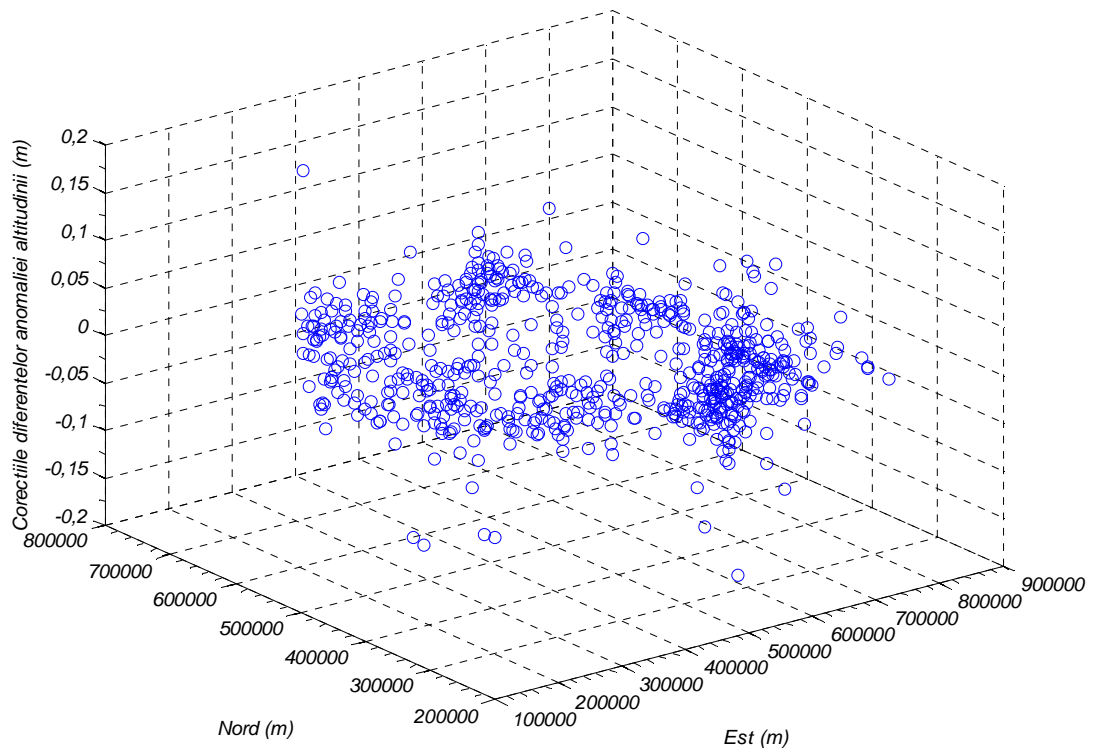


Figure 3. The distribution of the correction of the differences between the anomalies of the altitude interpolate in EGG97 and the anomaly of the altitude obtained by GNSS and levelling measurements.

The grid with dense cell spacing, 3000m, assures a precision for the interpolation of approximately 2 cm. The smooth surface of small dimension for one grid cell interpolated then with a cubic function generates a precise model of interpolation. The only inconvenient of this method, to correct the EGG97 model, is that the precision of the interpolation into the new grid created depends of the precision of its determination.

The grid of the quasigeoid on the Romanian territory, with cell spacing of 3000m, is presented in the following image:

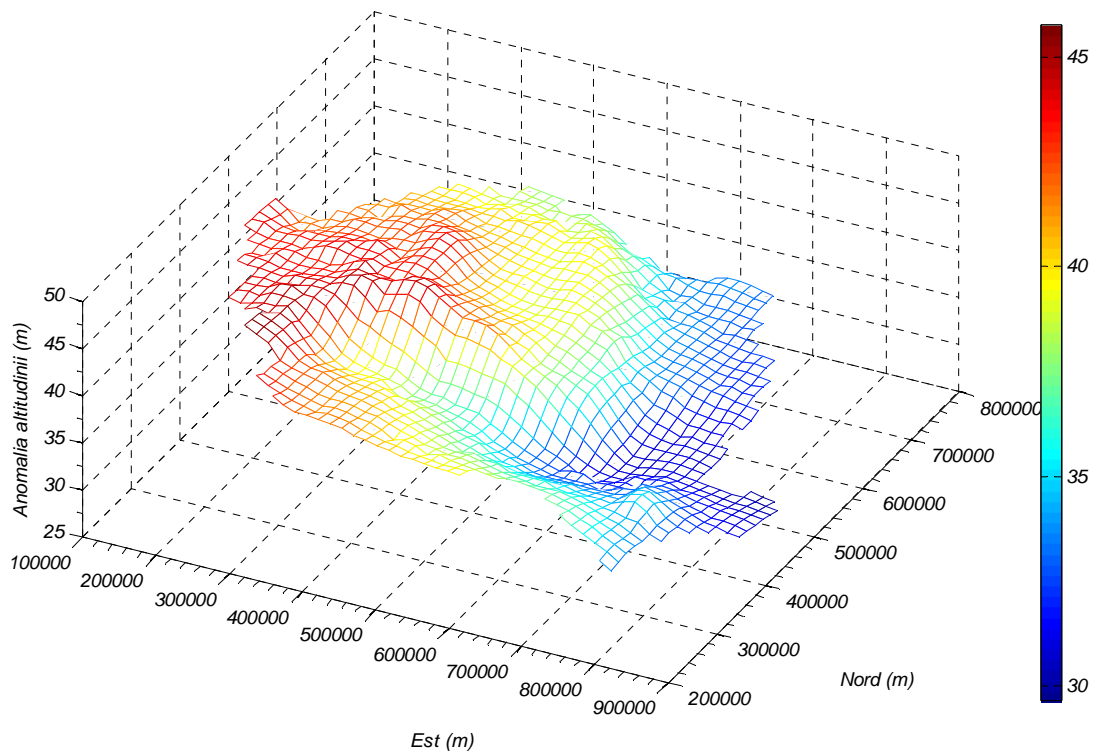


Figure 4. The grid of the quasigeoid model determined by the correction of the EGG97 model with values of the anomaly of the altitude calculated from GNSS and levelling measurements

The final product is represented by the regular rectangular network (grid) that contains in its nodes the values of the anomaly of the altitude. In order for these values to be used they have to be delivered in a binary or ASCII file that can be accessed by an application that will realise the interpolation into the grid nodes. The interpolation should be a cubic one for regular grid.

The ASCII (txt) file it is named „QVAGERO.txt”, in the header of it being presented the main characteristics of the regular grid.

The grid contains 12 lines and presents the following information:

- First line : name of the quasigeoid model – QVAGERO
- Second line : the date of the creation of the file)
- Third line : the date of the modification of the file

Dimension of the grid:

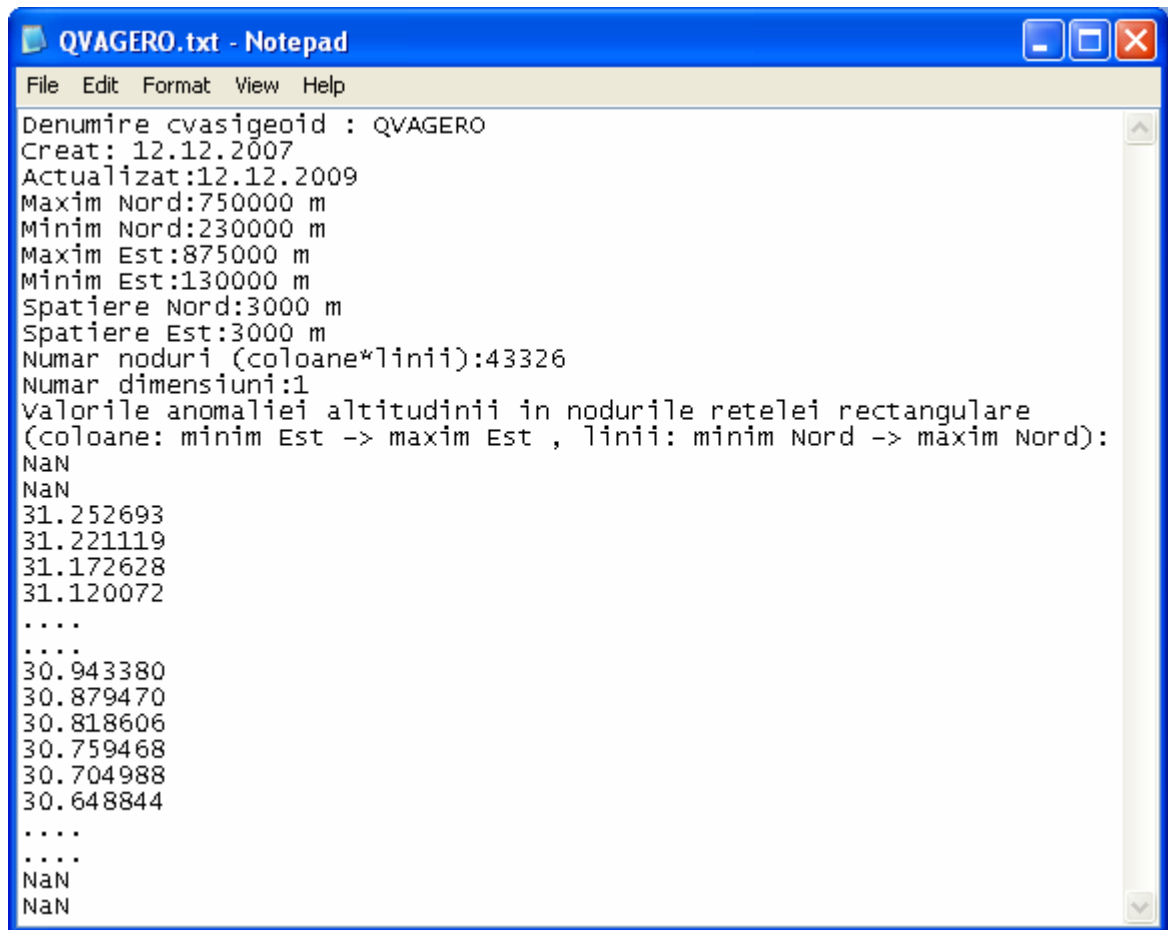
- Line four : Maximum limit of the grid for North direction – 750000m
- Line five : Minimum limit of the grid for North direction – 230000m
- Line six : Maximum limit of the grid for East direction – 875000 m
- Line seven : Minimum limit of the grid for East direction – 130000m
- Line eight : Cell spacing on North direction – 3000m
- Line nine : Cell spacing on East direction – 3000m
- Line ten : Number of nodes of the grid– 43326
- Line eleven : Number of the values in one node -1

- Line twelve: Explanation regarding the sense of browsing the grid, on the columns and lines directions.

According to the line twelve descriptions, the sense of the attribution of the anomalies of the altitude in grid is made from the lower left corner, following the first column, and ends in the upper right corner.

After the line twelve follows on one column, according to the sense in grid, the values of the anomalies of the altitudes. The unit of measure is meter.

An example for the file “QVAGERO.txt” is presented in the following image:



```

Denumire cvasigeoid : QVAGERO
Creat: 12.12.2007
Actualizat:12.12.2009
Maxim Nord:750000 m
Minim Nord:230000 m
Maxim Est:875000 m
Minim Est:130000 m
Spatiere Nord:3000 m
Spatiere Est:3000 m
Numar noduri (coloane*linii):43326
Numar dimensiuni:1
Valorile anomaliei altitudinii in nodurile retelei rectangulare
(coloane: minim Est -> maxim Est , linii: minim Nord -> maxim Nord):
NaN
NaN
31.252693
31.221119
31.172628
31.120072
....
....
30.943380
30.879470
30.818606
30.759468
30.704988
30.648844
....
....
NaN
NaN

```

Figure 5. The representation of the quasigeoid model in ASCII format

4. Conclusions

The geometric modelling of the quasigeoid surface represents one of its methods of generation. After the analysis of more methods of surfaces interpolation (nearest neighbour, natural neighbour, inverse distance weighting, Kriging, Radial basis function, Polynomial regression) the Delaunay Triangulation with bicubic spline interpolation method lead to the best results, also regarding the data used (the number of points and their distribution).

Given the current trends of GNSS technology development and also the change of the Romanian datum in 2009 through the Order no. 212/4.05.2009 of the General Director of the national Agency of Cadastre and land Registration, regarding the Adoption in Romania of the European Terrestrial Reference System 1989 and the introduction of the ETRS89 as official

system to be used in GNSS positioning, this paper presents the determination of the anomalies of the altitudes, the height of the quasigeoid on the GRS80 ellipsoid.

The code developed in Matlab application could be easily used in case of new conversion surface for the quasigeoid model. The completion of the initial ASCII file used at the quasigeoid modelling with coordinates and values of the anomaly of the altitude for new known points is the only operation that should be made for a new quasigeoid generation.

The actual algorithm for interpolation permit the using of the palnimetric coordinates in national reference system S-42, Stereographic 1970 projection. Its flexible way of development permits with small efforts the modification of it for direct interpolation of the ellipsoidal coordinates.

The generated model could be improved through the utilisation of other methods or by their combination.

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