# COMPARATIVE STUDY ON OBTAINING COORDINATES IN AREAS AWAY FROM CIRCLE OF ZERO STRAIN OF THE STEREOGRAPHIC 1970 SYSTEM

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***Abstract:*** *In areas far from the zero strain circle of the stereographic coordinate system 1970 is very important to reduce the distance correct, on the plan nationally used. Often people who perform topographic surveying work or cadastral work on small areas do not reduce the distance correctly. They are limited to use the topography formula reducing the distance on the horizon or work directly in coordinate without taking into account the deformations that may occur. From here consistent errors in the topographic works, and from this to the points of detail.*
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## 1. General considerations

Over time the land surveying professionals were on hand equipment that could determine the position of points to be represented on the plan. After the measurements were made calculations, reporting points and then joining the points which made objects.
Directions were measured with theodolites, initially less accurate, then over time, more precise.
Distances have always been the weak measurements. In the beginning the base were measured in geodetic networks with Invar wire and for detail points with the tape. In the '60 years have appeared instruments for distance measuring on radio waves. It was a big step forward, but for measurements above 10 km show a very thorny problem: determine the coefficient of refraction to bring the distance from a sinuous line to a straight line. To determine the coefficient of refraction was needed to determine temperature, humidity and pressure at the base ends. If the distance was great and the two heads were large differences between these values was necessary to measure the values of temperature, humidity and pressure during the line of the measurement. Or, this is as hard and difficult. For this reason it was easier to determine distances under 10 km where the values of temperature, humidity and pressure were relatively the same at both ends of the base. Note that people who make these measurements were well trained to reduce the slop distance, after correction the refraction and reduce to the ellipsoid and reduced to the plane of projection used in the area. In Romania a long time the projection Gauss - Kruger was used, then the stereographic 1970 projection. Distance between two points reduced to plan Gauss - Kruger or stereographic 1970 plan differs greatly in some areas. Note that in the pole (center) of the stereographic 1970projection, the distances are reduced by 25 cm/km, while in the projection Gauss - Kruger, for the same two points, the distance is greater than about 45 cm/km. Hence a difference of 70 cm/km for the same distance measured. Now the distance is measured with total stations that have lots of facilities including determining the coordinates of the stations and detail points in real time. It is easy for the user, but not always the programs in the menu of the total station reduced correctly the distance. Not always take into account all parameters

needs to reduce correct the distance. Most often the professionals working normally, since not work on large areas, use the formula to reduce the distance to a local plane of projection. To understand what adverse effect can produce the formula to reduce the distance to the plane of projection, we have the example in Mangalia, where deformation reaches over 60 cm/km.

## 2. Adjustment of the measurements using the least squares method

Method of least squares is known and it is not appropriate to detail here all the formulas. Currently, because of computer system no longer uses the Gauss scheme and the conditioned measurements but indirect measurements matrix treatment. Each measurement gives an equation, each new point gives two unknowns (x coordinate and y coordinate). Number of measurements (equations) must be greater than the number of unknowns to obtain the most probable value of the coordinate as close to real value. Equations are two or three kinds (the directions, distances and measurements GNSS). If we had studied all three types of equations: the direction and distance coordinate differences. Equations on the coordinate directions and the differences of coordinate (GNSS) are not interesting to study, so we will refer only to distance equations.

It starts from the slope distances measured by total station. These distances are already corrected by the coefficient of refraction because each station can determine the total pressure and temperature, and shows directly the direct distance between two points of the distance. The formula to reduce the distance on ellipsoid:

$$D_E = \sqrt{\frac{D^2 - (H_2^E - H_1^E)^2}{\left(1 + \frac{H_1^E}{R}\right)\left(1 + \frac{H_2^E}{R}\right)}} + \frac{D_E^3}{24\,R^2} \qquad (2.1)$$

After this reduction, the ellipsoid distance can be reduced to the plane of projection used, in our case the stereographic 1970, with the formula:

$$D_{ppr} = D_E + D_E \frac{x_m^2 + y_m^2}{4R_0^2} + D_E \frac{\Delta x^2 + \Delta y^2}{48R_0^2} \qquad (2.2)$$

Where:
- D is slope distance, measured by total station;
- $H_1^E$ , $H_2^E$ are ellipsoidal altitudes of the head base (can calculate approximate with the distance reduced to horizon or be extracted from the plans);
- R is the Gaussian average radius, calculated at mid-point;
- $x_m, y_m, \Delta x, \Delta y$ are the average and difference coordinates in stereographic 1970 system to the ends of the distance calculated approximate, with only small distance on the horizon.

With the distances reduced to the plane of projection can write distance equations.

Distance measured $D_{ij}^M$ it is corrected with $v_{ij}^D$ and becomes the corrected distance $D_{ij}$ .

$$D_{ij} = D_{ij}^{M} + v_{ij}^{D} \tag{2.3}$$

For a distance measured between two points we can write the equation:

$$D_{ij}^{M} + v_{ij}^{D} = D_{ij}^{0} + dD_{ij} \tag{2.4}$$

Where:
- $D_{ij}^{M}$ is the distance measured value;
- $v_{ij}^{D}$ is the correction of the measured distance, obtained from the compensation process;
- $D_{ij}^{0}$ is the distance determined from provisional coordinates calculated in the compensation process;
- $dD_{ij}$ correction applied to the distance determined from the provisional coordinates and determined in the compensation process.

If in equation (2.4) replace the right side with the equation:

$$dD_{ij} = A_{ij}dx_j + B_{ij}dy_j - A_{ij}dx_i - B_{ij}dy_i \tag{2.5}$$

representing the variation of distance for varying plane coordinates $d\,x_i, dy_i$, we obtain equations of corrections for distance:

$$v_{ij}^{D} = A_{ij}dx_j + B_{ij}dy_j - A_{ij}dx_i - B_{ij}dy_i + l_{ij}^{D} \tag{2.6}$$

Free term is calculated as follows:

$$l_{ij}^{D} = D_{ij}^{0} - D_{ij}^{M} \tag{2.7}$$

We have the equations with corrections on directions, distances and coordinates differences and we can write matrix relations:

$$v = A\,x + l \tag{2.8}$$

$$N = A^{T} P\,A \tag{2.9}$$

$$x = -N^{-1} A^{T} P\,l \tag{2.10}$$

where:
- v is the matrix of corrections;
- A is the matrix correction coefficients;
- x is the unknown matrix;
- P is the weight matrix.

Finally we obtain adjusted coordinates of points using the reduced distance on the stereographic 1970 plane.

### 3. Determination of the coordinates using distances incorrect reduced

The poligonation method is a method widely used by surveyor and geodesy specialists for stretching small areas. Basically it starts from a point with known coordinates that are stationary with total station and is aimed at another point which is also known coordinates. From the point stationed we see next point in poligonation, previously correlated with that point and thus can determine the orientation of departure. It also measured the distance to the next point and form the next point (from point to point with known coordinates and back again). Total station is then moved to the new point and we see the point where we started and next point in poligonation. The operation is repeated until it reaches the point of closing poligonation, which is also known position. From this point of the poligonation is aimed at the previous point and another point with known coordinates for closed orientation.

Basically we determine the coordinates step by step and finally closes on a known point. Coordinates provided by poligonation should be close to those of inventory, given the precision of the old network and accumulated errors ahead.

General formulas for determination are:

$$x_B = x_A + D_{0_{AB}} \cos\theta_{AB}$$
$$y_B = y_A + D_{0_{AB}} \sin\theta_{AB} \qquad (2.11)$$

The distance $D_0$ should be reduced to formulas (2.1) and (2.2), but many users are limited to the classical formula:

$$D_0 = D\cos\alpha = D\sin z \qquad (2.12)$$

Where:
- D is slope distance, measured by total station;
- α is the angle of slope measured with the total station between the ends base;
- z is the zenith angle measured with the total station between the ends base.

Orientation is transmitted as shown in Figure 1.

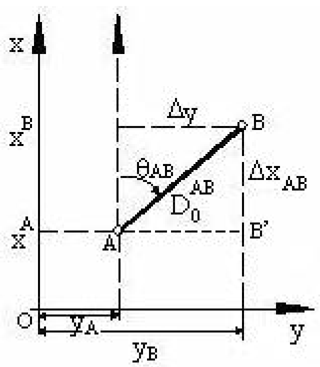


Figure 1. Determination of orientation

The poligonation calculated with formula (2.11) and the distance calculated by the formula (2.12) will not lead to a lower close on the final station coordinates. Function of distance from the zero strain circle that has radius of 201.7 km, the errors may be higher or lower.

## 4. Case study

We realized a geodetic network for national roads in Romania. For areas where we work was done to plant a monument every two kilometers. Between these monuments was conducted a poligonation and from these points we measured points for cross sections and a longitudinal profile of the road. Along the points roads we focused too points from the national network that churches, water towers, etc.. to compensate in a single network. Monuments were stationed too with GPS receivers (now GNSS). Network has been compensated by the method of least squares, indirect measurements of three types of equations: direction, distance and coordinate differences.
In the county of Constanta, where the distance from the circle of zero strain is higher, the difference between the distance reduced to stereographic 1970 plan of projection and local plan distance is more than 60 cm/km.

After the work, on the monuments located and computed by SC CORNEL & CORNEL TOPOEXIM SRL were stationed authorized person who made sporadic cadastre and topographic works. Their equipment is total stations and the method used for transmitting coordinates poligonation. Following the completion of the work accused the new geodetic network is not responding. We was summoned to OCPI Constanta and we discussed with the authorized persons. We understand that authorized persons have reduced the distance in a wrong way.

We rebuilt a new calculations with formulas used by authorized persons and calculated coordinate differences present in Table 1. We chose a section between two monuments located at 2 km, in Mangalia. From calculations results that in the studied area the reduction coefficient of the distance is 0.619 m/km. The area is at 377.2 kilometers from the projection center and at 175.5 kilometers of the circle of zero strain. Note that on the total distance of about 2210 meters of the poligonation the error have a total of 1.511 meters. In Table 2 is the differences of distances, reduced by formulas (2.1) and (2.2), correct and reduced to formulas (2.12), incorrectly.

## 5. Conclusions

Our case is real and has led to many discussions regarding the calculation of the coordinates. He was accused of lack of programs to compensate geodetic networks using the method of least squares. Now, with the help of ***excel*** programs is very easy to compute a network of more than 10 new points. Basically, in a few hours, without requiring the programmer can compensation a network and modify the mistakes in real time. It is obvious the error of principle that a wrong reduction of distances can induce in coordinates stations and in the points of detail. The tables are inconclusive and have realized real numbers, errors which may arise from non correct theory. The poligonation start from the point 7-39-52 (means National Road 7, National Road 39, kilometer 52), passing through 7-39-54 and closes in the point 159, end of the poligonation. The points 151 and 7-39-52 are considered old points with known position. The point 7-39-54 is a starting point and point 151 is for orientation. In both calculations are the same coordinates. Note that as we move away from the fixed points, the difference between the coordinates calculated with the two types of distances are greater, as is normal.

The conclusion is that wherever we perform topographic measurements is better to reduced correct the distance, thus eliminates any source called methodical errors. These errors do not result from equipment or operator of any environmental conditions but the use of inappropriate methods of calculation.

Table 1 Coordinates obtained using reduced distance from the stereographic projection plane 1970 and reduced using the horizon distance

| Den. pct | Coordonate obtined by reducing the distance to a local plan | | Coordonate obtined by reducing the distance to the Stereographic 1970 plan | | Coordinate differences (x-axis and y-axis) | | Total difference |
|---|---|---|---|---|---|---|---|
| | x [m] | y [m] | x [m] | y [m] | dx [m] | dy [m] | dt [m] |
| 151 | 257425.402 | 787825.481 | 257425.402 | 787825.481 | 0.000 | 0.000 | 0.000 |
| 7-39-52 | 257021.446 | 787776.516 | 257021.446 | 787776.516 | 0.000 | 0.000 | 0.000 |
| 152 | 256724.374 | 787741.181 | 256724.197 | 787741.206 | 0.177 | -0.025 | 0.179 |
| 153 | 256482.79 | 787740.353 | 256482.464 | 787740.416 | 0.326 | -0.063 | 0.332 |
| 154 | 256202.524 | 787727.527 | 256202.021 | 787727.625 | 0.503 | -0.098 | 0.512 |
| 155 | 255829.365 | 787511.739 | 255828.598 | 787511.761 | 0.767 | -0.022 | 0.767 |
| 156 | 255651.392 | 787361.987 | 255650.492 | 787361.944 | 0.900 | 0.043 | 0.901 |
| 157 | 255408.146 | 787182.664 | 255406.849 | 787182.578 | 1.297 | 0.086 | 1.300 |
| 7-39-54 | 255264.244 | 787054.707 | 255262.865 | 787054.546 | 1.379 | 0.161 | 1.388 |
| 158 | 255175.995 | 787004.041 | 255174.562 | 787003.848 | 1.433 | 0.193 | 1.446 |
| 159 | 255080.867 | 786918.465 | 255079.376 | 786918.217 | 1.491 | 0.248 | 1.511 |

Table 2 Distance reduced to the stereographic 1970 plan of projection and the distance reduced to a local plan

| From | To | Distance reduced to a local plan | Distance reduced to stereographic 1970 plan | Difference between the two distances |
|---|---|---|---|---|
| | | [m] | [m] | [m] |
| 7-39-52 | 152 | 299.167 | 299.352 | 0.185 |
| 152 | 153 | 241.587 | 241.736 | 0.149 |
| 153 | 154 | 280.557 | 280.731 | 0.174 |
| 154 | 155 | 431.056 | 431.323 | 0.267 |
| 155 | 156 | 232.595 | 232.739 | 0.144 |
| 156 | 157 | 302.205 | 302.392 | 0.187 |
| 157 | 7-39-54 | 192.565 | 192.684 | 0.119 |
| 7-39-54 | 158 | 101.757 | 101.820 | 0.063 |
| 158 | 159 | 127.956 | 128.035 | 0.079 |
| | suma | 2209.441 | 2210.810 | 1.369 |

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