THE VERTICAL ATMOSPHERIC PRESSURE GRADIENT ESTIMATION IN THE LOWER TERRESTRIAL ATMOSPHERE

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Abstract: *In this article, the authors present different modalities for vertical atmospheric pressure gradient estimation for the terrestrial low atmosphere, using the data and the formulae from the Standard Atmosphere (SA). The vertical atmospheric pressure gradient is used in geodetic engineering for different purposes: testing the barometers and meteorological stations, establishing of the predicted values in barometric leveling, rapid estimation of the height differences based on the atmospheric pressure differences, the determination of the reference atmosphere pressure in a specific location for prediction of the specific atmospherically phenomena, the estimation of the refraction coefficient, and so forth… In the article, are presented various simulated calculus for different heights above the mean sea level, focusing especially on the specific terrestrial measurements.*

Keywords: *geodetic leveling, barometric leveling, atmospheric corrections, refraction coefficient, vertical atmospheric pressure gradient.*

1. Introduction

Knowing the vertical gradients of the atmospheric pressure used in geodetic specific processes (atmospheric pressure *P*, air temperature *t* and water vapor pressure *e*) is an important aspect, if not essential, in the study of the interaction between the propagation medium and the electromagnetic radiation, radiation that is the carrier of the measurement signal.

Beside the atmospheric parameters above mentioned, the atmospheric pressure *P*, by its numerical value, contributes to the correction process of the EDM distances [2]. In addition to this important and practical role, the atmospheric pressure parameter plays other important roles through its vertical distribution, especially in the lower atmosphere: through the vertical gradient $\frac{dP}{dh}$.

The determination of the vertical atmospheric pressure gradient $\frac{dP}{dh}$ is a difficult task in the absence of an atmospheric model. In addition, even into the limits of an atmospheric model, the simulation calculus showed that the numerical values of the vertical pressure gradient is not constant, it is in a continued variation, being a function of the atmospheric pressure value itself *P*, the air temperature *t* and the *height* above the mean sea level. In this context, the simulated calculus presented in this article follows different scenarios (using different versions of the standard atmosphere), function of the formulae variables.

2. The utilization of the vertical atmospheric pressure gradient

The numerical value of the vertical atmospheric pressure gradient permits the realization of different objectives in practical geodesy:

- The testing of some atmospheric models. For example, measuring the atmospheric pressure in both ends of an *EDM* distance permits the obtaining of two sets of values: measured and estimated values (by using the vertical atmospheric pressure gradient). The more the estimated values are closer to the measured ones, the more the atmospheric model is closer to the standard one.
- The testing of the meteorological stations. This objective is the more important as the calibration processes are usually difficult and expensive. Knowing the vertical atmospheric pressure gradient enables the quick diagnostic of a meteorological station (with a integrated barometer) by the estimation of the resolution power.
- Establishing of a predicted values for the barometric leveling. This values offer a continuous feedback in the field, showing right away the eventual anomalies, either from wrong measurements, or the deviation of the current atmospheric medium from the standard one.
- The rapid estimation of the level differences through barometric leveling. This estimation is instantaneous if the atmospheric pressure is known in at least two points.
- The estimation of the reference atmospheric pressure in a specific location. This reference value enables quick weather predictions, obtaining in this way a good decisional support.
- The determination of the refraction coefficient *k*. This coefficient is essential in the correction process of the measured angles (especially the zenith ones) from the refraction effect, through the refraction angle. The refraction coefficient *k* can be calculated by complicated formula [3], where the principal variables are both the numerical values and vertical gradients of the atmospheric parameters.

3. The determination of the vertical atmospheric pressure

It is very difficult to realize a precise determination of the vertical pressure gradient. Instead, is relative ease the *estimate* this gradient, this estimation could afford the user a key to unlock the majority of the problems listed in the previous section.

For the estimation process of the vertical pressure gradient $\frac{dP}{dh}$, any of the following barometric leveling formulae can be used [3]:

- the formula of the isotherm atmosphere
- the formula of the polytrope atmosphere
- the *Babinet* formula for the polytrope atmosphere
- the *Laplace* formula for the polytrope atmosphere
- the *Standard Atmosphere 1972* formula for the vertical distribution of the atmospheric pressure

All of the above mentioned formulae work in an atmospheric model frame. This frame is a standard one, but in different versions. But all of these versions are standard versions so is not a surprise that all these formulae give similar results in the estimation of the vertical atmospheric pressure gradient.

In the following, it is presented two different independent strategies for the determination of the vertical pressure gradient $\frac{dP}{dh}$: one in a slightly complex form, where the air temperature *t* is a variable beside the atmospheric pressure *P*, and one in a simple form, where only the pressure parameter is used as a variable.

A. The vertical atmospheric pressure gradient in the isotherm and polytropic atmosphere

Both, the isotherm and polytropic atmosphere are versions of the standard atmosphere, and the isotherm is only a particular form of the polytrope one. The isotherm formula has an advantage: the air temperature occur only once, so the formula is easy to be used in simulated calculus:

$$\Delta z(m)=(-7991.02 - 31.96 \cdot t) \cdot \ln(\frac{P}{P_0}) \qquad (1)$$

In the formula *1*, the atmospheric pressure varies in the [700mmHg-780 mmHg] range, the air temperature is a variable in the $[-5^0 C \div 40^0 C]$ range, and the maximum height is considered to be in the 250 m. The vertical pressure gradient can be now extracted from table *1*:

Table 1.

P [mmHg]/ t [C]	***700***	***710***	***720***	***730***	***740***	***750***	***760***	***770***	***780***	***Average***
-5	0.880	0.892	0.905	0.917	0.930	0.943	0.955	0.968	0.980	*0.930*
-4	0.876	0.889	0.901	0.914	0.926	0.939	0.951	0.964	0.976	*0.926*
-3	0.873	0.885	0.898	0.910	0.923	0.935	0.948	0.960	0.972	*0.923*
-2	0.869	0.882	0.894	0.907	0.919	0.931	0.944	0.956	0.969	*0.919*
-1	0.866	0.878	0.891	0.903	0.915	0.928	0.940	0.952	0.965	*0.915*
0	0.862	0.875	0.887	0.899	0.912	0.924	0.936	0.949	0.961	*0.912*
1	0.859	0.871	0.884	0.896	0.908	0.920	0.933	0.945	0.957	*0.908*
2	0.856	0.868	0.880	0.892	0.905	0.917	0.929	0.941	0.953	*0.905*
3	0.852	0.865	0.877	0.889	0.901	0.913	0.925	0.938	0.950	*0.901*
4	0.849	0.861	0.873	0.885	0.898	0.910	0.922	0.934	0.946	*0.898*
5	0.846	0.858	0.870	0.882	0.894	0.906	0.918	0.930	0.942	*0.894*
6	0.843	0.855	0.867	0.879	0.891	0.903	0.915	0.927	0.939	*0.891*
7	0.839	0.851	0.863	0.875	0.887	0.899	0.911	0.923	0.935	*0.887*
8	0.836	0.848	0.860	0.872	0.884	0.896	0.908	0.920	0.932	*0.884*
9	0.833	0.845	0.857	0.869	0.880	0.892	0.904	0.916	0.928	*0.880*
10	0.830	0.842	0.853	0.865	0.877	0.889	0.901	0.913	0.925	*0.877*
11	0.827	0.838	0.850	0.862	0.874	0.886	0.897	0.909	0.921	*0.874*
12	0.824	0.835	0.847	0.859	0.871	0.882	0.894	0.906	0.918	*0.871*
13	0.820	0.832	0.844	0.856	0.867	0.879	0.891	0.902	0.914	*0.867*

P [mmHg]/ t [C]	*700*	*710*	*720*	*730*	*740*	*750*	*760*	*770*	*780*	*Average*
14	0.817	0.829	0.841	0.852	0.864	0.876	0.887	0.899	0.911	*0.864*
15	0.814	0.826	0.838	0.849	0.861	0.872	0.884	0.896	0.907	*0.861*
16	0.811	0.823	0.834	0.846	0.858	0.869	0.881	0.892	0.904	*0.858*
17	0.808	0.820	0.831	0.843	0.855	0.866	0.878	0.889	0.901	*0.855*
18	0.805	0.817	0.828	0.840	0.851	0.863	0.874	0.886	0.897	*0.851*
19	0.802	0.814	0.825	0.837	0.848	0.860	0.871	0.883	0.894	*0.848*
20	0.799	0.811	0.822	0.834	0.845	0.857	0.868	0.879	0.891	*0.845*
21	0.797	0.808	0.819	0.831	0.842	0.853	0.865	0.876	0.888	*0.842*
22	0.794	0.805	0.816	0.828	0.839	0.850	0.862	0.873	0.884	*0.839*
23	0.791	0.802	0.813	0.825	0.836	0.847	0.859	0.870	0.881	*0.836*
24	0.788	0.799	0.810	0.822	0.833	0.844	0.855	0.867	0.878	*0.833*
25	0.785	0.796	0.808	0.819	0.830	0.841	0.852	0.864	0.875	*0.830*
26	0.782	0.794	0.805	0.816	0.827	0.838	0.849	0.861	0.872	*0.827*
27	0.780	0.791	0.802	0.813	0.824	0.835	0.846	0.857	0.869	*0.824*
28	0.777	0.788	0.799	0.810	0.821	0.832	0.843	0.854	0.866	*0.821*
29	0.774	0.785	0.796	0.807	0.818	0.829	0.840	0.851	0.862	*0.818*
30	0.771	0.782	0.793	0.804	0.815	0.826	0.837	0.848	0.859	*0.815*
31	0.769	0.780	0.791	0.802	0.813	0.823	0.834	0.845	0.856	*0.813*
32	0.766	0.777	0.788	0.799	0.810	0.821	0.832	0.843	0.853	*0.810*
33	0.763	0.774	0.785	0.796	0.807	0.818	0.829	0.840	0.850	*0.807*
34	0.761	0.771	0.782	0.793	0.804	0.815	0.826	0.837	0.848	*0.804*
35	0.758	0.769	0.780	0.790	0.801	0.812	0.823	0.834	0.845	*0.801*
36	0.755	0.766	0.777	0.788	0.799	0.809	0.820	0.831	0.842	*0.799*
37	0.753	0.764	0.774	0.785	0.796	0.807	0.817	0.828	0.839	*0.796*
38	0.750	0.761	0.772	0.782	0.793	0.804	0.814	0.825	0.836	*0.793*
39	0.748	0.758	0.769	0.780	0.790	0.801	0.812	0.822	0.833	*0.790*
40	0.745	0.756	0.766	0.777	0.788	0.798	0.809	0.820	0.830	*0.788*
Average	*0.809*	*0.820*	*0.832*	*0.843*	*0.855*	*0.867*	*0.878*	*0.890*	*0.901*	

In table *1*, the vertical pressure gradient values are given with exchanged sign and are expressed in mmHg/10m. For example, in accord with table 1, for the following values: $t=12^0C$ and *P=760mmHg,*the vertical pressure gradient has the value: $\frac{dP}{dh} = -0.894 mmHg/10m. = -0.089 mmHg/1m.$ That means that the atmospheric pressure decrease with approximated value of -0.9 mmHg for each 10 meters, or with -0.09 mmHg for each meter till the limit of 250 m. above mean sea level.

In table *2*, it is presented similar calculus for the maximum height 750m. above mean sea level.

Table 2.

P [mmHg]/ t [C]	*700*	*710*	*720*	*730*	*740*	*750*	*760*	*770*	*780*	*Average*
-5	0.852	0.865	0.877	0.889	0.901	0.913	0.925	0.938	0.950	0.901
-4	0.849	0.861	0.873	0.885	0.898	0.910	0.922	0.934	0.946	0.898
-3	0.846	0.858	0.870	0.882	0.894	0.906	0.918	0.930	0.942	0.894
-2	0.843	0.855	0.867	0.879	0.891	0.903	0.915	0.927	0.939	0.891
-1	0.839	0.851	0.863	0.875	0.887	0.899	0.911	0.923	0.935	0.887
0	0.836	0.848	0.860	0.872	0.884	0.896	0.908	0.920	0.932	0.884
1	0.833	0.845	0.857	0.869	0.881	0.892	0.904	0.916	0.928	0.881
2	0.830	0.842	0.854	0.865	0.877	0.889	0.901	0.913	0.925	0.877
3	0.827	0.838	0.850	0.862	0.874	0.886	0.898	0.909	0.921	0.874
4	0.824	0.835	0.847	0.859	0.871	0.882	0.894	0.906	0.918	0.871
5	0.820	0.832	0.844	0.856	0.867	0.879	0.891	0.903	0.914	0.867
6	0.817	0.829	0.841	0.852	0.864	0.876	0.887	0.899	0.911	0.864
7	0.814	0.826	0.838	0.849	0.861	0.873	0.884	0.896	0.907	0.861
8	0.811	0.823	0.835	0.846	0.858	0.869	0.881	0.893	0.904	0.858
9	0.808	0.820	0.831	0.843	0.855	0.866	0.878	0.889	0.901	0.855
10	0.805	0.817	0.828	0.840	0.851	0.863	0.874	0.886	0.897	0.851
11	0.802	0.814	0.825	0.837	0.848	0.860	0.871	0.883	0.894	0.848
12	0.800	0.811	0.822	0.834	0.845	0.857	0.868	0.879	0.891	0.845
13	0.797	0.808	0.819	0.831	0.842	0.854	0.865	0.876	0.888	0.842
14	0.794	0.805	0.816	0.828	0.839	0.850	0.862	0.873	0.884	0.839
15	0.791	0.802	0.813	0.825	0.836	0.847	0.859	0.870	0.881	0.836
16	0.788	0.799	0.811	0.822	0.833	0.844	0.856	0.867	0.878	0.833
17	0.785	0.796	0.808	0.819	0.830	0.841	0.853	0.864	0.875	0.830
18	0.782	0.794	0.805	0.816	0.827	0.838	0.849	0.861	0.872	0.827
19	0.780	0.791	0.802	0.813	0.824	0.835	0.846	0.858	0.869	0.824
20	0.777	0.788	0.799	0.810	0.821	0.832	0.843	0.855	0.866	0.821
21	0.774	0.785	0.796	0.807	0.818	0.829	0.840	0.852	0.863	0.818
22	0.771	0.782	0.793	0.804	0.815	0.826	0.838	0.849	0.860	0.815
23	0.769	0.780	0.791	0.802	0.813	0.824	0.835	0.846	0.857	0.813
24	0.766	0.777	0.788	0.799	0.810	0.821	0.832	0.843	0.854	0.810
25	0.763	0.774	0.785	0.796	0.807	0.818	0.829	0.840	0.851	0.807
26	0.761	0.772	0.782	0.793	0.804	0.815	0.826	0.837	0.848	0.804
27	0.758	0.769	0.780	0.791	0.801	0.812	0.823	0.834	0.845	0.801
28	0.755	0.766	0.777	0.788	0.799	0.809	0.820	0.831	0.842	0.799
29	0.753	0.764	0.774	0.785	0.796	0.807	0.817	0.828	0.839	0.796
30	0.750	0.761	0.772	0.782	0.793	0.804	0.815	0.825	0.836	0.793
31	0.748	0.758	0.769	0.780	0.790	0.801	0.812	0.822	0.833	0.790
32	0.745	0.756	0.766	0.777	0.788	0.798	0.809	0.820	0.830	0.788
33	0.743	0.753	0.764	0.774	0.785	0.796	0.806	0.817	0.827	0.785
34	0.740	0.751	0.761	0.772	0.782	0.793	0.804	0.814	0.825	0.782

P [mmHg]/ t [C]	***700***	***710***	***720***	***730***	***740***	***750***	***760***	***770***	***780***	***Average***
35	0.738	0.748	0.759	0.769	0.780	0.790	0.801	0.811	0.822	*0.780*
36	0.735	0.746	0.756	0.767	0.777	0.788	0.798	0.809	0.819	*0.777*
37	0.733	0.743	0.754	0.764	0.775	0.785	0.795	0.806	0.816	*0.775*
38	0.730	0.741	0.751	0.762	0.772	0.782	0.793	0.803	0.814	*0.772*
39	0.728	0.738	0.749	0.759	0.769	0.780	0.790	0.801	0.811	*0.769*
40	0.725	0.736	0.746	0.757	0.767	0.777	0.788	0.798	0.808	*0.767*
Average	*0.786*	*0.797*	*0.808*	*0.819*	*0.830*	0.842	0.853	0.864	*0.875*	**0.830**

The values from the table *2* are similar with the values from the table *1*. On the basis of air temperature and atmospheric pressure, the values of the vertical pressure gradient can be extracted from the tables *1* and *2*, function by the height range where the wanted location fits.

B. The vertical atmospheric pressure gradient in the Standard Atmosphere 1972

The start point of the determination of the vertical pressure gradient is the vertical distribution of the atmospheric pressure in the Standard Atmosphere 1972 [1]:

$$P = P_0 \times (1 - 2.26 \times 10^{-5} \times H)^{5.225} \quad (2)$$

In the formula *2*, the atmospheric pressure varies in the [700mmHg - 780 mmHg] range and the vertical pressure gradient is calculated for the maximum height of 250 m. above the mean sea level.

Table 3.

P [mmHg]	***-dP/dh [mmHg/10m.]***
700	0.822
710	0.833
720	0.845
730	0.857
740	0.868
750	0.880
760	0.892
770	0.904
780	0.915

In the table *3*, it can be observed that for a atmospheric pressure of 760 mmHg, the vertical gradient is -0.892mmHg./10m. This value is almost identical with the value obtained in the same conditions (the same atmospheric pressure) from the table *1*.

For the maximum height of 750 m. above mean sea level, the same algorithm is applied, the results being presented in the table *4*.

Table 4.

P [mmHg]	*-dP/dh [mmHg/10m.]*
700	0.801
710	0.812
720	0.823
730	0.835
740	0.846
750	0.858
760	0.869
770	0.881
780	0.892

For the same value P=760 mmHg., the vertical pressure gradient has the value of dP/dh=-0.869mmHg./10m. Again, this value is very similar with the value obtained under the same conditions (the same value for the atmospheric pressure) from the table *4*.

4. Conclusions

The vertical atmospheric pressure gradient is a number that reflect the vertical distribution of this atmospheric parameter. This parameter is involved in an entire series of specific measurements processes in modern geodesy. Knowing of this gradient even only by estimation, permits quick solutions for the majority of the problems listed in the section 2.

In the present article, by two different formulae (for different versions of the standard atmosphere) under the same conditions and in an independent way, similar values for the vertical atmospheric pressure gradient are presented. The almost identical values obtained by different approached and in an independent manner, validates the correctness of the algorithm used in the calculus process.

The estimated values of the vertical atmospheric pressure gradient can be easily extracted from the tables 1, 2, 3 and 4, function of different variables measured in the field, and can be used in the geodetic processes where its presence is necessary.

5. References

1. *Klein Baltink H.; Derks H. J. P.; van Lammeren A. C. A. P.; Ambrosius B. A. C.; van der Hoeven A. G. A.; van der Marel H.; Kleijer F.; Kosters A. J. M. (1999): Chapter 2: Water Vapour from GPS Tropospheric Delay Estimates in GPS Water Vapour Meteorology, Beleids Commissie Remote Sensing (BCRS), 3-11.*
2. *Plopeanu M., Îndrumător de lucrări practice la MED, Editura Conspress, Bucureşti 2006*
3. *Plopeanu M., Note de curs BMGU, 2012*