COMPARISONS OF GLOBAL GEOPOTENTIAL MODELS WITH TERRESTRIAL GRAVITY DATA IN LIBYA

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Abstract: *This study compares a series of global geopotential models (GGMs) using available terrestrial data from the territory of Libya. The anomalies of free air were determined in points of gravimetric survey of Libya by using the model of European Improved Gravity 6C2 (EIGEN-6C2), the model of the Earth Gravitational Model 2008 (EGM2008), as well as model of the Earth Gravitational Model 1996 (EGM96). The points of gravimetric survey of Libya are related to International Terrestrial Reference Frame (ITRF2000), Geodetic Reference System 1980 (GRS80), International Gravity Standardization Net 1971 (IGSN71) and the levelling network of Libya. Free air anomalies were evaluated at the points of gravimetric survey. Along with all mentioned above the results gained by using said models were separately compared with the data obtained by using the Earth Gravitational Model (EGM96).*

Keywords: *Geopotential Model, Gravity, Anomaly, Spherical Harmonics, Geoid.*

1. Introduction

The beginning of the 20th century was a time of Italian occupation of Libya and the time when the first geodetic works were done. Until 1940s Italian's had conducted triangulation surveys in conjunction with their topographic mapping. In the 1950s and 1960s, the first major geodetic network, a singular triangulation chain of about 1900 km long, was established by the American Army Map Service (AMS). This triangulation network was connected to the European Datum 1950 (ED50) with fundamental point in Potsdam (Figure 1, left).

The second major geodetic work was in 1976, when 45 first order Doppler points were distributed around a country at about every 250 km, and 16 of them were linked with AMS network by ground traversing and geometric levelling. This work was completed by the National Geographic Institute (IGN), using satellite observation (JMR devices) and astronomical observations (Figure 1, right).

In the early 1980s surveying Department of Libya (SDL) started its largest project, the national mapping project. This project was to map the whole territory of Libya and it was conducted by two companies:

- Aero - Service Corporation (ASC) established 670 Doppler points and 942 first order traverse stations (Figure 2).
- Pol-Service Geokart conducted 7468 levelling bench marks and 63 first order gravimetric stations, and established 4 tide gauges along Mediterranean coast (Figure 3).

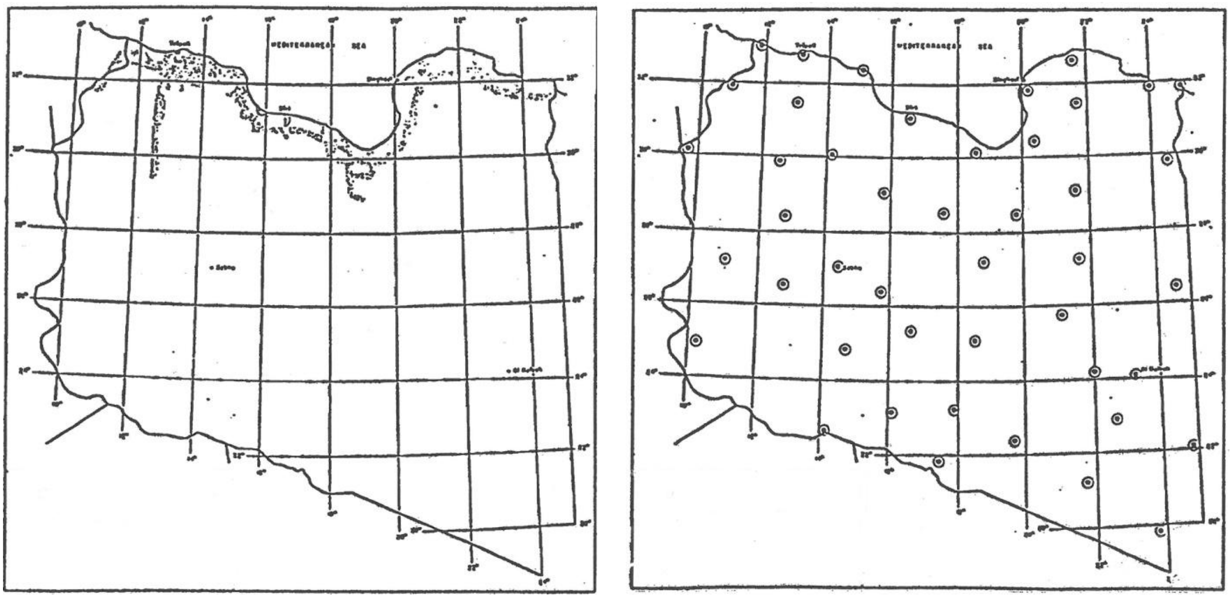


Figure 1. Left: AMS first order triangulation network [4];
Right: IGN first order Doppler network [4];

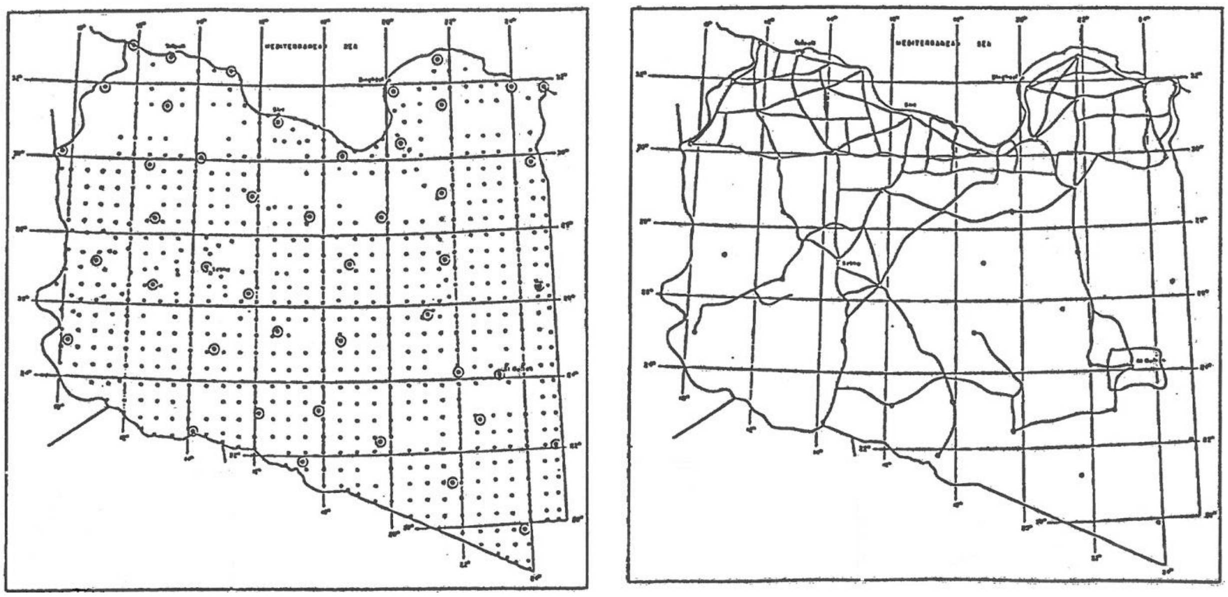


Figure 2. Left: ASC 670 second order Doppler and IGN 45 first order Doppler [4];
Right: ASC first order traverse network [4];

In the last years Libyan Petroleum Institute (LPI) started *compiling* the gravity data and collecting the data from different sources in order to make the Libya gravity data base for research purposes.

The total gravity station coverage represents about 35% to 40% of the total area of Libya (Figure 4). The overall gravity stations coverage is not uniform.

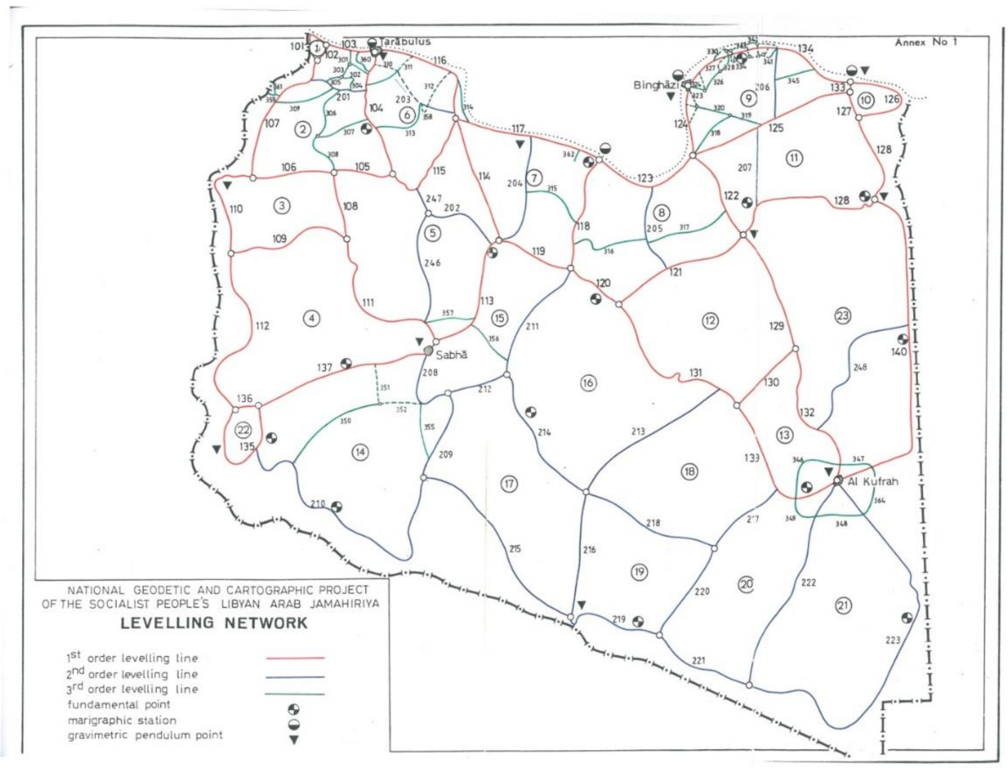


Figure 3: Levelling network by Pol-Service [8]

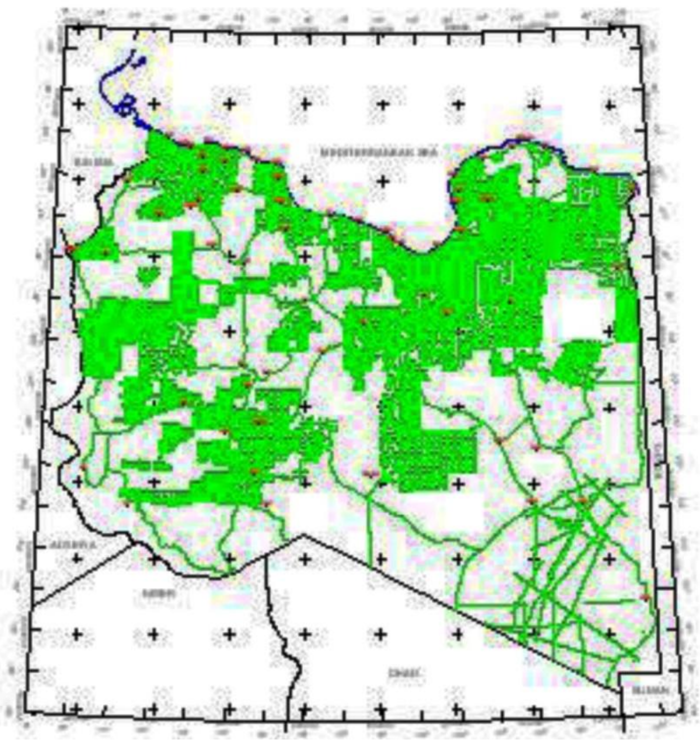


Figure 4: Gravity stations in map of Libya

Up to now, there were four attempts to compute the Libyan geoid and all of them were before 2000.

The first attempt, a geoid map based on 19 levelled Doppler, were computed by IGN. In its computations IGN used, as the only source of data, the deflections of the vertical obtained from the comparison of the astronomical coordinates and their respective WGS72 to the levelled Doppler points above the same ellipsoid. From the result of these computations, IGN drew the following map of Libyan geoidal undulations (Figure 5) [9].

A second geoidal map was computed by ASC. Two geoidal maps with relation to two different datums were computed, one with relation to the local datum, European Libyan Datum 1979 (ELD79) and the other one with relation to WGS72 datum (Figure 6) [9].

The third computation of the gravimetric geoid for Libya, by A. Lyszkowicz and A. A. Wahiba, was conducted in 1997. The data used are:

- 5′ x 5′ gridded value of Bouguer anomalies based on African Gravity Project.
- Global 5′ x 5′ digital terrain model [2].
- The OSU91A and EGM96 geopotential models.

Two geoid/quasi geoid models were computed, first model is referred to OSU91A and second one to EGM96 global model, (Figure 7) [5].

The fourth geoid determination for Libya, by A. M. Swassi, was conducted in 2000. He used method based on the remove-restore technique with respect to 360 x 360 spherical harmonic reference model OSU91A with the terrain effect consideration (Figure 8) [9].

From 2008 when EGM08 was published up-to date, along with the mentioned, more than 20 GGM of different degree and order were created.

Aiming to determine the most adequate GGM for the needs of modeling geoid/quasi geoid in Libya, within the frames of this research, 3 models (Table 1) were tested: EGM96, EGM2008 high degree geopotential model, as well as the EIGEN-6C2. [1].

Table 1: List of global geopotential models

Model	Maximal degree	Data
EIGEN-6C2	1949	S(GOCE,GRACE,LAGEOS),G,A
EGM08	2190	S(GRACE),G,A
EGM96	360	EGM96S,G,A

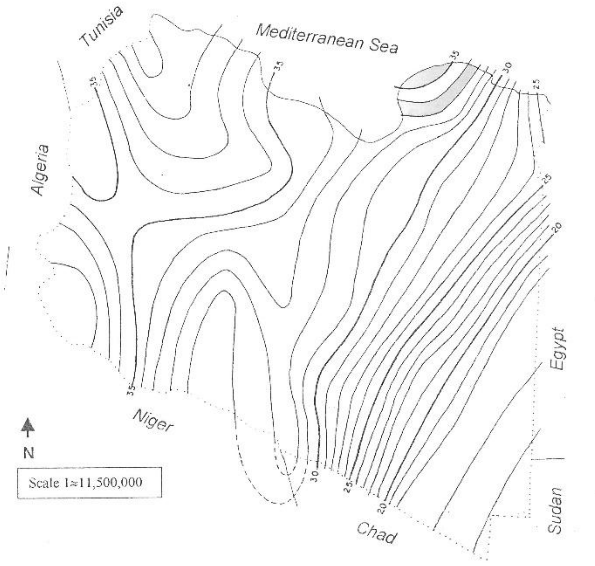


Figure 5: ING Libyan geoidal undulation map (WGS72 Datum) [9]

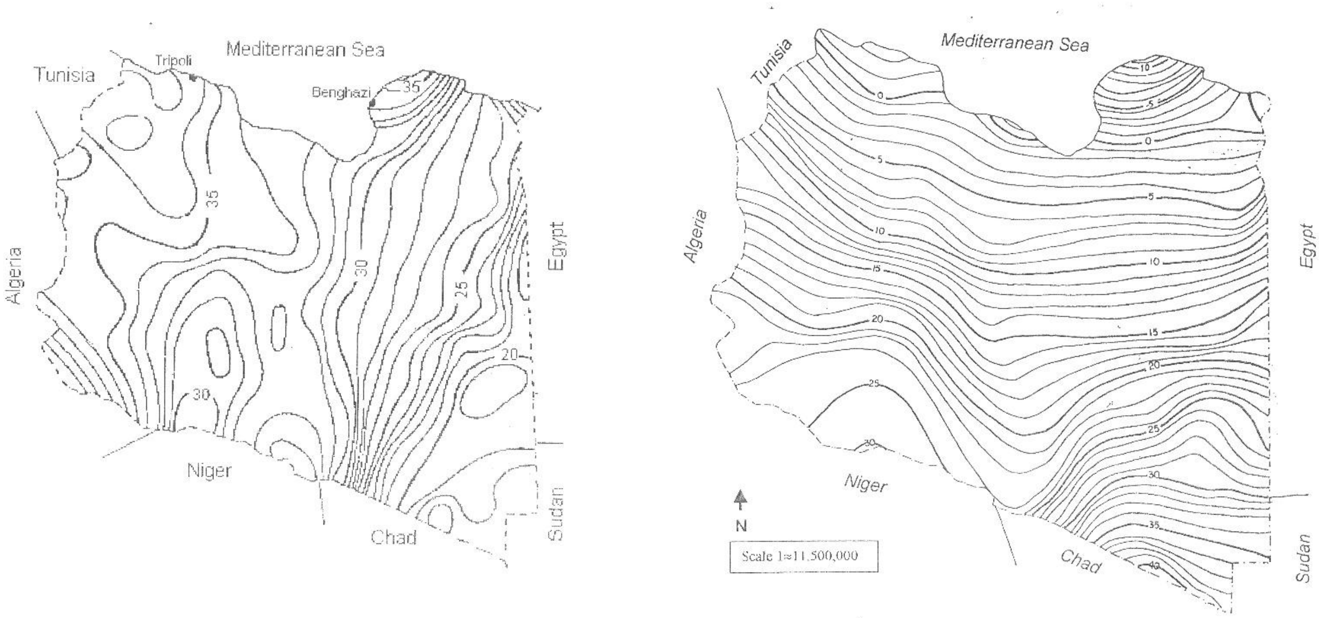


Figure 6: Left: ASC Libyan geoidal undulation map (ELD79 Datum);
Right: ASC Libyan geoidal undulation map (WGS72 Datum) [9];

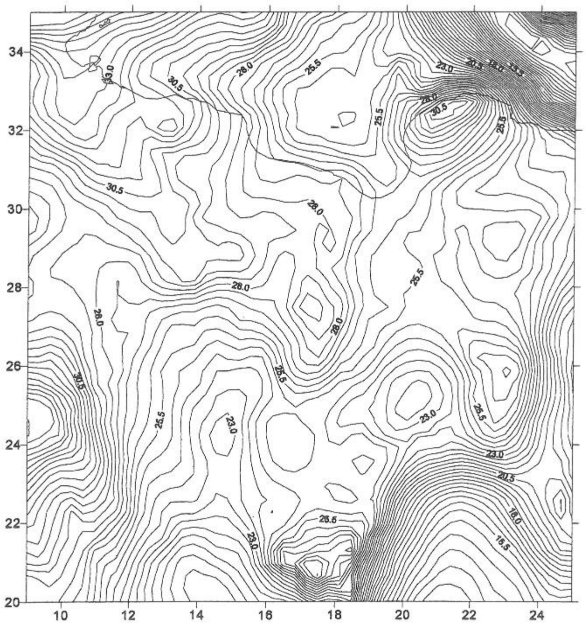


Figure 7. Geoid (quasi geoid) for the area of Libya [5]

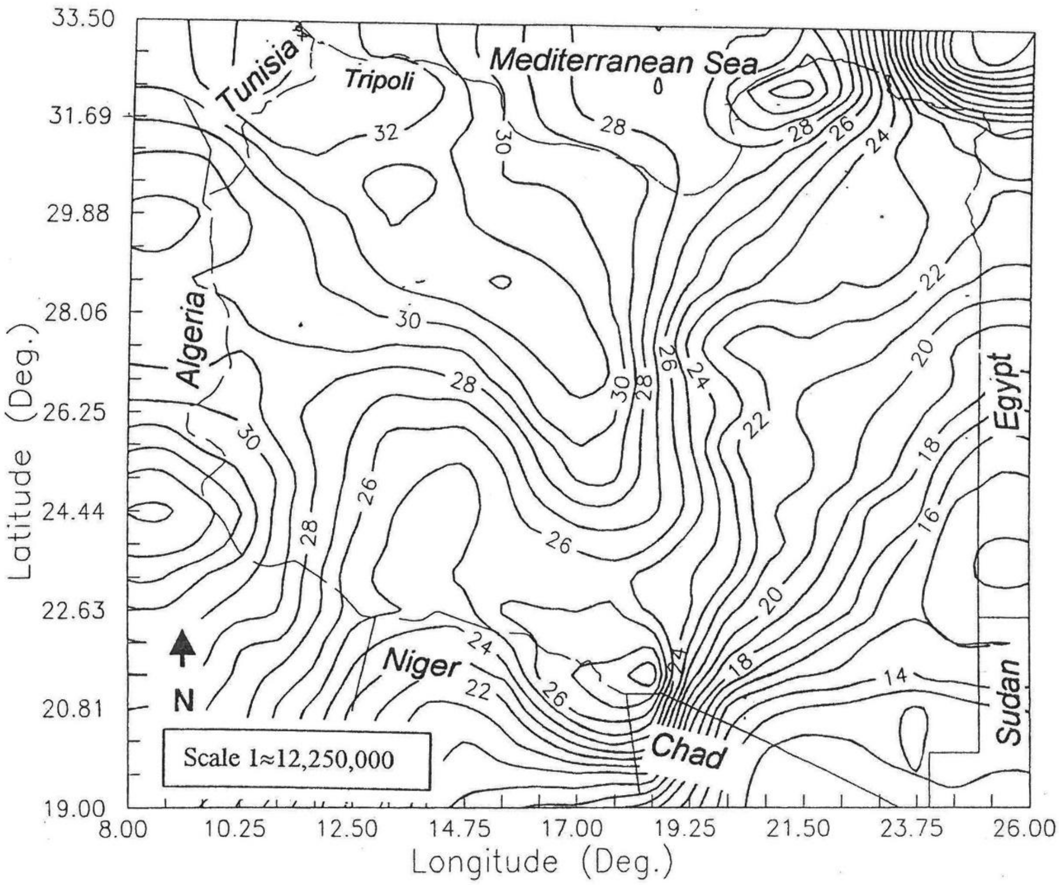


Figure 8. The geoidal undulation map with 1 m interval [9]

2. Parameters for quality evaluation and mutual comparision of global geopotential models

It is possible to determine the free air anomalies using the following expression [3]:

$$\Delta g = g_P - \gamma_Q, \tag{1}$$

where g_P is determined gravity value in point *P*, and γ_Q is the value of normal gravity in point *Q* at teluroid.

Gravity value, present in the equitation (1) for the free air anomaly is related to the International Gravity Standardization Net 1971 system (IGSN71) [6], and the value of normal gravity is obtained by calculating within the frame of Geodetic Reference System 1980 (GRS80) [7].

On the other hand, it is possible to determine the free air anomaly by using GGM coefficient. Real and normal potential, *W* and *U*, can be determined by equations [11]:

$$\begin{aligned} W(r,\theta,\lambda) = \frac{GM^{REF}}{a^{REF}} \sum_{n=0}^{N_{\max}} \left(\frac{a^{REF}}{r}\right)^{n+1} \sum_{m=0}^{n} \left(\overline{C}_{nm}^{ELL} \cos m\lambda + \overline{S}_{nm}^{ELL} \sin m\lambda\right) \overline{P}_{nm}(\cos\theta) + \\ + \frac{1}{2}\left(\omega^{REF}\right)^2 r^2 \sin^2\theta, \end{aligned} \tag{2}$$

$$U(r,\theta,\lambda) = \frac{GM^{REF}}{a^{REF}} \left(\sum_{n=0(2)}^{8} \left(\frac{a^{REF}}{r}\right)^{n+1} \overline{C}_n^{REF}\, \overline{P}_n(\cos\theta) \right) + \frac{1}{2}\left(\omega^{REF}\right)^2 r^2 \sin^2\theta, \tag{3}$$

where

$$\begin{Bmatrix} \overline{C}_{nm}^{ELL} \\ \overline{S}_{nm}^{ELL} \end{Bmatrix} = \left(\frac{GM^{GGM}}{GM^{REF}}\right) \left(\frac{a^{GGM}}{a^{REF}}\right)^n \begin{Bmatrix} \overline{C}_{nm}^{GGM} \\ \overline{S}_{nm}^{GGM} \end{Bmatrix}, \tag{4}$$

GM is the gravity constant times total mass of the Earth, a is the semi-major axis - Equatorial Radius of the Earth, n and m are the degree and order of the GGM, $N_{\max}$ is the GGM maximum degree, r, θ and λ are spherical coordinates, $\overline{P}_{nm}$ and $\overline{P}_n$ are the Legendre function, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the coefficient of spherical harmony development and ω is the angle speed of the Earth rotation. Labeled as *REF* mentioned values relate to adopted reference system (in this paper work GRS80), and as *GGM* they relate to global geopotential model, and as *ELL* those witch are related to spherical harmonic coefficients that result after being scaled as it is shown in the equation 4.

By using shown equations it is possible to determine at the ellipsoid vertical which passes thru the point *P* the point *Q* in which the relation:

$$W_P = U_Q, \tag{5}$$

is satisfied and then the free air anomaly can be shown as:

$$\Delta g_{P,GGM} = g_{P,GGM} - \gamma_Q, \tag{6}$$

where, in rectangular coordinates *x*, *y* и *z*, said values can be determined out of expression:

$$g_{P,GGM} = \left|grad\, W_P\right|, \quad \gamma_Q = \left|grad\, U_Q\right| \tag{7}$$

By forming the differences by using the expression:

$$R_{\Delta g} = \Delta g_{P,GGM} - \Delta g_P, \tag{8}$$

the parameters by which the GGM quality and adequacy can be estimated are defined.

3. Available data

The data collected and used in this paper were obtained from two sources, the Surveying department of Libya (SDL) and the Libyan Petroleum Institute (LPI). The Libyan Gravity Project (LGP) compiled all available onshore gravity data for the whole Libya by the Libyan Petroleum Institute (LPI) under consulting Robertson Research Group, UK, 2001 [10]. The gravity data was gridding at a cell size of 2 minutes (approximately 4 km). Gravity data base of Libya consists of geodetic latitude and longitude, heights above sea level, and gravity values. All gravity values are related to IGSN71. The sources and quantities of gravimetric data used are:

- 12386 gravity points from SDL forming the first and second order leveling network of Libya.
- 13435 gravity points from LPI from different concessions of oil exploration in Libya.

All the gravity points (25821 points) are relatively irregularly positioned throughout the territory of Libya, which surface is 1.76 million sq. km^2, the terrain of Libya is mostly flat and up the sea level. Libya's lowest point is at 47m below sea level and the highest point is at 2267m. The Figure 9 shows the distribution of these gravimetric data. The accuracy of gravimetric survey is not bigger than 0.1 mgal.

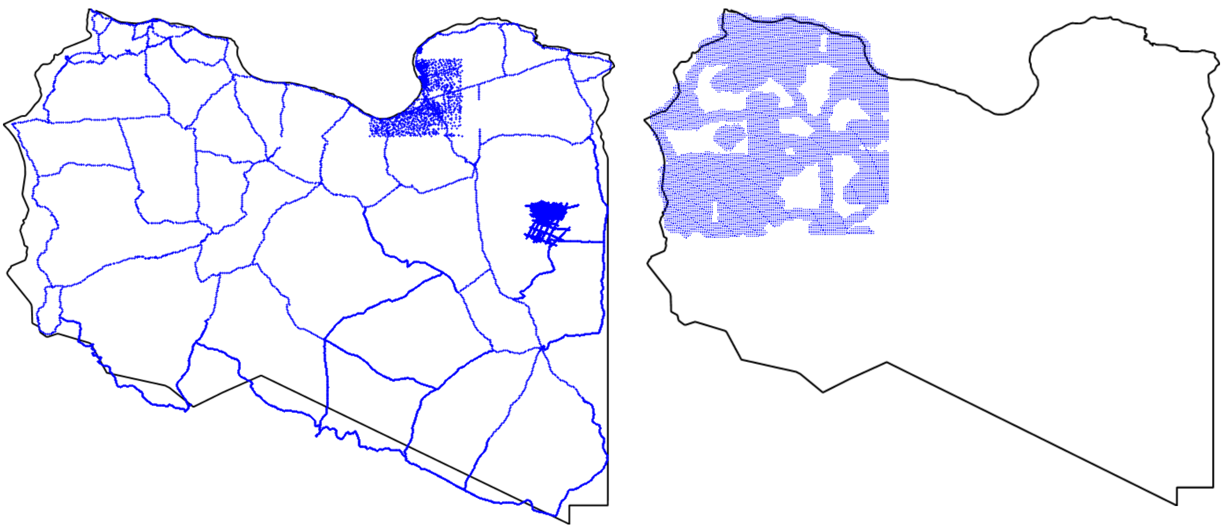


Figure 9. Left: Gravity data along leveling lines.
Right: Gravity data in north-west Libya.

4. Numerical researches

Using the equations presented in the previous section, the free air anomalies were determined by applying GMM for all the points of gravimetric survey. By applying equation (8) the difference sets $R_{\Delta g}$ were formed.

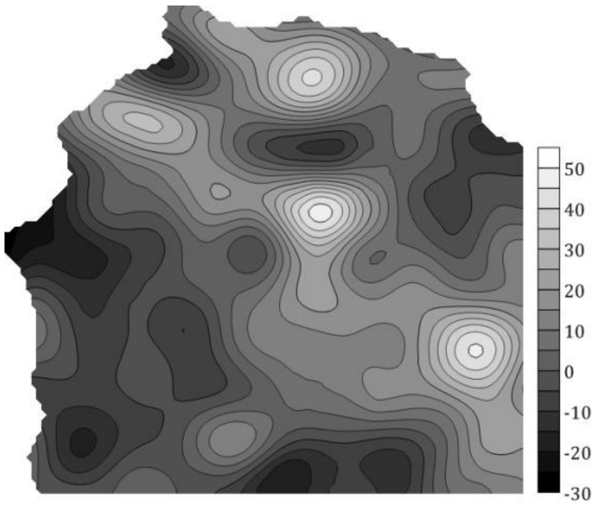
The determination results are shown in Figure 10 and their basic statistic data are shown in Table 2.

Concerning the free air anomalies one can notice as follows:

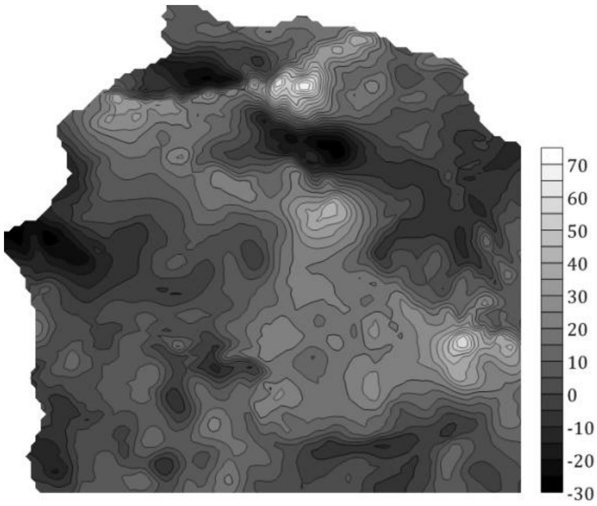
- Using EGM08 the gravity anomalies which approximate the real anomalies with an average differences value of 1.79 mgal and a standard deviation of 6.84 mgal are obtained.
- Using EIGEN-6C2 the gravity anomalies which approximate the real anomalies with an average differences value of 2.01 mgal and a standard deviation of 6.33 mgal are obtained.
- Also, there is a high concordance between results obtained by EGM08 and EIGEN-6C2. The set of 11523 anomalies differences was determined by these two models by applying the expression

$$R_{\Delta g,GGM} = \Delta g_{EGM08} - \Delta g_{EIGEN-6C2}, \tag{1}$$

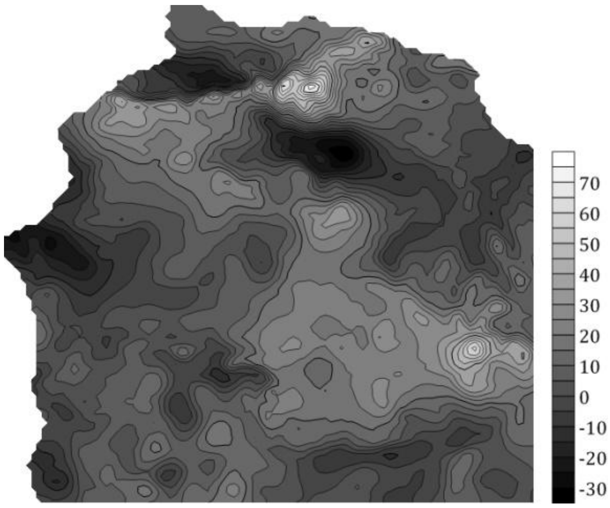
and the average value of this differences of 0.22 mgal were obtained, with a standard deviation of 1.96 mgal,



(a) EGM96

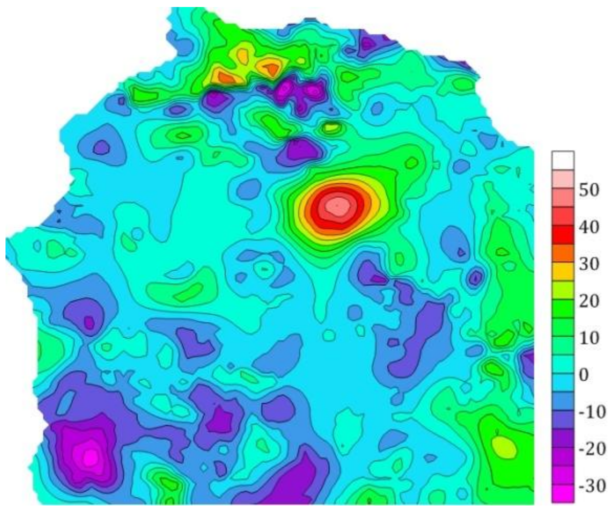


(b) EGM08

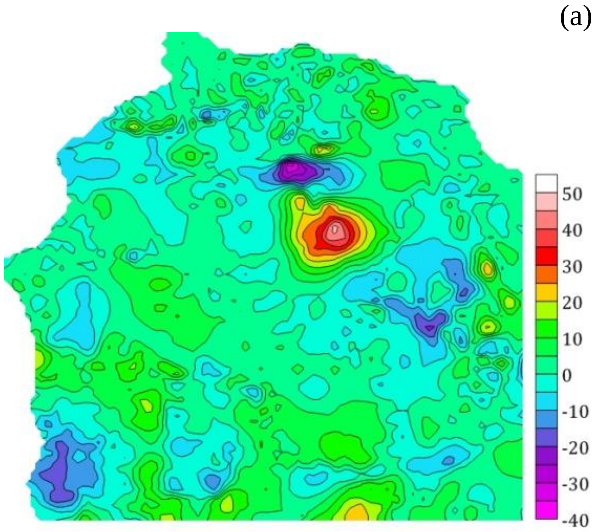


(c) EIGEN-6C2

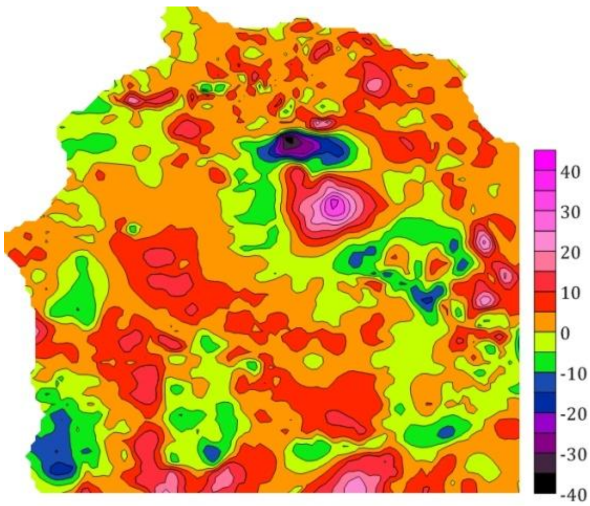
Figure 10. Free air anomalies Δg derived by applying GGMs (equidistance 10 mgal)



(a) EGM96



(b) EGM08



(c) EIGEN-6C2

Figure 10. Differences $R_{\Delta g}$ (equidistance 10 mgal)

5. Conclusion

In the frame of this paper the numeric researches that are related to the quality of several existing global geopotential model for the territory of Libya are shown. The EGM96, EGM08, and EIGEN-6C2 models were tested. The coefficients of mentioned models were used up to the maximum degree and order.

The greatest agreement with the terrestrial data of the free air anomalies is achieved by using the EIGEN-6C2 model. The set of the terrestrial differences anomalies of free air and the anomalies that follow out of applying EIGEN-6C2 model has an average value of 0.22 mgal, with the standard deviation of 1.96 mgal.

Together with all mentioned it can be said that the results of determining pointed to a great accordance between the EIGEN-6C2 and EGM08 models as well as that by using all of these tested models far better approximations of gravity field in the territory of Libya than the one that follows after using the global EGM96 model are obtained.

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