HAUSDORFF DISTANCE FOR THE DIFFERENCES CALCULATION BETWEEN 3D SURFACES

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Abstract: *In order to obtain the 3D surface model, many data sources and methods have been developed, the most popular one being the Delaunay triangulation. In this paper the Hausdorff distance accuracy in the differences evaluation process between 3D mesh surfaces, was tested, as well as the ALS data precision. The aim of this research is to determine the horizontal and vertical differences between two 3D surfaces. To obtain the results, the roof surface of the "Department of Terrestrial Measurements and Cadastre" building, from Iasi – City area, was created, based on ALS data and precise measurements made with a total station.*

Keywords: *3D surface, Hausdorff distance, ALS, evaluation*

1. Introduction

In order to obtain the 3D surface model, many data sources and methods have been developed, the most popular one being the Delaunay triangulation. The result is a TIN network that consist in a geometric component (defines the nodes tridimensional positions) and a topological one (defines the connectivity between the nodes), the so called, mesh surface. Often, some meshes contain additional information, such as: the normal vectors of the triangular surfaces, the colours associated to each node and the texture coordinates.

In order to obtain the differences between two 3D mesh surfaces, most algorithms are based on geometric measurements of distance or curvature. These algorithms were introduced to measure and highlight the errors caused by a mesh simplification, as a representation of a complex 3D surface, consume the computer resources and it is not always necessary. An overview of the techniques used to evaluate the error introduced by the mesh simplification process, can be found in (Cignoni *et al.*, 1998).

In this paper, the Hausdorff distance was used for the differences evaluation between two 3D triangle mesh surfaces.

The Hausdorff distance was implemented in 2002 in the "Metro tool" software, displaying the errors numerical values and their distribution as a histogram and also a visualisation of errors at a local level, by using a color palette (Cignoni *et al.*, 1998), in the "MeshDev" software (Michael Roy *et. al.*, 2002) and in the "CloudCompare" software (Cloud Compare, 2012). The last software can process 3D point clouds, but also mesh surfaces. First, was design to compare two point clouds (for exemple a point cloud resulted by measurements made with a laser scanner) or a point cloud with a mesh surface.

2. Presentation of the Study Area, Materials and Equipment

2.1. Presentation of the Study Area

The „Department of Terrestrial Measurements and Cadastre" building, from the Technical University „Gheorghe Asachi" of Iasi, has two building parts with different heights, both with a regular shape that is rectangular parallelepiped.

(a) (b)

Fig. 1 –The roof surface of the (a) north building part, (b) south building part

2.2. Materials and Equipment

The main materials for this task are the aerial laser scanner and the standard surveying tools. The topographic measurements were made using the Leica TC(R) 405 total station, having the data processed with the TopoSys software, while the 3D points were handled using Leica Cyclone v.6.0 and Cloud Compare software.

2.3 The Hausdorff distance

Hausdorff Distance - named after Felix Hausdorff, is the most famous metric for comparing two mesh surfaces, providing a global comparison.

Considering two surfaces *S* and *S'*, the distance between *S* and *S'* is $d_S(S,S')$, defined as:

$$d_S(S,S') = \max_{p \in S} d(p,S'), \qquad (1)$$

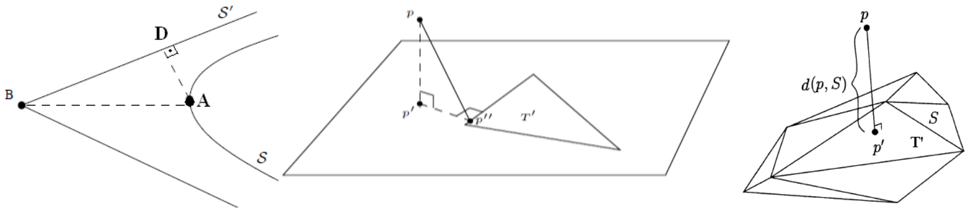
where $d(p,S') = \min_{p' \in S'} d(p,p')$ and $d(p,p')$ is the Euclidean distance between two points in R^3.

This distance is not symmetric, for exemple $d(S,S') \neq d(S',S)$. The $d(S,S')$ distance will be called the *forward distance*, and the $d(S',S)$ the *backward distance*. The term Hausdorff symmetric distance $d_S(S,S')$, will be introduced, defined as:

$$d_S(S,S') = \max[d(S,S'), d(S',S)]. \qquad (2)$$

The symmetrical distance offers a more accurate measurement of the differences between two surfaces, because the one-side distance can lead to an underestimation of the distance values between the two surfaces, as shown in Fig. 2. In this case, d(S,S') will be a lot smaller than d(S',S) distance, because d(A,D)≤ d(B,S).

When the orthogonal projection *p'* of the point *p* on the triangle surface *T'*, is inside the triangle, the distance from the point to the triangle is nothing but a point to plan distance. When the point projection is outside the triangle *T'***,** the point to triangle distance, is the distance from the point *p* to the closest point *p''* that belongs to a triangle side, as you can see in *Fig. 2*.

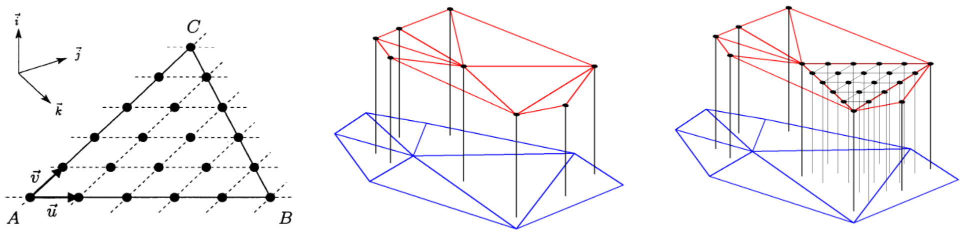


(a) (b) (c)

Fig. 2 – The distances between S and S' surfaces, (b) The distance between **p** şi **T'** with p' situated outside the triangle (Aspert N. *et al.*, 2002), (c) The distance between **p** şi **T'** with p' situated inside the triangle (Roy M. *et al.*, 2004)

Even if the distance $d(p,S')$ can be calculated analytically for any point *p*, a sampling of the triangle surface it is necessary, in order to obtain the maximum for $p \in S$. Thus, each triangle which belongs to *S* surface is sampled, computing the distances between each nod of each sample and *S'* surface (Fig. 3a).

The differences between the two surfaces formed by triangles, calculated using the Hausdorff distance, are graphically shown in Fig. 3.



(a) (b) (c)

Fig. 3 – (a) The sampling scheme performed on a triangle (Roy M. *et al.*, 2004), The distances calculation between two mesh surfaces, (b) at local level, only in the triangle vertices, (c) at global level, both in the triangle vertices and in different points on its surface

Having a set of distances, the mean distance E_m between two surfaces, is calculated using the following formula:

$$d_m(S,S') = \frac{1}{|S|}\int_S d(p,S')ds. \tag{3}$$

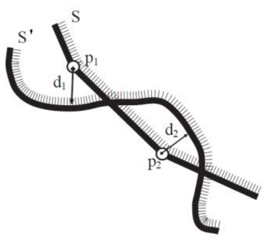


Fig. 4 –Signed distances between S and S' surfaces (S is the sampled curve) (Cignoni P. *et al.*, 1998)

If the *S'* surface is orientable, the distance between point *p* (which belongs to *S* surface) and the *S'* surface, can be said, only informally speaking, that is positive, if the *p* closest point, namely $p' \in S'$ is in the outer space with respect to *S*, and negative otherwise (Fig. 4). Or, in other words, if *Np* is the normal vector to *S* in point *p* and $p' \in S'$ is the nearest point, the sign of the distance is given by the relation $N_p(p'-p)$.

The distance accompanied by its sign was introduced in the "*Cloud Compare*" software for a independent evaluation of the areas that belong to the first surface and are situated inside or outside the space, relative to the second surface.

3. Results and discussion

3.1 Total Station Surveying

For the building roof located in the North side, a total of 567 detail points, uniformly distributed over the roof surface, as well as on its edge, was measured from a single point station. Two points located in the corners of the roof, have been materialized, for the station orientation, as can be seen in Fig. 5. For the building roof located in the South side, a number of 105 detail points has been measured, from a single point station, with orientation on a single point with known coordinates (Fig. 5). Both the station coordinates and the points coordinates used for orientation, were determined by GNSS technology using a S82-South V rover.

Uniting the measured detail points on the edge of the building, the two roofs limits were obtained, marked in red colour in Fig. 5.

To assess the accuracy of the roofs edges obtained based on ALS data, were superimposed over those obtained with precision using a total station measurements, the differences ranging from the minimum of 1 cm up to a maximum of 60 cm, for the Noth building path, namely from 1 cm up to a maximum of 70 cm, for the South building part.

It should also be noted that two points that were classified as "high vegetation", and which by their horizontal position belong to the roof plan, actually represents a point measured on the pilaster with the antenna of the GNSS reference station, and a second one, measured on a flue of the building ventilation system (Fig. 5, 6). Horizontal distance calculated based on the two sets of coordinates is about 70 cm, this explaining very clearly the differences between the two limits.

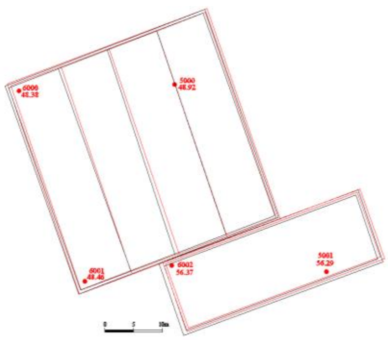
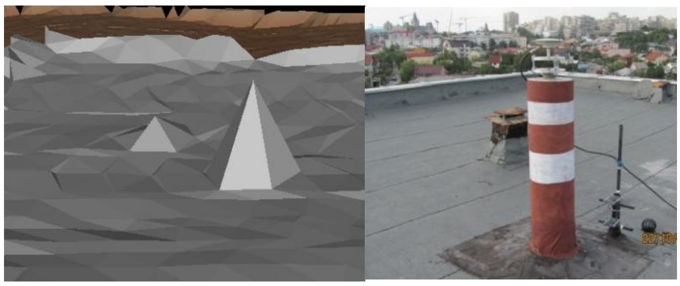


Fig. 5 – The roof boundary and edges of the “Cadastre” building, with red being marked the ones obtained based on total station measurements and with black colour those obtained based on ALS data

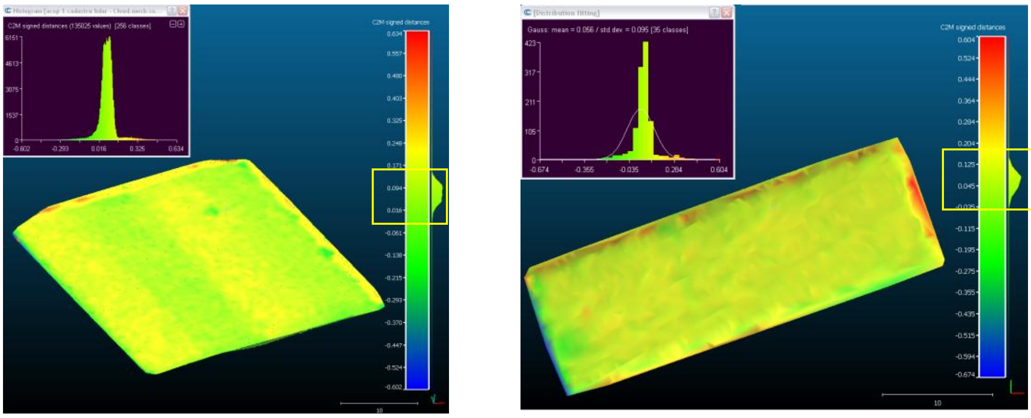


(a) (b)

Fig. 6 –(a)The Delaunay triangulation applied for the ALS data, the two peaks representing the pilaster of the GNSS antenna and a flue of the building ventilation system, (b) digital image

3.2 The ALS horizontal precision using the Hausdorff distance

Were created two *txt file, one containing the detail points coordinates resulted after the total station measurements, and the other one containing the ALS data. Then were imported into the „*CloudCompare v.2.4*” software, where, first were created two triangular surfaces by the Delaunay triangulation. Then, the two surfaces were compared, considering as reference surface the one resulted from total station measurements, using as comparison metric the sign Hausdorff distance and the octree level 8.



(a) (b)

Fig. 7 – The differences between the surface resulted based on ALS data and that obtained based on GNSS measurements (a) the roof surface of the North building part, (b) the roof surface of the south building part

For the roof of the building north part, the differences were of maximum **63.4 cm**, and for the roof of the building south part, the differences were of maximum **67.4 cm**, representing the horizontal translation between the two data sets (Fig. 7).

3.2 The ALS vertical precision using the Hausdorff distance

To determine the ALS data vertical precision, the two surfaces generated based on GNSS and ALS data, should be compare regardless the translation between them, or should be compare only the double coverage area. To obtain this area, the two meshes should be cut after an outline that defines the boundary of the double coverage area. This cannot be done directly in the "*CloudCompare*" software, that's why a regular grid was built, which have the same dimensions for both surfaces, and there is no translation between them (each node of the grid belonging to the first surface correspond to a node of the grid that belongs to the second surface).

In order to achieve the purpose, the roof detail points have been interpolated. Various methods have been chosen, in order to find the most accurate one for the roof surface generation. Were used the interpolation methods existing in the Matlab programming language library, both for the interpolation of a data set distributed in a regular rectangular grid, as well as, of a data set randomly distributed.

Taking into account that the ALS points density is 4-5 pct/m^2 and the points determined by GNSS measurements are located at distances of about 1-2m, the grid size of 0.5m, was chosen (Chirilă C. *et. al.*, 2013).

The statistical results obtained for the 0.5 m grid size, using the four interpolation methods, and the testing performed on the three control points (the station point and the two orientation points) in the case of ALS data interpolation, are presented in Table 1, and the results obtained in the case of GNSS interpolation, are presented in Table 2.

The average of the deviations in absolute value from the mean was calculated with the following relation:

$$\theta = \frac{1}{n}\sum_{i=1}^{n}\left|x - \bar{x}\right| \;, \tag{4}$$

where $\bar{x}$ is the mean average of the individuals determinations and *n* is the determinations total number.

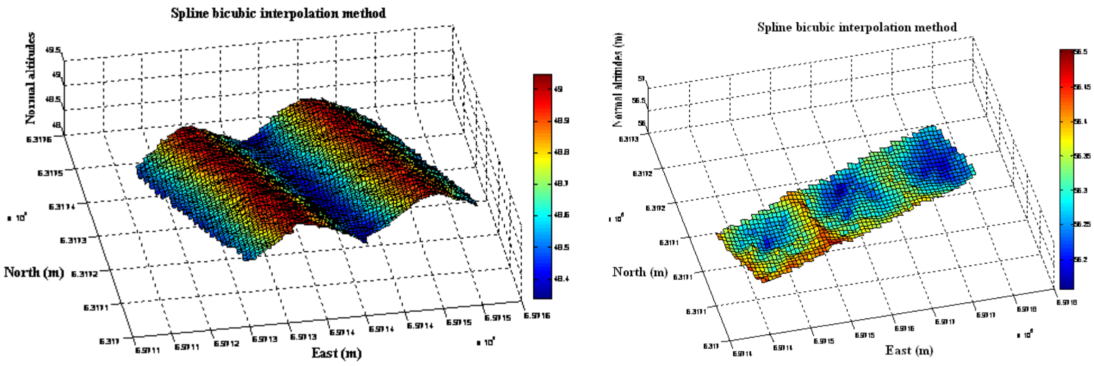
Table 1 – The statistical results of the ALS data set interpolation, through the existing methods in the Matlab programming language library (0.5 m)

The interpolation method	Mean deviation of transformation [cm]	Maximum value of deviation [cm]	Average of the deviations in absolute value from the mean [cm]
Linear	1.65	13.22	1.20
Nearest neighbor	1.54	14.00	0.54
Spline bicubic	**1.38**	**9.11**	**0.99**
„v4” (1 m grid size)	2.34	13.52	1.76

Table 2 – The statistical results of the GNSS data set interpolation, through the existing methods in the Matlab programming language library (0.5 m)

Interpolation method	Mean deviation of transformation [cm]	Maximum value of deviation [cm]	Average deviations in absolute value from the mean [cm]	Mean deviations in the control points [cm]
Linear	0.59	5.00	0.37	5.0
Nearest neighbor	0.07	1.00	0.01	6.0
Spline bicubic	**0.28**	**1.04**	**0.20**	**5.0**
„v4” (1 m grid size)	0.46	3.44	0.30	3.5

The spline bicubic interpolation method was chosen for the grid generation. The graphic representation of the surfaces corresponding to the building roofs, is presented in Fig. 8.



(a) (b)

Fig. 8 – The surface interpolated by the spline bicubic method, with the grid size of 0.5 m, based on ALS data, (a) the roof of the north building part, (b) the roof of the south building part

Then, were calculated the Hausdorff distances using the *"CloudCompare v2.4"* software, between the common nodes corresponding to the two surfaces (ALS and GNSS), yielding the maximum difference of **17.9 cm,** the mean difference of 7.53 cm, and the standard deviation of 3.34 cm, for the north building part (4535 common nodes) and the maximum difference of **17.3 cm,** the mean difference of 6.64 cm, and the standard deviation of 2.74 cm, for the south building part (1198 common nodes) (Fig. 9).

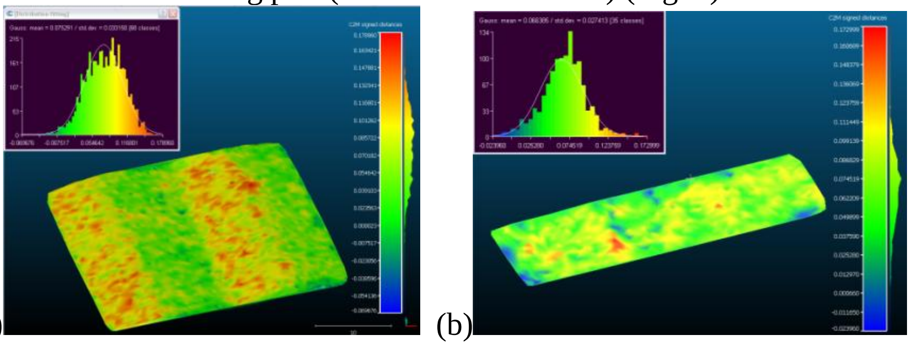
(a) (b)


Fig. 9 – The differences between the roof surface of (a) the north building part, (b) the south building part, resulted from ALS data and the one resulted based on GNSS measurements, for the 0.5 m grid size

A second comparison was made between the surfaces obtained by the ALS and GNSS data approximation, using the least squares method, with the help of the "*Leica Cyclone v.6.0*" software, and those obtained by Delaunay triangulation. The roof surface of the North building part was approximated with four planes, and the roof surface of the South building part was approximated by a single plane.

In the case of the roof surface belonging to the North building part, the maximum positive difference between the surface obtained by approximating the ALS data and the one obtained by Delaunay triangulation is **20.5 cm**, the mean difference is 0.37 cm, and the standard deviation is 3.89 cm, as can be seen in Fig. 10 a. The maximum negative difference between the surface obtained by approximating the GNSS data and the one obtained by Delaunay triangulation is **21.22 cm**, the mean difference is 0.06 cm, and the standard deviation is 6.26 cm, as can be seen in Fig. 10 b.

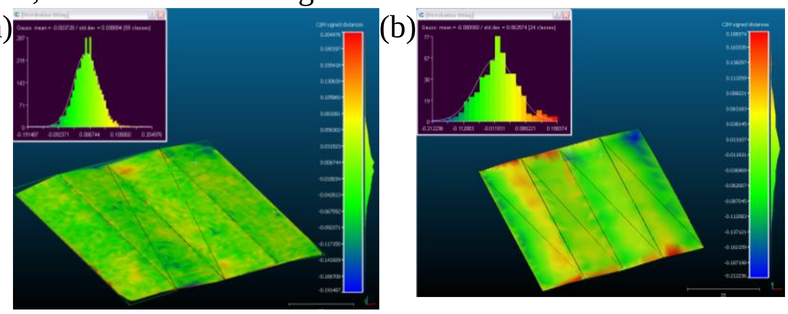
(a) (b)


Fig. 10 – The differences between the roof surface of the North building part, resulted by interpolating the (a) ALS , (b) GNSS data, using the Delaunay triangulation, and the one resulted by points approximation using the „Leica Cyclone v.6.0” software

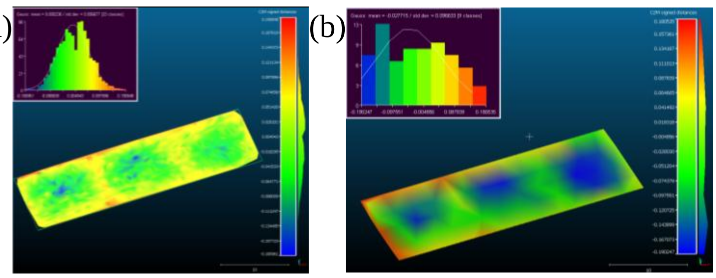
(a) (b)


Fig. 11 – The differences between the roof surface of the South building part, resulted by interpolating the (a) ALS , (b) GNSS data, using the Delaunay triangulation, and the one resulted by points approximation using the „Leica Cyclone v.6.0” software

In the case of the roof surface belonging to the south building part, the maximum positive difference between the surface obtained by approximating the ALS data in the *„Leica Cyclone v.6.0"* software, and that obtained by means of Delaunay triangulation is **19.1 cm**, the maximum negative is 18.1 cm, the mean difference is 0.24 cm and the standard deviation of 5.67 cm (Fig. 11). The maximum positive difference between the surface produced by the approximation of the GNSS data and that obtained by means of Delaunay triangulaţia is 18.05 cm, the maximum negative difference is **19 cm**, the mean difference is 2.77 cm and standard deviation of 9.66 cm (Fig. 11).

To verify the results obtained in the "*CloudCompare*" software, have been approximated the ALS data and the GNSS data, corresponding to the roof of the South building part, with a plan by the least squares method, by a created program in the Matlab programming language. The purpose of this experiment is to calculate the differences between the elevation of each point and the elevation of the point defined by the same position within the XOY plan and belongs to the approximated plan, and to compare them with those obtained in the "*CloudCompare"* software.

The starting point was the implicit equation of the plan, having the following form:

$$A\cdot x+B\cdot y+C\cdot z+D=0, \tag{5}$$

where *A*, *B* and *C* represents the plan parameters, respectively, the plan normal components, along the three axes.

The distance from point $P_i(x_i, y_i, z_i)$ to the plan **π** defined by the relation (6) is:

$$d(P_i,\pi)=\frac{|Ax_i+By_i+Cz_i+D|}{\sqrt{A^2+B^2+C^2}}. \tag{6}$$

For a given data set $\{(x_i, y_i, z_i)\}_{i=1}^{n}$, have to be determined the *A, B* and *C* parameters, so that, the plan that best fits the points it is obtained by the minimisation of the distances (perpendiculars) drawn from each point of z_i normal altitude, to the Ax_i+By_i+C plan, or in other words, the sum of the squared errors between the z_i points normal altitudes and the plan corresponding values is minimized:

$$\sum_{i=1}^{n} d(P_i,\pi)^2 \to \min. \tag{7}$$

The following function is considered:

$$F_i(A,B,C)=\sum_{i=1}^{n}\left[\left(Ax_i+By_i+C\right)-z_i\right]^2, \tag{8}$$

In order to obtained the final values of the plan normal parameters (*A*, *B* and *C*) the Gauss-Newton method is used.

To aply the least square principle, it is considered $F_i=0$. The partial derivates of the function with respect to the arguments A, B and C, are:

$$\begin{aligned}\frac{\partial F_i}{\partial A}&=2Ax_i^2+2Bx_iy_i+2Cx_i-2x_iz_i=0,\\ \frac{\partial F_i}{\partial B}&=2By_i^2+2Ax_iy_i+2Cy_i-2y_iz_i=0,\\ \frac{\partial F_i}{\partial C}&=2C+2Ax_i+2By_i-2z_i=0.\end{aligned} \tag{9}$$

The gradient of the function *F*, has to satisfie the condition $\nabla F_i=(0,0,0)$:

$$(0,0,0)=\nabla F_i=2\sum_{i=1}^{n}\left[\left(Ax_i+By_i+C\right)-z_i\right]\left(x_i,y_i,1\right).$$

The general equation system written in matrix form is:

$$A_{3,3} \cdot X_{3,1} = V_{3,1}, \tag{10}$$

or detailed:

$$\begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} 1 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} x_i z_i \\ \sum_{i=1}^{n} y_i z_i \\ \sum_{i=1}^{n} z_i \end{bmatrix}.$$

where: $A_{3,3} = \begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} 1 \end{bmatrix}; \quad X_{3,1} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \quad and \quad V_{3,1} = \begin{bmatrix} \sum_{i=1}^{n} x_i z_i \\ \sum_{i=1}^{n} y_i z_i \\ \sum_{i=1}^{n} z_i \end{bmatrix}.$

The unknowns vector result from solving the equations system (11):

$$X = \left(A^T A\right)^{-1} A^T V. \tag{11}$$

By determination of the unknowns parameters, the plan equation that best fits the considered points, is obtained.

The coordinates inventory corresponding to the GNSS and ALS data, were imported into Matlab and the equations system (11) was solved using a personal script.

Were calculated analytically the differences between the altitude of each point and the altitude of the point belonging to the plan approximated in Matlab, using the ***A, B*** and ***C*** parameters obtained by solving the equations system (11). The same diferences were calculated using the „*CloudCompare*” software.

The obtained results, after the analitical calculation of differences, in Matlab and “*CloudCompare*“ software, are centralized in Table 3.

Table 3 - Altitudes differences obtained in the "CloudCompare” software and Matlab

Altitudes differences [m]	**Plan obtained based on GNSS data**		**Plan obtained based on ALS data**	
	Matlab	**CloudCompare**	**Matlab**	**CloudCompare**
Maximum positive difference	0.209028	0.209012	0.218740	0.218741
Maximum negative difference	0.163282	0.163280	0.192616	0.192616
Standard deviation	0.096217	0.096203	0.065907	0.065907

It can be observed, that, results obtained in „*CloudCompare*” software are almost identical to those obtained by means of analytical calculation, therefore, it can be said that, a high degree of confidence can be given to this software, in the process of differences between two 3D surfaces calculation.

4. Conclusions

The Hausdorff distance and the "*CloudCompare*" software, offers a comprehensive assessment of the differences between two 3D mesh surfaces by using a color palette, the user being able to quickly and accurate identify the errors of the 3D surfaces that was compared with a reference surface considered with no errors.

To assess the accuracy of the roofs edges obtained based on ALS data, were superimposed over those obtained with precision using total station measurements, the differences ranging from the minimum of 1 cm up to a maximum of 60 cm, for the Noth building path, namely from 1 cm up to a maximum of 70 cm, for the South building part. These differences are clearly explained by the fact that, the horizontal distance between the ALS point measured on the pilaster with the antenna of the GNSS reference station and those determined by geodetic measurements is about 70 cm.

The maximum differences between the roof surface of the North building part, respectively the South part, belonging to the „Cadastre" building, resulted by ALS and GNSS data interpolation, calcultated using the Hausdorff distance in „*CloudCompare*" software, have appropriate values with those obtained into AutoCad software.

From these experimental studies it was observed that the maximum vertical difference between the two data sets LSA and GNSS was approximately 17.5 cm, value which is within the accuracy mentioned in the literature.

It was observed that, through the approximating process of the surfaces with the most probably geometric shapes, it lose much of fidelity representation, the differences between the real and the approximated surface reaching about 20 cm. Currently existing software for point cloud processing, use this method to create a final 3D model of the scanned object.

When we want to compare two 3D surfaces regardless the translation between them, or to compare only the double coverage area, the two meshes should be cut after an outline that defines the boundary of the double coverage area. This cannot be done directly in the "*CloudCompare*" software, although the software has a cutting function by a polygon defined interactively by the user, because, by this operation, the triangles situated outside the cutting polygon and also the ones that intersect the polygon, are deleted.

5. References

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