ISSUES REGARDING THE GEODETIC SUPPORT NETWORK RESTORATION FROM COJOCNA EXPERIMENTAL DIDACTIC STATION PERIMETER, BELONGING TO USAMV CLUJ-NAPOCA

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Abstract: After H.G. published in M.O. No. 199 / 8.V.2000 of passing a land area of 667.1 hectares, owned by the state in the administration of M.E.N. for USAMV Cluj-Napoca, is required the restoration of the geodetic support network in the area. In essence, this paper addresses two issues related to the physical realization of geodetic support network and the analysis of stochastic functional model, used to determine the coordinates of the points of the network. The support network will be used to locate and monitor various existing and future objectives of the resort area and also will be a geodesic polygon in which students from Land Measurements and Cadastre department within U.S.A.M.V. Cluj-Napoca will conduct their annual practice. To achieve the necessary precision for positioning and staking various objectives of the resort, the geodetic support network would be restored through modern GPS methods.

Keywords: geodetic support network, global positioning, stochastic functional model

1. Introduction

The need to restore the network support from Cojocna Farms 3 and 4 occurred as a result of Government Decision published in the O.M. no.199/8.05.2000 regarding the passage of 667.1 ha of agricultural land from the state property to the management of Ministry of National Education for USAMV Cluj-Napoca.

Restoration of the support network aims to locate and monitor various existing and future objectives from Cojocna didactic resort, and the achievement of a geodesic polygon in which students from USAMV Cluj-Napoca will conduct their annual practice.

After following the recognition phase of the field were identified four old landmarks from the Vth ordn of the national triangulation, without targeting signals, with a low accuracy of localization and does not cover the entire surface of the resort. It should, therefore, restoring the network and increase the accuracy of localization points.

2. General aspects

To ensure the necessary accuracy of the works, in general, the design of the support the network is realized in three steps, measured and independently compensated. For the same value of the accuracy coefficient it can be written the following relations:
where:

\[ m_2 = \frac{m_3}{k}; m_1 = \frac{m_2}{k} = \frac{m_3}{k^2} \] (1)

Taking into account relation (1), the total average error of points positioning will be given by the expression:

\[ M = m_3 \sqrt{\frac{1}{k^4} + \frac{1}{k^2}} + 1 = m_3Q \] (2)

Considering known the total average error of panimetric positioning of the network and coefficient of increasing the measurements accuracy, can be calculated (Cristescu, 1978), the mean square errors of the three steps with the following relations.

According to the calculated accuracies of the three steps will be chosen the tools and methods.

To achieve the necessary precision of positioning and plotting the various objectives of the resort, the geodetic support network will be restored through modern GPS.

GPS project planning is the optimal choice of measurement methods, apparatus required, and planning observations.

When planning observations in a GPS project we must take into account several factors:

- Satellites configuration;
- Number and type of the available receivers;
- Economics.

Satellite measurements planning session is done with special programs delivered by companies that provide topographic apparatus.

The first step in choosing a optimum period for the measurements, which will be subdivided into work sessions. The optimum period is characterized by a sufficient number of visible satellites and a PDOP value that could not be lower <6.

In determining the relative positioning of the work session, will consider four factors:

- The length of the base;
- The number of visible satellites; satellite constellation geometry (PDOP);
- The signal / noise for satellite signal;
- The second phase of planning for the distribution of observations refers to receivers on teams and scheduling points for each team. It is usually drawn up a table which provides the team in the session must be stationary at one point.

The minimum number of sessions in a grid of \( p \) points and the use of such receivers is determined by the relationship:

\[ s = \frac{(p - n)}{(r - n)} \] (4)

where

\( n \) - number of connection points between sessions.
\( S \) - number of working sessions.

Sessions must be chosen in order to contact at least four common satellites at an elevation of more than 150 in all items included in a session and the factor PDOP is not greater than 4 for the entire measurement period.

During field measurements must be provided the following requirements:

- Proper centering the antenna in the point station;
Data processing from the GPS equipment is done with different software of different companies, the vast majority use the Bursa-Wolf transformation models and Molodenski-Badecas.

The Bursa-Wolf (Bursa, 1962; Wolf, 1963) is a transformation model for the tridimensional transcalculation of the cartesian coordinates using seven parameters. Transformation involves three translational constants ($\Delta X$, $\Delta Y$, $\Delta Z$), three rotation elements (RX, RY, RZ) and scale factor ($\Delta L$). The matrix can be written as:

$$
\begin{bmatrix}
X_{KA} \\
Y_{KA} \\
Z_{KA}
\end{bmatrix} = 
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix} + 
\begin{bmatrix}
1 + \Delta L & R_z & -R_y \\
-R_z & 1 + \Delta L & R_x \\
R_z & -R_y & 1 + \Delta L
\end{bmatrix}
\begin{bmatrix}
X_{WGS} \\
Y_{WGS} \\
Z_{WGS}
\end{bmatrix}
$$

(5)

where:

$X_{WGS}$, $Y_{WGS}$, $Z_{WGS}$: geocentric cartesian coordinates of the global datum (WGS-84);

$X_{KA}$, $Y_{KA}$, $Z_{KA}$: cartesian coordinates of the local datum (Krasovski-1940).

The linear corrections equations in matrix treatment of a point are present as follow:

$$
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & 0 & X_{WGS} & 0 & -Z_{WGS} & Y_{WGS} \\
0 & 1 & 0 & Y_{WGS} & Z_{WGS} & 0 & -X_{WGS} \\
0 & 0 & 1 & Z_{WGS} & -Y_{WGS} & X_{WGS} & 0
\end{bmatrix}
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix} = 
\begin{bmatrix}
X_{KA} - X_{WGS} \\
Y_{KA} - Y_{WGS} \\
Z_{KA} - Z_{WGS}
\end{bmatrix}
$$

(6)

To determine the 7 parameters of transformation requires at least 3 points with known coordinates on both systems.

The solution to determine the unknown parameters is obtained by applying the theory of least squares method for solving matrix (Moldoveanu, 2002):

$$
X = (A^T P A)^{-1} A^T P L
$$

(7)

unde:

$X$ – the vector of the unknown parameters;

$A$ – configuration matrix of coefficients;

$L$ – vector of the free terms;

$P$ – matrix of the observation weights.

The accuracy of determining the zonal parameters of transformation is expressed by the mean square error $s_0$ (standard deviation):

$$
S_0 = \sqrt{V^T P V} / \sqrt{3n - 7}
$$

(8)
where:

n - number of points with known coordinates bet on both systems.

From geocentric coordinates, calculated with relation (5) will calculate longitude and latitude or altitude ellipsoidal iteratively on Krasovski ellipsoid.

\[
\lambda = \arctg \left( \frac{Y}{X} \right)
\]  

(9)

Next is: calculated the spherical latitude (\(\phi_0\)), the large radius of curvature (N), ellipsoid height (h) and the geodesic latitude (\(\phi\)) (Soler and Hoth, 1988).

\[
\phi_0 = \arctg \left( \frac{Z}{\sqrt{X^2 + Y^2}} \right)
\]  

(10)

\[
N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}
\]  

(11)

\[
h = \sqrt{X^2 + Y^2} - N \cos \phi
\]  

(12)

\[
\phi = \arctg \left( \frac{Z}{\sqrt{X^2 + Y^2} \left( 1 - e^2 \frac{N}{N + h} \right)} \right)
\]  

(13)

Geodetic coordinates (\(\phi\) and \(\lambda\)) will turn in Stere 70 rectangular coordinate, using the constant coefficient method.

3. Case study

The restored geodesic network from the Cojocna resort consists of 15 points, from which 4 are points of order V from the old national network (Fig. 1).

Fig. 1. Network drawing ([Δ] - points from the old network (501, 7, 12, 503); [+] - points from the new network)
Materialization of the points was achieved with Feno landmarks where the plastic part was replaced with a concrete one (Fig. 2).

Making the measurements was done with the following receivers: Stonex S9 GNSS L1, L2, Magellan ProMark 3 L1 și Leica (SR 20 L1) and for making the measurements using the static method was used and the ROMPOS system. Positioning the points with GPS receivers was made as follows:
- Landmarks 501, 502 respectively 503, 504 were stationed with GPS STONEX S9 GNSS L1, L2. The same session was interrupted due to low battery, so two sets of coordinates appear on the same points (501, 502 and 503, 504);
- Points 13 and 14 were stationed with GPS MAGELLAN PROMARK 3, the whole session was continuously for 8 hours;
- Points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 were stationed with GPS LEICA SR 20.
- Point 8 (9) was stationed 2 times keeping still the device to perform a comparative analysis on it;

Coordinate transformations used in Romania are based on Helmert zonal type parameter with 7 parameters or 4 parameters or coordinate transformation with model distortion.

4. Results and discussions

Positioning accuracy of network points on the three stages are calculated according to equations (2) and (3), the total error in the positioning stage M-3 are shown in table 1.

<table>
<thead>
<tr>
<th>m_M/m_{max}</th>
<th>k</th>
<th>Q</th>
<th>m_3 [mm]</th>
<th>m_2 [mm]</th>
<th>m_1 [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>2</td>
<td>1.15</td>
<td>34.9</td>
<td>17.5</td>
<td>8.7</td>
</tr>
<tr>
<td>20%</td>
<td>1.5</td>
<td>1.28</td>
<td>31.2</td>
<td>20.8</td>
<td>13.9</td>
</tr>
</tbody>
</table>

The results of post processing with GNSS SOLUTIONS software and the values of transcalculated coordinates in the national system using TransDat 4.04 are shown in Table 2.
Table 2

<table>
<thead>
<tr>
<th>Old landmark</th>
<th>Point</th>
<th>B</th>
<th>L</th>
<th>H</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>501</td>
<td>46°44'14.88796&quot;N</td>
<td>23°54'01.02622&quot;E</td>
<td>475.759</td>
<td>582571.809</td>
<td>416090.801</td>
<td>435.573</td>
</tr>
<tr>
<td></td>
<td>502</td>
<td>46°44'14.88802&quot;N</td>
<td>23°54'01.02629&quot;E</td>
<td>475.766</td>
<td>582571.811</td>
<td>416090.802</td>
<td>435.58</td>
</tr>
<tr>
<td>18</td>
<td>503</td>
<td>46°43'10.98822&quot;N</td>
<td>23°55'27.89919&quot;E</td>
<td>543.169</td>
<td>580573.779</td>
<td>417907.853</td>
<td>503.017</td>
</tr>
<tr>
<td></td>
<td>504</td>
<td>46°43'10.98823&quot;N</td>
<td>23°55'27.89933&quot;E</td>
<td>543.182</td>
<td>580573.779</td>
<td>417907.856</td>
<td>503.03</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Vector</th>
<th>DX</th>
<th>DY</th>
<th>DZ</th>
<th>Length [m]</th>
<th>Stationed time</th>
<th>Satellites</th>
<th>PDOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1-BAIA</td>
<td>-0.001</td>
<td>-0.008</td>
<td>-0.012</td>
<td>104916.173</td>
<td>3h51'30&quot;</td>
<td>8</td>
<td>1.7</td>
</tr>
<tr>
<td>S1-DEVA</td>
<td>-0.084</td>
<td>0.018</td>
<td>-0.017</td>
<td>122052.934</td>
<td>6h03'30&quot;</td>
<td>9</td>
<td>1.7</td>
</tr>
<tr>
<td>S1-BACA</td>
<td>0.004</td>
<td>0.008</td>
<td>0.004</td>
<td>231376.326</td>
<td>3h51'30&quot;</td>
<td>8</td>
<td>1.7</td>
</tr>
<tr>
<td>S2-BAIA</td>
<td>0.015</td>
<td>0.004</td>
<td>0.009</td>
<td>107290.087</td>
<td>3h51'30&quot;</td>
<td>9</td>
<td>1.4</td>
</tr>
<tr>
<td>S2-DEVA</td>
<td>0.050</td>
<td>-0.003</td>
<td>0.044</td>
<td>121705.227</td>
<td>6h03'30&quot;</td>
<td>9</td>
<td>1.4</td>
</tr>
<tr>
<td>S2-BACA</td>
<td>0.024</td>
<td>0.029</td>
<td>0.023</td>
<td>229415.680</td>
<td>3h51'30&quot;</td>
<td>8</td>
<td>1.4</td>
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</table>

Researches made, after determining the positioning of the network points, was targeted the framing precision of points 501 and 503 (old landmarks), numbered S1 and S2.

The first case were processed points S1 and S2 using as reference points the stations BAIA, BACA, DEVA (ROMPOS-EUREF). In Table 3 are presented the accuracies of points determination, length, stationed time with the GPS, number of satellites and PDOP. These data were extracted from the process after processing.

Table 4

<table>
<thead>
<tr>
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<td>1.6</td>
</tr>
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<td>0.029</td>
<td>0.023</td>
<td>229415.680</td>
<td>3h51'30&quot;</td>
<td>9</td>
<td>1.4</td>
</tr>
</tbody>
</table>

In the second case were processed the points S1 and S2 using as reference points the permanent stations Cluj, Mure, Dej, with smaller vector length. The results are shown in Table 4.
M. Ortelecan, I. Pop, N. Pop, R. Joldis, T. Salagean
Issues regarding the geodetic support network restoration from Cojocna experimental didactic station perimeter, to USAMV Cluj-Napoca

Table 4

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<th>PDOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1-CLUJ</td>
<td>-0.015</td>
<td>-0.009</td>
<td>0.013</td>
<td>24082.204</td>
<td>3h46'10&quot;</td>
<td>11</td>
<td>1.3</td>
</tr>
<tr>
<td>S1-MURE</td>
<td>0.033</td>
<td>0.095</td>
<td>0.022</td>
<td>54768.762</td>
<td>3h49'00&quot;</td>
<td>11</td>
<td>1.3</td>
</tr>
<tr>
<td>S1-DEJ</td>
<td>0.058</td>
<td>-0.036</td>
<td>0.018</td>
<td>45356.206</td>
<td>8</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>S2-CLUJ</td>
<td>-0.018</td>
<td>-0.014</td>
<td>0.011</td>
<td>26168.747</td>
<td>10</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>S2-MURE</td>
<td>0.026</td>
<td>0.060</td>
<td>-0.082</td>
<td>52350.776</td>
<td>8</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>S2-DEJ</td>
<td>-0.047</td>
<td>-0.031</td>
<td>-0.055</td>
<td>47434.852</td>
<td>10</td>
<td>1.6</td>
<td></td>
</tr>
</tbody>
</table>

In case 3 was chosen only one reference station located at a small distance from points S1 and S2. Similar to the previous version, the results are noted in Table 5.

Table 5

<table>
<thead>
<tr>
<th>Vector</th>
<th>DX</th>
<th>DY</th>
<th>DZ</th>
<th>Length [m]</th>
<th>Stationed time</th>
<th>Satellites</th>
<th>PDOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1-CLUJ</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>24082.210</td>
<td>3h46'10&quot;</td>
<td>11</td>
<td>1.3</td>
</tr>
<tr>
<td>S2-CLUJ</td>
<td>-0.001</td>
<td>-0.002</td>
<td>-0.001</td>
<td>26168.755</td>
<td>3h49'00&quot;</td>
<td>11</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Comparing the coordinate values of points S1 and S2 when dealing with short vector (Cluj) and the ones obtained from three vectors were obtained the differences shown in Table 6.

Table 6

<table>
<thead>
<tr>
<th></th>
<th>DX</th>
<th>DY</th>
<th>DZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long vectors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>-0.188</td>
<td>0.004</td>
<td>-0.095</td>
</tr>
<tr>
<td>S2</td>
<td>-0.164</td>
<td>0.019</td>
<td>-0.042</td>
</tr>
<tr>
<td>Short vectors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>-0.039</td>
<td>-0.029</td>
<td>0.003</td>
</tr>
<tr>
<td>S2</td>
<td>-0.039</td>
<td>-0.033</td>
<td>0.002</td>
</tr>
</tbody>
</table>

5. Conclusions

- The coordinate values of points S1 and S2 obtained by post-processing just a short vector (Cluj) are the most accurate.
- The difference between the coordinates of points S1 and S2 obtained by one vector compared to the ones obtained from three vectors is smaller for the short vectors.
- The difference between the known coordinates of the points S1 and S2 from the V order triangulation network and the ones determined by Global Positioning are obtained due to poor determination accuracy of V order points from the old triangulation network.
- We recommend using the nearest permanent stations to reduce the stationed time, so we increases the accuracy of determining the points that positively affects productivity and reduce costs.
- It is recommended the use of dual frequency receivers to eliminate some of the errors of ionosphere.
8. Bibliography