VELOCITY COMPONENT ANALYSIS ON A GPS NETWORK

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***Abstract**: For the error analysis of the GPS velocity component we have to consider many factors inclusive to determine the stochastic model. Trough this article we propose to do analyses of the influence of different types of noise: white noise, flicker noise and random walk and see how much we can eliminate the errors by a better statistical modeling if we have enough information to eliminate the uncertainties. Also the atmospheric delay and signal scattering are unwanted sources of noise.*

We used a spectral analysis to infer characteristics and density great enough to see the behavior of the time series.

The GNSS receiver extract the velocity information from the Doppler measurements that, in turn, are directly estimated from the Phase Lock Loop (PLL) output or obtained by differentiating carrier phase observations. The variance and bias of Doppler estimates are related to the carrier-to-noise density ratio, the user dynamics and the PLL parameters.

***Keywords:** error analysis, velocity component, white noise, flicker noise, random walk*

1. Introduction

High accuracy velocity determination is important for many field of positioning. In order to improve the accuracy of GPS velocity determination, researchers have made some investigations on GPS velocity determination [10].

GPS receivers extract velocity information from the Doppler measurements that are usually estimated in two different ways. The first option is the use of raw Doppler measurements. [8],[9] that are the direct output of the Phase Lock Loop (PLL) filter. The second method uses time-differenced GNSS carrier phase measurements.

For many application of Global Positioning System for a better understanding and more precise results we need to have good knowledge of the time series error spectrum.

The precision of the estimates is often assessed by their repeatability defined by the weighted mean square – RMS – scatter of individual coordinates components – north, east and vertical – about a linear trend. Horizontal station velocities are then determined either by linear regression of individual coordinates components or by simultaneous estimation of position and velocities. In estimating velocities, several assumptions are made which are predicted by the non continuous nature of the observation [11].

Determining the Uncertainties of GPS parameter estimates need's rigorous estimate of uncertainties that requires full knowledge of the error spectrum, both temporal and spatial correlations which is never possible. Full knowledge of the errors would require knowing the magnitude and correlations of every error at every epoch and place. Also sufficient approximations are often available by examining time series phase and/or position, and reweighting data. The implication of this statement it can be understand that we shouldn't try to be too rigorous; rather we need to understand and account for the dominant errors at the frequencies we are most interested in.

The noise contained within GPS station coordinate time series it is best fit by using a linear trend to the coordinate's time series and then model the noise properties of the residuals. It has been done different studies that shown that spectrum of errors of a time series of GPS it is optimal characterized by a stochastic process following a power-law as:

$$S(f) \simeq \frac{1}{f^{\alpha}} \qquad \alpha \in [0,2] \qquad (1.1)$$

With:
- $S(f)$the power spectrum
- α spectral index.

Following this model, researchers demonstrated that the noise is colored with mainly three components: white noise, flicker noise and random walk [4]. White noise is independent of frequency, and is generally associated with hardware noise or measurement errors. Clearly, white noise contains little or no geophysical information. However, it is useful to have a good knowledge of the white noise statistics to enable efficient filtering. For example, GPS time series can be filtered using a Kalman filter that requires a priori knowledge of the noise statistics [2]. To smooth the GPS time series it can be used a complex noise model. A power law noise model means that $S(f)$ in not flat but is governed by long range dependencies. If the probability density function of the noise is Gaussian or has a different density function with a finite value of variance, its fractal properties can be described by the Hurst parameter (H) [5].

Different number of assumptions is used in all statistical methods. This assumption generally aimed at the "flexibility "of the model from computation and theoretically point of view. We can understand that the model is a simplification of reality and is validity it best case the model is a "very good "approximation [6].

If we have H<0.5 the process behaves as a Gaussian variable, otherwise the process exhibits long-range dependence; while the case H=0.5 corresponds with white noise. H is connected directly with α by the relation:

$$\alpha = 2H - 1, \qquad \alpha \leq 2 \qquad (1.2)$$

Following the definition flicker noise corresponds to $\alpha = 1$ or H=1, the random walk to $\alpha = 2$ or H=3/2 and with noise is related to $\alpha = 0$ (H=0.5). So, we can state that random walk and the flicker noise are classified as long-term dependency phenomena.

2. Data analysis strategy

In general, all nonmodeled physical phenomena or neglected correlated noise content generate processes that affect velocity uncertainty estimation. Noise sources result from mismodeling of orbits [1], atmospheric effects – tropospheric delay - correlation through estimated parameters within the GPS data processing, and station-dependent effects like monument instability or near-field multipath [3].

When rates are estimated without any a priori data covariance information, the noise content is usually assumed to be uncorrelated, thus simplifying the covariance matrix into a diagonal matrix. Assuming that the sampling rate (ΔT) is constant and the number of points N is large, then the trend variance is approximated by:

$$\sigma_r^2 \approx \frac{12 * a^2}{T * T^2} = \frac{12 * a^2}{\Delta T^{2} * (N^3 - N)} \qquad (2.1)$$

where a represents the uncorrelated white noise amplitude and T is the time series length.

Assuming that the covariance matrix reflects time-dependent positions, for example by supposing that station monuments move following a random walk process, then the formal uncertainty of the estimated velocities is approximated by:

$$\sigma_r^2 \approx \frac{b^2}{T} = \frac{b^2}{\Delta T * (N-1)} \qquad (2.2)$$

where b represents the random walk noise amplitude. Equation 2.2 shows that in the presence of heavily correlated time series, velocity uncertainties are significantly augmented with respect to those from uncorrelated time series. In this case, the addition of more (correlated) positions by extending the observation span barely reduces the formal rate uncertainty.

Moreover, changing the sampling interval, while keeping the observation span constant, does not affect the estimated formal uncertainties at all. For all these global noise analyses with nonreprocessed data, a combination of white and flicker noise is commonly used to describe the stochastic properties of the position time series [7].

The impact white noise on the velocity uncertainties can be computed with:

$$(\sigma_r^2)_w = \frac{12 * \sigma_W^2}{\Delta t^2 m(m-1)(m+1)} \cong \frac{12 * \sigma_W^2}{mT^2} \qquad (2.3)$$

For flicker noise, the velocity uncertainty can be computed with:

$$(\sigma_r^2)_f = \frac{1.78 * \sigma_f^2 \Delta t^{0.22}}{T^2} \cong \frac{9 * \sigma_f^2}{16T^2} \qquad (2.4)$$

For random walk noise, the velocity uncertainty can be computed with:

$$(\sigma_r^2)_{rw} = \frac{\sigma_{rw}^2}{\Delta t(m-1)} \cong \frac{\sigma_{rw}^2}{T} \qquad (2.5)$$

Where:

$(\sigma_r^2)_w$ - is the velocity uncertainty that white noise caused,
$(\sigma_r^2)_f$ - is the velocity uncertainty that flicker noise caused,
$(\sigma_r^2)_{rw}$ - is the velocity uncertainty that random walk caused,
T – is the total observation span in years,
Δt - is sample interval,
m – is the observation number,
σ_w - is white noise amplitude
σ_f - is flicker noise amplitude
σ_{rw} - is random walk amplitude

Equation (2.3), (2.4), (2.5) shows clearly that different noises have different impact on velocity estimates, and so it is necessary to estimate different noise components exactly.

3. Processing and Results

The experiment was conducted it in Tileagd area between Oradea and Alesd city. We have used 6 GPS receiver Trimble R4 and Trimble R8 model and it was determined 20 points. The experiment was conducted for a period of 11 months. The necessary time for processing GPS baseline increased exponentially with the number of station.

The elevation cut-off angle is set to $\mathbf{10^0}$, for avoiding mismodelling of low elevation troposphere and phase center variations – PCV – of relative to absolute antenna calibration. Logging interval was set to 5 seconds. Charier phase was weighted by an elevation-dependent model for each station.

A "typically good" antenna environment for the phase noise should look like the pattern describe in fig.1.

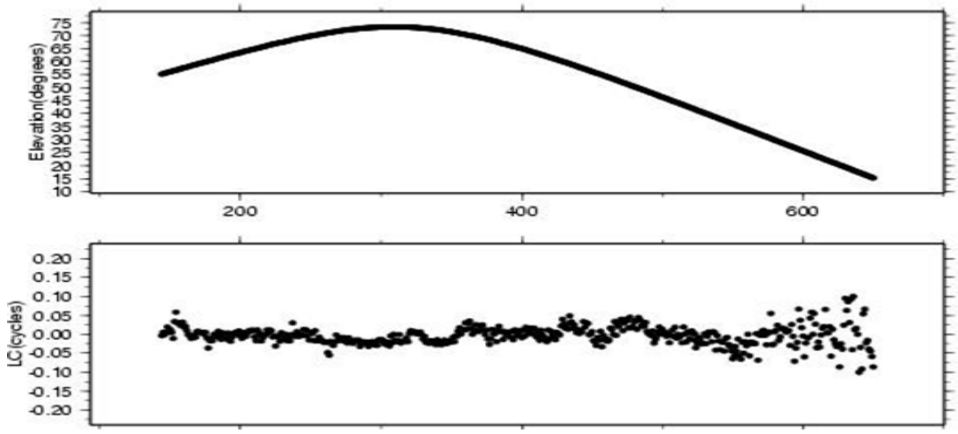

Fig. 1 – elevation angle and phase residuals for a single satellite

In the figure is presented LC phase residuals after removing clock drift and the best fit geometric model and clock. It is presented also the elevation angle of the observation. The overall rms is 6 mm (0.03 cycles), but the scatter increases from 2 mm at the zenith to 13 mm at low elevation, due both to increased multipath and atmospheric effects. At high elevations the dominant errors have correlation times of about 20 minutes.

Code bias corrections are applied for the whole period using monthly tables from the Astronomical Institute of University of Bern. In this case, they are fixed using the Melbourne-Wubbena wide-lane to resolve L1–L2 cycles and then the estimation it is used to resolve L1 and L2 cycles.

The figure 2 is the time series analysis of daily observation.

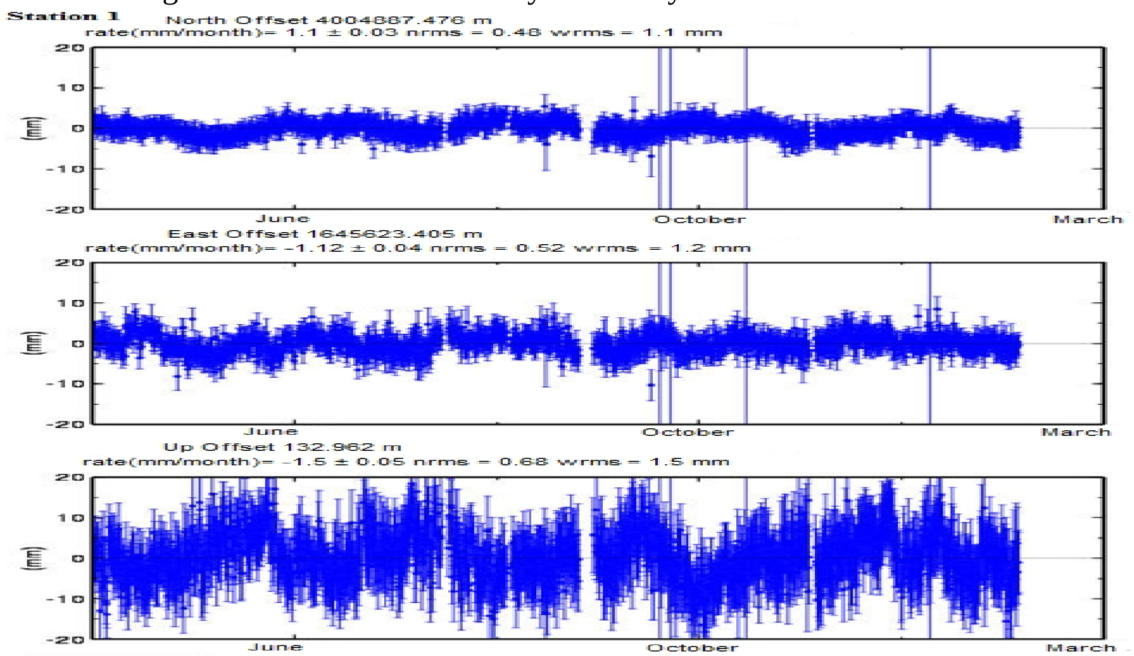

Fig.2 Characterizing the Noise in Daily Position Estimates

It can be observe that the wrms is approximately 1.2 mm horizontal and 1.5 mm vertical, and the normalized rms is approximately 0.5. Under a white noise assumption the formal error after scaling by the nrms is 0.02mm/month horizontal, 0.1mm/month vertical.

But the correlation time for independent observations is ~100 days, so if we again apply our rule-of-thumb, and take only every 100th point as independent, we would scale the velocity uncertainties by a factor of 10, to get 0.2 mm/yr horizontal and 0.1 mm/yr vertical uncertainties for the velocity estimates.

We will analyze three ways for correlation time that are considerate rigours.

The first is the spectral analysis of the time to estimate an error model that it is presented in figure 3.

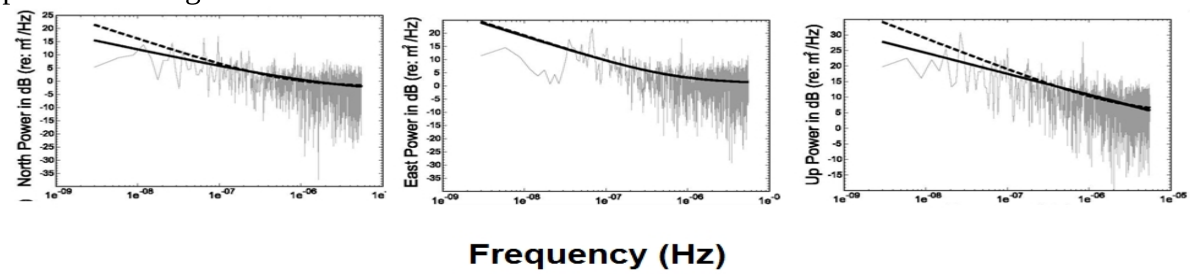

Fig.3 The spectral analysis of the time to estimate an error model

Where:

- Solid line is the mean,
- Dashed line is Maximum-Likelihood Estimation

The high-frequency (short-period) noise on the right-hand side has nearly constant amplitude for periods up to about approximately 4 days implying that it can be characterized by a white-noise error model. The most important feature for the purpose of getting realistic velocity estimates is the increase in amplitude for periods longer than 4 days. This increase implies temporally correlated noise. If we can represent the spectrum as a power law then a slope of -2 implies that the noise is a random walk. The slopes from GPS time series spectra at periods greater than 4 days are typically between -0.5 and -1.5, leading analysis to conclude that the noise is most like “flicker” noise, which has a slope of -1. Note that for periods greater than 100 days, the spectral amplitudes are quite variable, reflecting our inability to estimate them reliably a small number of samples. This is critical since long-period noise is most correlated with an estimate of the slope of the time series: if there is an anomalously large long-period noise signal, proper characterization of the high-frequency noise in the time series will not be very useful for getting a reliable estimate of the velocity uncertainty.

By using this method we are confronting the next two problems: it is computationally intensive and the second one it is that no model captures reliably the lowest-frequency part of the spectrum.

The second method for obtaining realistic uncertainties is the assumption where we consider that the amplitude of the long period source of noise is proportional to the white noise. If we are using time series white noise can be easily computed but the flicker noise it is a little harder to compute.

In figure 4 it is presented the results for north, east and vertical with white and flicker noise. It can be seen that there is a correlation, and a regression line fit to the points we will obtain about 50% of the appropriate flicker noise component to apply for a given wrms.

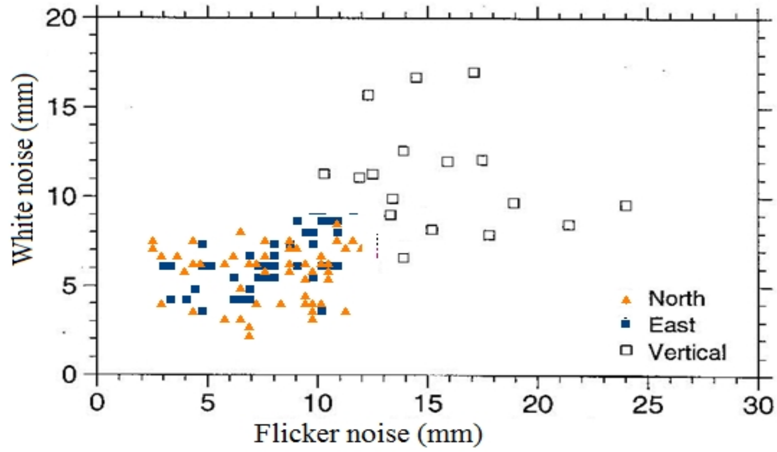

Fig.4 Noise statistics to predict flicker noise

The third method is the “realistic sigma” algorithm for velocity uncertainties that it is used in the experiment. It is a first order Gauss-Markov process. It is assuming that the noise can be represented by a Gauss-Markov process. A Gaussian process it is a random process that can be describe by the mean and variance. With the help of the central limit theory we can hypothesize that the ensemble of errors is has a Gaussian distribution. However no single instantaneous source of error is necessary Gaussian. These assumptions can be tested by the distribution of velocity residuals from an appropriately modeled GPS solution. For a better understanding of the realistic sigma algorithm – the effect of averaging on time-series noise we present the figure 5.

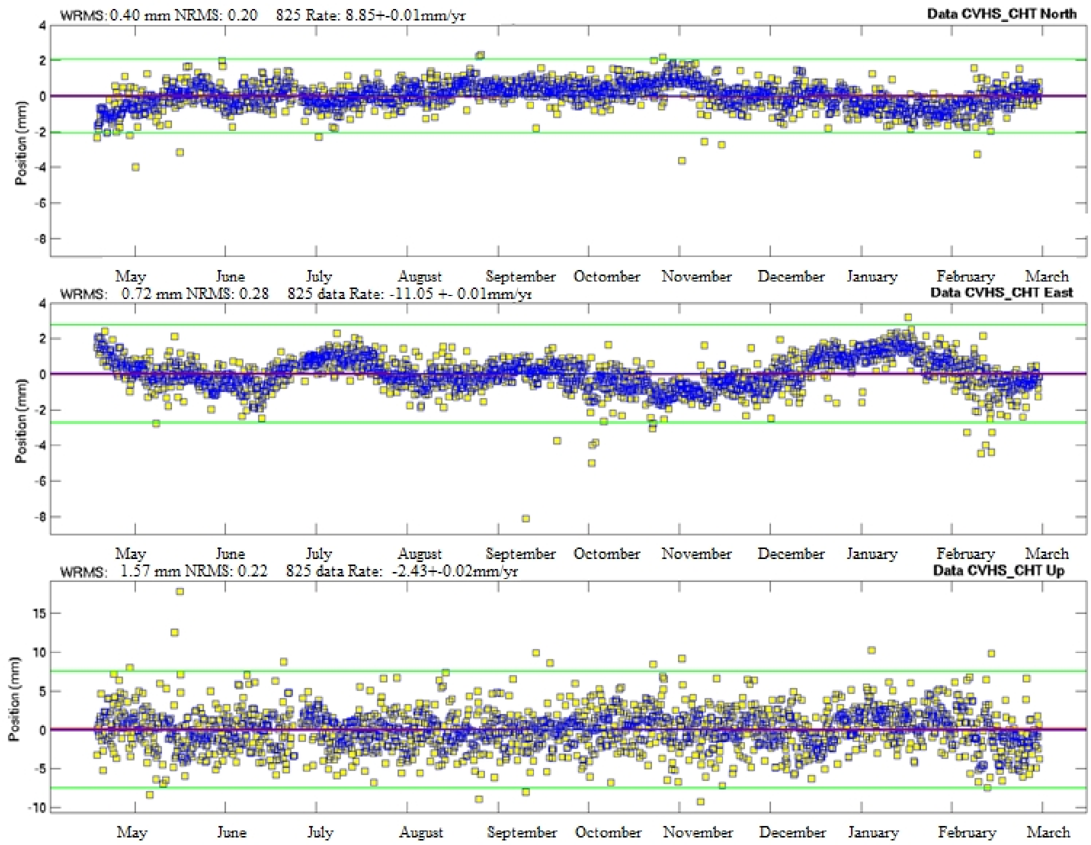

Fig.5 Understanding realistic sigma

Where:

- yellow squares are daily values, with nrms of approximate 0.3,
- blue points are 4-day averages. It can be seen the reducing of the short-term noise considerably.

Next we will do a time series analysis with one day and 100 days averaging – figure 6.

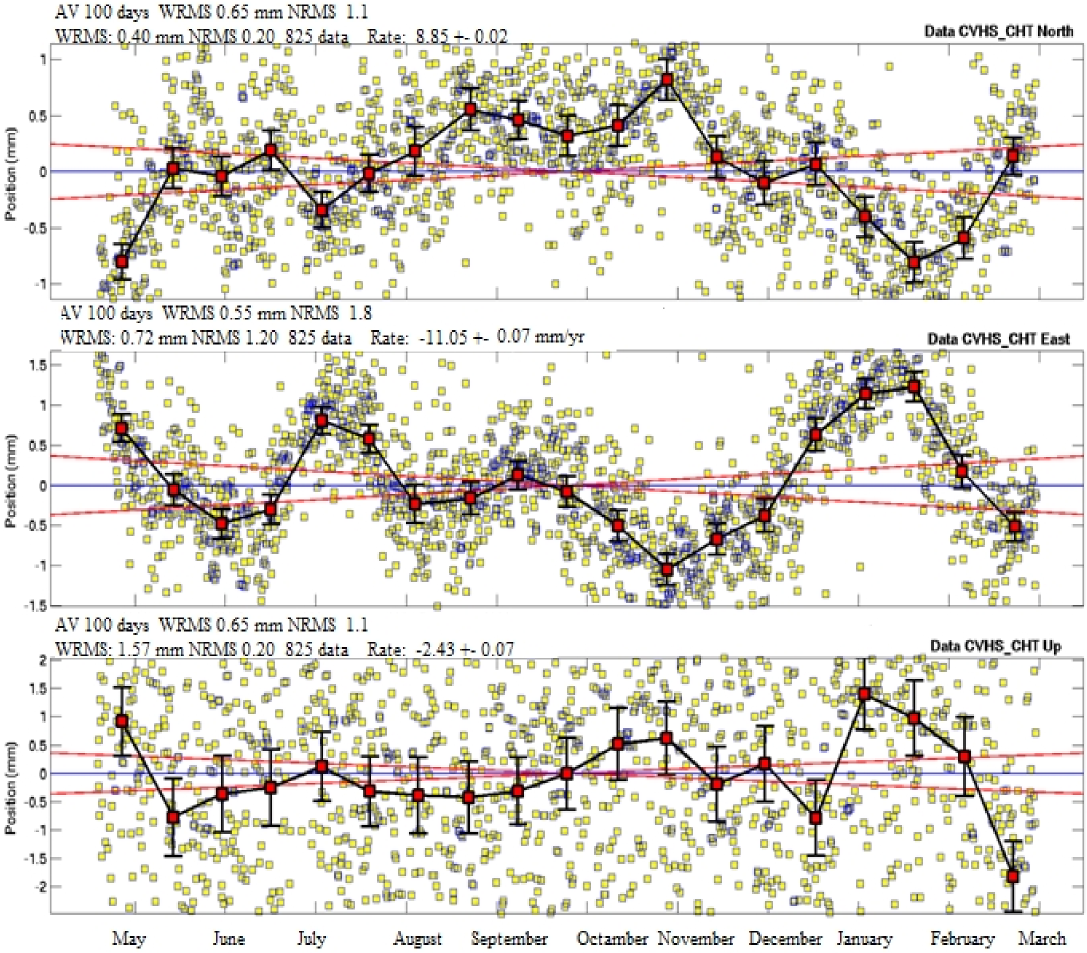

Fig.6 Realistic-sigma results

In the figure we can see the realistic sigma, so the red lines with symbols show the range of rates allowed by the more realistic sigma's estimated with the algorithm. The straight red lines show the 68% probability bounds of the velocity based on the results of applying the algorithm.

4. Conclusion

In this paper, we mainly focused on the scientific aspects of the GPS velocity determination and the influence of white noise, flicker noise and random walk with different computational algorithm. We have to understand that this is not the only influences for determining the velocity because we have to pay attention to residuals atmospheric like ionospheric and tropospheric delays - precipitable water vapor, dry and wet zenith delays – multipath.

We performed an analysis of the position time-series for all stations with the aim of providing station velocities. Also we conducted an analysis of the entire spectrum of noise – white, flicker and random walk – that it is crucial for determining the velocity component.

We used a first order Gauss-Markov process for obtaining the "realistic sigma" algorithm for velocity uncertainties to model the stochastic variation because it can be directly implemented in a Kalman filter

Also we can observe that correlated noise content is dependent on time series length but mainly on data time period.

5. References

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