THE STUDY OF OBLIQUE MERCATOR PROJECTION FOR LARGE SCALE MAPPING OF THE TERRITORY OF THE REPUBLIC OF MOLDOVA

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Abstract: *Sometimes the shape, direction and size of the country territory leads to use a single zone for certain map projection, where the central axe have to coincide with direction of the largest expanse of territory angularly to the axial meridian. Taking in account that the territory of Republic of Moldova is extended from South-East to North-West it was proposed to use as original meridian the central line along the country fitting of the 2,5 degree zone. This paper presents the results of study of Oblique Mercator map projection for large scale mapping of territory of our country in order to reduce deformations on the margin of the zone. Comparative study between TMM (Transverse Mercator for Moldova) map projection, currently used for large scale mapping, and Oblique Mercator map projection sowed the reduction of deformations on the margins of the zone from 16 to 8 cm/km.*

Keywords: *reference system, map projection, central meridian, linear deformations.*

1. Introduction

Starting from 1999 a new World Geodetic System WGS84 and Universal Transverse Mercator map projection UTM for navigation and small scale mapping from 1:25 000 to 1:500 000 was adopted by Government decision (Fig. 1).

Late in 2001 a new reference system MOLDREF99 based on European Terrestrial Reference System (ETRS89) and Transversal Mercator for Moldova (TMM) map projection was established to provide large scale mapping from 1:500 to 1:10 000 (Fig. 1).

However, the deformations on the margins of the 3.5 degree zone are not fitting requirements for 1:500 mapping. In this case for marginal zones a new scale coefficient has to be used in order to reduce deformations. This circumstance complicates many geodetic applications: geodetic works, cadastral surveying, GIS, mapping, etc.

In order to provide a unique map projection for large scale mapping, including 1:500 scale, for all territory of Republic of Moldova Oblique Mercator map projection was proposed. But taking in account that geodetic surveying and GIS software at that time was not supporting this projection the TMM map projection was adopted by Land Relation and Cadastre Agency.

Nowadays Oblique Mercator Projection is included in most geodetic and GIS applications and could be adopted for the territory of Republic of Moldova extended from South-East to North-West. The coordinate transformations of existing data sets from TMM to Oblique Mercator could be done trough geodetic coordinates on ellipsoid GRS80.

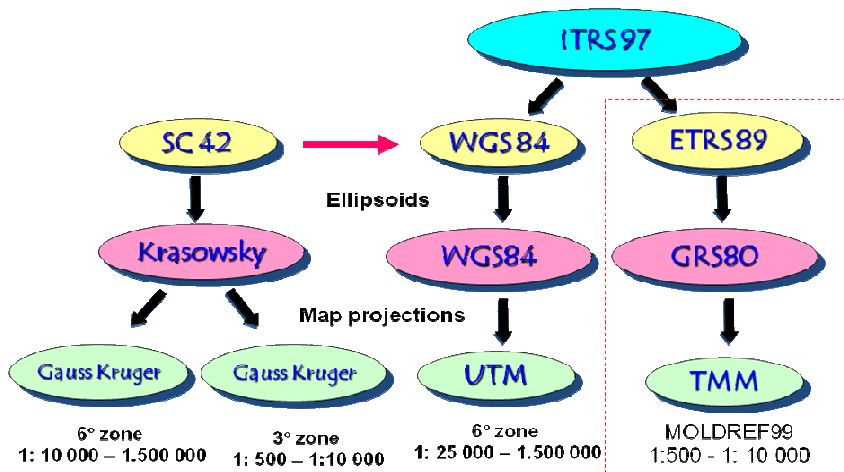

Fig. 1. Reference systems and map projection used in Republic of Moldova

2. Definition of Oblique Mercator map projection parameters

Taking in account that the territory of Republic of Moldova is extended from South-East to North-West it was proposed to use as initial line central to the map area of given azimuth α_c passing through a defined centre of the projection with latitude ϕ_c and longitude λ_c.

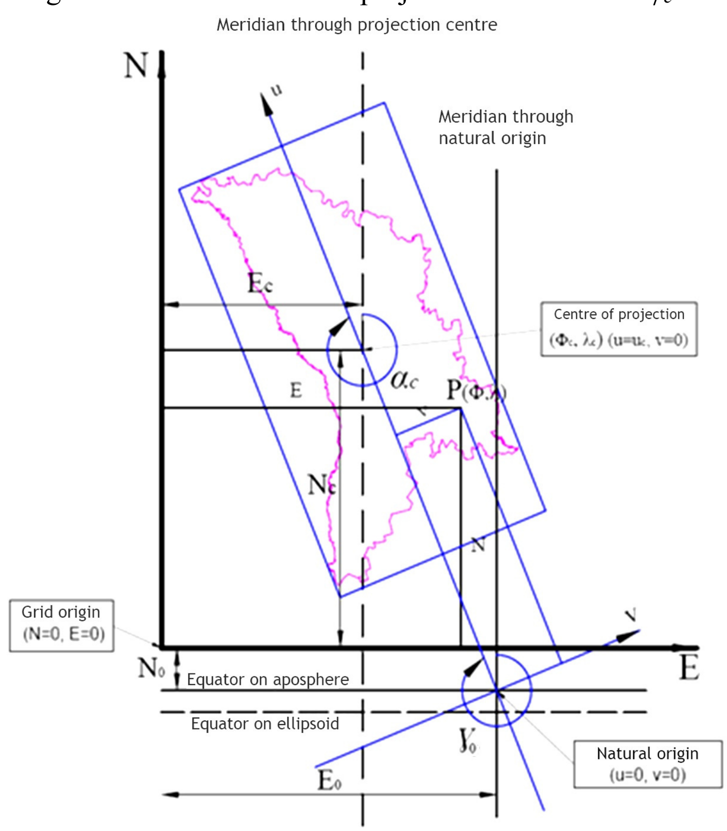

Fig. 2. Oblique Mercator map projection design

The point where the projection of these line cuts the equator on the aposphere is the origin of the *u, v* coordinate system. The axis *u* is parallel to the centre line and the *v* axis is perpendicular to this line. In applying the Hotine formulae modified by Snyder, the first set of coordinates computed are referred to the *u*, v coordinate axes defined with respect to the azimuth of the central line. These coordinates are then rectified to the usual Easting and Northing by applying an orthogonal transformation (Fig. 2).

To ensure that all coordinates in the map area have positive grid values, false coordinates are applied. These may be given values $E(y)_c$, $N(x)_c$ if applied at the Oblique Mercator map projection centre or be applied as false easting $E(y)_0$ and false northing $N(x)_0$ at the natural origin Hotine Oblique Mercator map projection (Fig,1).

The defining parameters for the Oblique Mercator projection are:

- latitude of centre of the projection $\varphi_c = 47°10'$;
- longitude of centre of the projection $\lambda_c = 28°30'$;
- azimuth (true) of the centre line $\alpha_c = 339°57'27.00''$;
- rectified bearing of the centre line $\gamma_c = 338°55'50.65''$;
- scale factor at the centre of the projection $k_c = 0{,}99998$;
- false Easting at the natural origin $E(y)_0 =$ 2 200 000 m;
- false Northing at the natural origin $N(x)_0 =$ -4 800 000 m.

From these defined parameters the constants for the map projection could be calculated using the following formulas:

$$B = (1 + e^2 cos4(\varphi_c) / (1 - e^2))^{0.5};$$
$$A = aB\, kc\, (1 - e^2)^{0.5} / (1 - e^2 sin2(\varphi_c));$$
$$t_0 = tan(\pi/4 - \varphi_c/2) / ((1 - e\, sin(\varphi_c)) / (1 + e\, sin(\varphi_c)))e/2;$$
$$D = B(1 - e^2)^{0.5} / (cos(\varphi_c)\, (1 - e^2 sin2(\varphi_c))^{0.5});$$
$$F = D + (D^2 - 1)^{0.5};$$
(1)
$$H = F\, t_0^B;$$
$$G = (F - 1/F)/2;$$
$$\gamma_0 = asin(sin(\alpha_c)/D);$$
$$\lambda_0 = \lambda_c - (asin(G\, tan(\gamma_0))) / B.$$

Then compute the u_c, v_c coordinates for the centre point ϕ_c, λ_c:

$$u_c = (A/B)\, atan((D^2 - 1)^{0.5} / cos(\alpha_c))\, SIGN(\varphi_c); \quad (2)$$
$$v_c = 0.$$

To compute $E(y), N(x)$ from a given ϕ, λ the following formulae are used:

$$t = tan(\pi/4 - \varphi/2) / ((1 - e\, sin(\varphi)) / (1 + e\, sin(\varphi)))e/2;$$
$$Q = H / t^B;$$
$$S = (Q - 1/Q)/2;$$
$$T = (Q + 1/Q)/2;$$
$$V = sin(B(\lambda - \lambda_0)); \quad (3)$$
$$U = (-V\, cos(\gamma_0) + S\, sin(\gamma_0)) / T;$$
$$v = A\, ln((1 - U)/(1 + U)) / 2B;$$
$$u' = A\, atan((S\, cos(\gamma_0) + V\, sin(\gamma_0)) / cos(B(\lambda - \lambda_0))) / B;$$
$$u = u' - u_c;$$
$$N(x) = u\, cos(\gamma_c) - v\, sin(\gamma_c) + x_c;$$

$E(y) = v\cos(\gamma_c) + u\sin(\gamma_c) + y_c.$

To compute ϕ, λ from a given $E(y), N(x)$ the following formulae are used:

$$
\begin{aligned}
u' &= (x - x_c))\cos(\gamma_c) + (y - y_c))\sin(\gamma_c) + u_c;\\
v' &= (y - y_c))\cos(\gamma_c) - (x - x_c))\sin(\gamma_c);\\
Q' &- e^{-(Bv'/A)}, \text{ were } e \text{ is a base of natural logarithms;}\\
S' &= (Q' - 1/Q')/2;\\
T' &= (Q' + 1/Q')/2;\\
V' &= \sin(B u'/A);\\
U' &= (V'\cos(\gamma_c) + S'\sin(\gamma_c))/T';\\
t' &= (H/((1 + U')/(1 - U'))^{0.5})^{1/B}; \qquad (4)\\
c &= \pi/2 - 2\,\mathrm{atan}(t');\\
\varphi &= c + \sin(2c).(e^2/2 + 5e^4/24 + e^6/12 + 13e^8/360) +\\
&\quad \sin(4c).(7e^4/48 + 29e^6/240 + 811e^8/11520) +\\
&\quad \sin(6c).(7e^6/120 + 81e^8/1120) +\\
&\quad \sin(8c).(4279e^8/161280);\\
\lambda &= \lambda_0 - \mathrm{atan}((S'\cos(\gamma_c) - V'\sin(\gamma_c))/\cos(B u'/A))/B.
\end{aligned}
$$

Using GRS80 ellipsoid and defined parameters for the Oblique Mercator projection the following constants were calculated according to formulae (1):

$B =$	1.000719681	$H =$	1.003109903
$A =$	6384183.617	$G =$	1.076966178
$t_0 =$	0.394163927	$\gamma_0 =$	-0.235364244
$D =$	1.469644905	$\lambda_0 =$	0.756712553
$F =$	2.546611083		

For example forward calculation for φ=48°23´58,8568" and λ=27°45´37,8705" gives the Easting and Northing values $E(y)$= 185345.256 m, $N(x)$= 417297.501 m.

3. Oblique Mercator map projection linear relative deformations analyses

For Oblique Mercator scale factor and linear relative deformations could be calculated using the following formulae (Table 1):

$$k = A\cos(Bu/A)(1 - e^2\sin^2\varphi)^{1/2} / \{a\cos\varphi\cos[B(\lambda - \lambda_0)]\} \qquad (5)$$

$$D = (k-1)10^5 \text{ cm/km} \qquad (6)$$

Table 1. Calculated Oblique Mercator linear relative deformations for territory of Moldova

φ/λ	26°40´	27°00´	27°30´	28°00´	28°30´	29°00´	29°30´	30°00´	30°10´
48°30´	5,08	1,44	-1,58	-1,68	1,17	6,99	15,81	27,64	32,25
48°00´	9,37	4,55	-0,21	-1,99	-0,76	3,49	10,79	21,15	25,3
47°30´	14,67	8,65	2,12	-1,37	-1,8	0,85	6,6	15,47	19,12
47°10´	18,77	11,92	4,2	-0,44	-2	-0,44	4,26	12,12	15,45
47°00´	20,98	13,73	5,4	0,17	-1,95	-0,94	3,23	10,58	13,74
46°30´	28,3	19,8	9,64	2,63	-1,21	-1,86	0,7	6,49	9,14
46°00´	36,63	26,85	14,83	6,02	0,43	-1,92	-1	3,21	5,34

45°30′	45,96	34,89	20,98	10,33	2,96	-1,12	-1,87	0,72	2,33

From Table 1 results that there are two lines with zero deformations situated on both sides of the central line. Between these two lines the values of deformations are negative form 0 to -2 cm/km. Outside of these lines values of deformations are positive from 0 to +8 cm/km. The zero deformation lines are about 40 km distance from the central line and the isolines are parallel (Fig. 3)

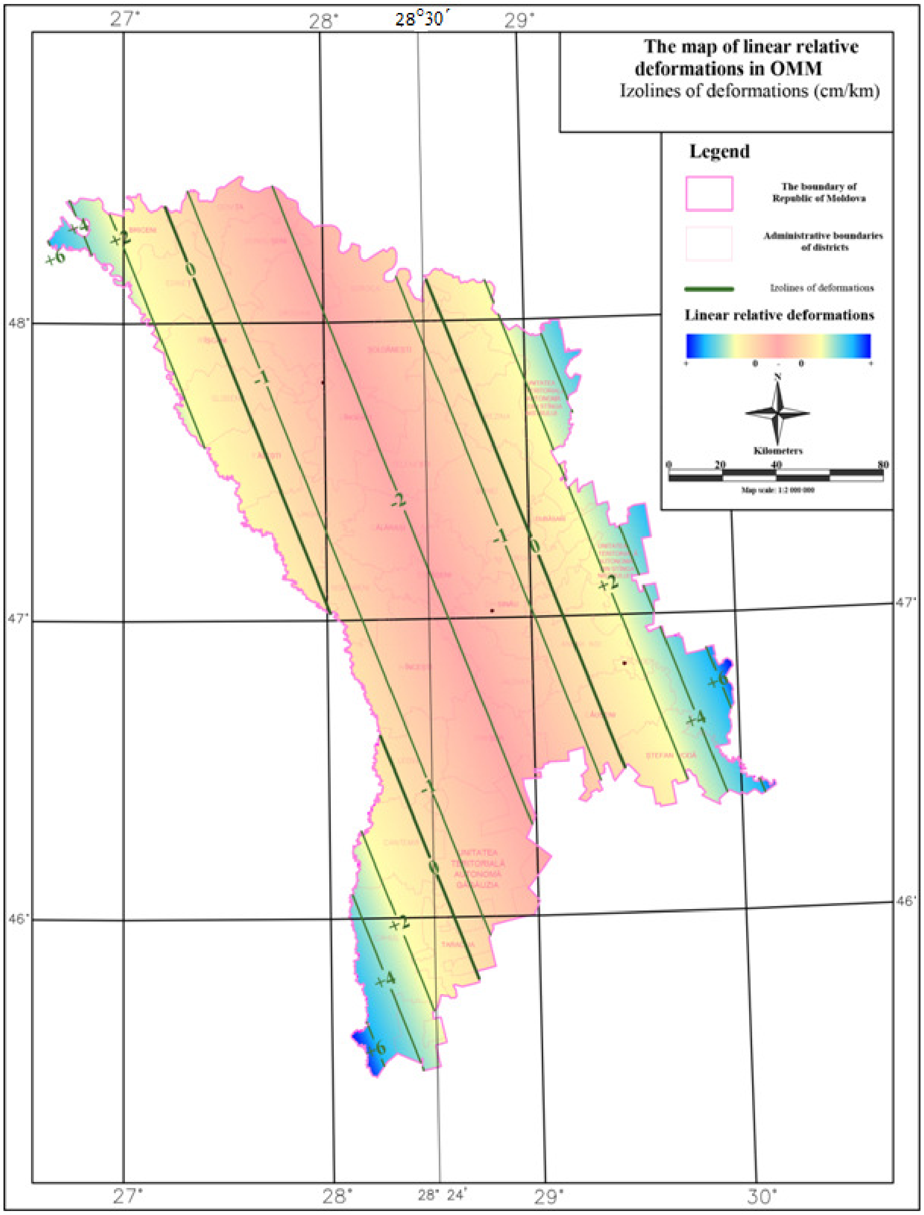

Fig. 3. Isolines of relative deformations for Oblique Mercator map projection.

The diagram of relative line deformations shows the relative linear deformations distance dependency (Fig. 4).

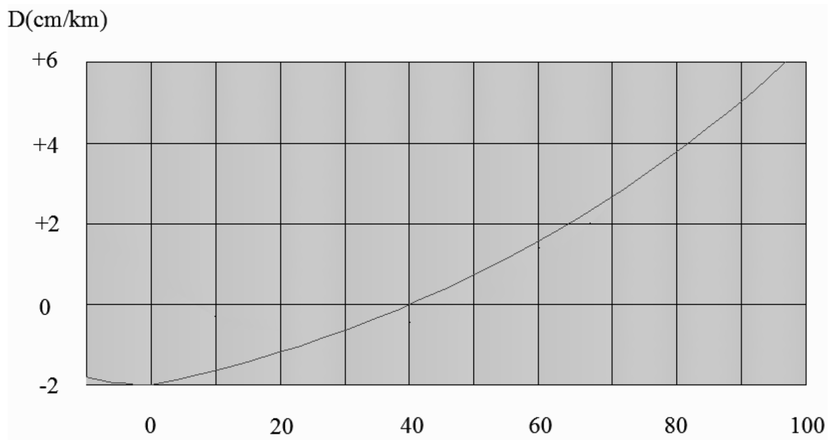

Fig. 4. Diagram of relative linear deformations for Oblique Mercator projection

4. Comparison with Transverse Mercator for Moldova map projection

The parameters for the Transverse Mercator projection for territory of Moldova are:
- longitude of centre line of the projection λc = 28°24′;
- scale factor at the centre of the projection k_c = 0,99994;
- false Easting at central meridian E_0 = 200 000 m;
- false Northing at the equator N_0 = -5 000 000 m.

Relative linear deformations are calculated using the following formulae (Fig. 5):

$$\mu = [1-\cos^2\phi \sin^2(\lambda-\lambda_0)]^{-1/2} \;; \qquad (7)$$

$$D = (\mu-1)10^5 \text{ cm/km}. \qquad (8)$$

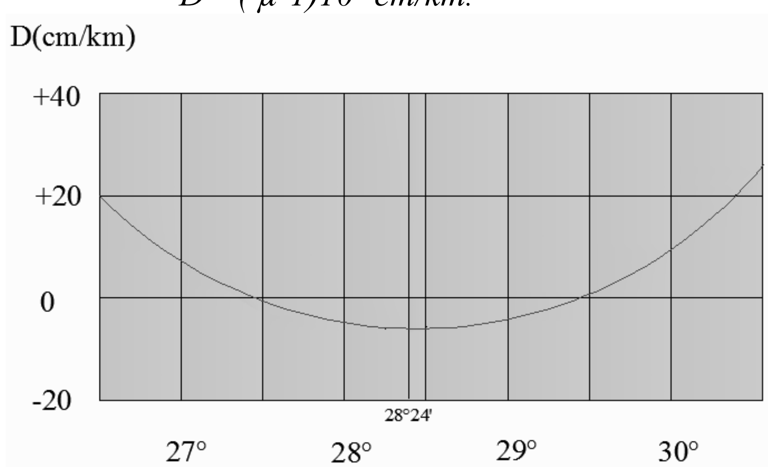

Fig. 5. Diagram of relative linear deformations for Transverse Mercator projection

From figure 5 results that there are two lines with zero deformations situated on both sides of the central line. Between these two lines the values of deformations are negative form 0 to -6 cm/km. Outside of these lines values of deformations are positive from 0 to +16 cm/km. The zero deformation lines are about 35 km distance from the central line and the isolines are parallel (Fig. 6).

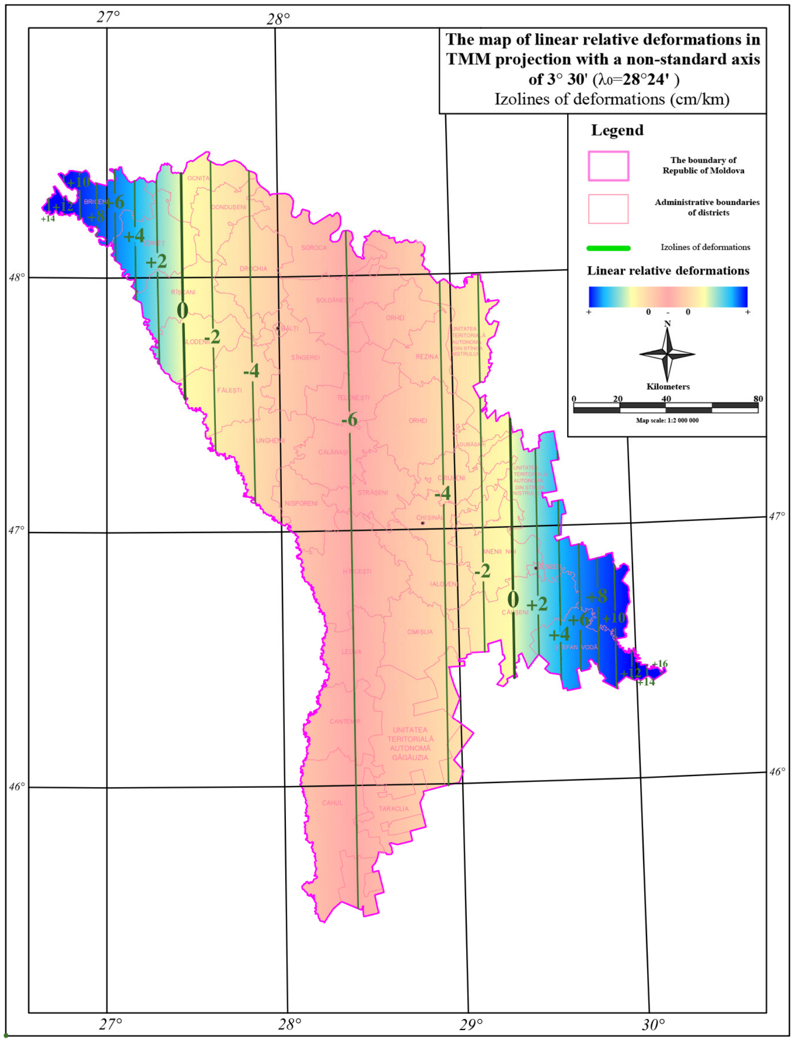

Fig. 6. Isolines of relative deformations for TMM map projection.

Comparison of TMM (Transverse Mercator for Moldova) map projection, currently used for large scale mapping, and Oblique Mercator map projection sowed the reduction of relative linear deformations on the margins of the zone from 16 to 8 cm/km.

5. Conclusions

Taking in account that the territory of Republic of Moldova is extended from South-East to North-West it was proposed to use as original meridian the central line along the country fitting of the 2,5 degree zone in order to provide a unique Oblique Mercator map projection for large scale mapping, including 1:500 scale.

Additional the parameters of Oblique Mercator map projection were defined and the constants were calculated for GRS80 ellipsoid. An example of forward calculation of Easting and Northing values from ellipsoidal longitudinal and latitudinal is given.

Comparative study between TMM map projection, currently used for large scale mapping and Oblique Mercator map projection sowed significant reduction of linear deformations on the margins of the zone from 16 to 8 cm/km.

The results of the study could be used for argumentation of new map projection for large scale mapping to be adopted by Land Relation and Cadastre Agency.

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