MODELLING THE NOISE IN GPS COORDINATE TIME SERIES

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***Abstract**: By investigating the velocity uncertainties in GPS coordinate time series we have to be aware of the fact that we also need good knowledge of the models that best describe the presents of the noise in the GPS time series coordinates. For a proper noise mode we have to take into account all the stochastic effects and to be able to classify the types of the noise source that will affect our results like: white noise, flicker noise, and random walk.*

The paper is presenting a case study in which we have used spectral analysis and Maximum Likelihood Estimation to best describe the presence of the noise. In the study we have estimated both annual and semiannual signals and also the GPS draconitic period harmonics which capture the unmodelled periodic effect.

***Keywords:** velocity uncertainties, noise analysis, spectral analysis, Maximum Likelihood*

1. Introduction

The determination of the high accuracy of velocity in very important in geodesy and near field of research, were the position offers lots of information regarding the motion of different objects – constructions and tectonic plates. From the beginning of GPS system researchers were interested in improving the accuracy of velocity determination (Wang, Zhang, & Huang, 2008).

To obtain more precise results by using Global Positioning System – GPS – we need to have good knowledge concerning the error spectrum from coordinates time series (Nistor & Buda, 2014b).

The estimation problem is influenced by the choice of weight matrix, which is able to describe the realistic influence the noise characteristics in which we need to obtain minimum variance estimators through the functional model (Amiri-Simkooei, Tiberius, & Teunissen, 2008). The indicator of the precision in the estimation process is defined by their repeatability which represents the weighted root mean square – WRMS- scatter of the individual coordinates – north, east and up – defined by a linear trend. The velocity is estimated by using linear regression applied to individual coordinates or simultaneous with the points coordinates. In velocity estimates (Zhang et al., 1997) stated that we have to use a few assumption which is then predicted by the non-continuous nature of the observations.

To be able to determine the uncertainties of GPS parameter estimates in a rigors way, it is necessary to have full knowledge of error spectrum – both temporal and spatial correlation. The problem is that in this case we need to know the magnitude of the correlation of every error at every epoch and place. By using the weight matrix we can reweight the data by examining the coordinate's time series, but this is very hard to achieve and we need only to understand and account the dominant error at the frequencies we are interested in.

The noise that is contained in the GPS coordinate time series is best identified by using a linear trend and then model the noise properties of the residuals. The spectrum of errors of a time series of GPS is defined optimal by stochastic process that follows a power-law:

$$S(f) \simeq 1/f^{\alpha} \qquad \alpha \in [0,2] \tag{1}$$

With:

S(f) the power spectrum and α the spectral index.

By taking into account this equation, researchers demonstrated that noise is colored mainly by this three components: white noise, flicker noise and random walk (Langbein & Johnson, 1997). The measurement errors or hardware noise is account as being white noise and is independent of frequencies. The most important feature of white noise is that it doesn't contain geophysical information but to be able to filter this type of noise we have to know the white noise statistics. The problem is that inside the GPS coordinates time series the white noise is not the only type of noise and we have to use more complex types of noise to proper estimate the velocity and their uncertainties, like power law noise model. We have to understand that in the case of power law when the index is fixed to integer value, α = -1 we are dealing with flicker noise, α = -2 the noise that best describe the data is random walk. Also in the case of power law we can compute the spectra index by using different methods, but the most recommended one is maximum likelihood method. By using the power law noise model the S(f) is not flat but is governed by long range dependencies (Nistor & Buda, 2014b). Different number of assumptions is used in all statistical methods. This assumption is generally aimed at the "flexibility "of the model from computation and from a theoretical point of view. We can understand that the model is a simplification of reality and is validity in the best case scenario is a "very good "approximation" (Nistor & Ionascu, 2013).

2. Additions

White noise

This is the case where $k = 0$. It can be seen that if $k = 0$, $\gamma_0 = 1, \gamma_n = 0\ (n > 0)$ and therefore $T = I$ and $J_0 = I$. That is, the covariance matrix is scalar and, since $\Lambda T^{-k/4} = 1$ is independent of time.

The transformation matrix is:

$$T = \begin{bmatrix} \gamma_0 & 0 & 0 & \dots & 0 \\ \gamma_1 & \gamma_0 & 0 & \dots & 0 \\ \gamma_2 & \gamma_1 & \gamma_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_n & \gamma_{n-1} & \gamma_{n-2} & \cdots & \gamma_0 \end{bmatrix} \tag{2}$$

Is one-sided power spectrum density is:

$$S(f) = 2\frac{1}{f_s} \tag{3}$$

Random walk noise

This is the case where $k = -2$. If $k = -2$ then $\gamma_n = 1$ for any n and so:

$$T = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ 1 & 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix} \tag{4}$$

The covariance matrix is therefore equal to:

$$J_{-2} = \begin{bmatrix} \Delta T_1 & \Delta T_1 & \Delta T_1 & \dots & \Delta T_1 \\ \Delta T_1 & \Delta T_2 & \Delta T_2 & \dots & \Delta T_2 \\ \Delta T_1 & \Delta T_2 & \Delta T_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta T_1 & \Delta T_2 & \Delta T_3 & \dots & \Delta T_n \end{bmatrix} \tag{5}$$

and is exactly the same as the matrices described in (Johnson & Wyatt, 1994; Mao, Harrison, & Dixon, 1999; Zhang et al., 1997).

Flicker noise

This is the case where $k = -1$. In this case the covariance matrix J_{-1} was approximated by (Zhang et al., 1997). The constants in this matrix were chosen so that the power spectrum of random-walk noise and flicker noise cross at a period of one year, when we are using a sampling interval of one day and equal amplitude ($b_{-1} = b_2$). But this is the earlier covariance matrix and the major difference is the scaling of the amplitude, because this is not exactly the same as that derived from the above transformation matrix. From the power-spectrum equations it can be estimated the scaling between the 'new' and 'old' amplitudes. This is:

$$P_{old} = \frac{b_{old}^2 f^{-1}}{2\pi^2} \tag{6}$$

$$P_{new} = \frac{b_{new}^2 f^{-1}}{\pi\sqrt{f_s x 24 x 60 x 60 x 365.25}} \tag{7}$$

Therefore

$$b_{new} = \frac{(f_s x 24 x 60 x 60 x 365.25)^{1/4}}{\sqrt{2\pi}} b_{old} \quad (8)$$

If it is used a sampling frequency of one per day (in Hz) then:

$$b_{new} = 1.7440\, b_{oid} \quad (9)$$

In any study if it is used the new matrix, the covariance matrix from (Zhang et al., 1997) must be scaled before being used for the determination of flicker noise.

3. PROCESSING AND RESULTS

The experiment was performed in Oradea city by using 4 GNSS receiver – Trimble R4. Daily observation are presented in fig. 1. The time is expressed in months. The starting year was 2009.

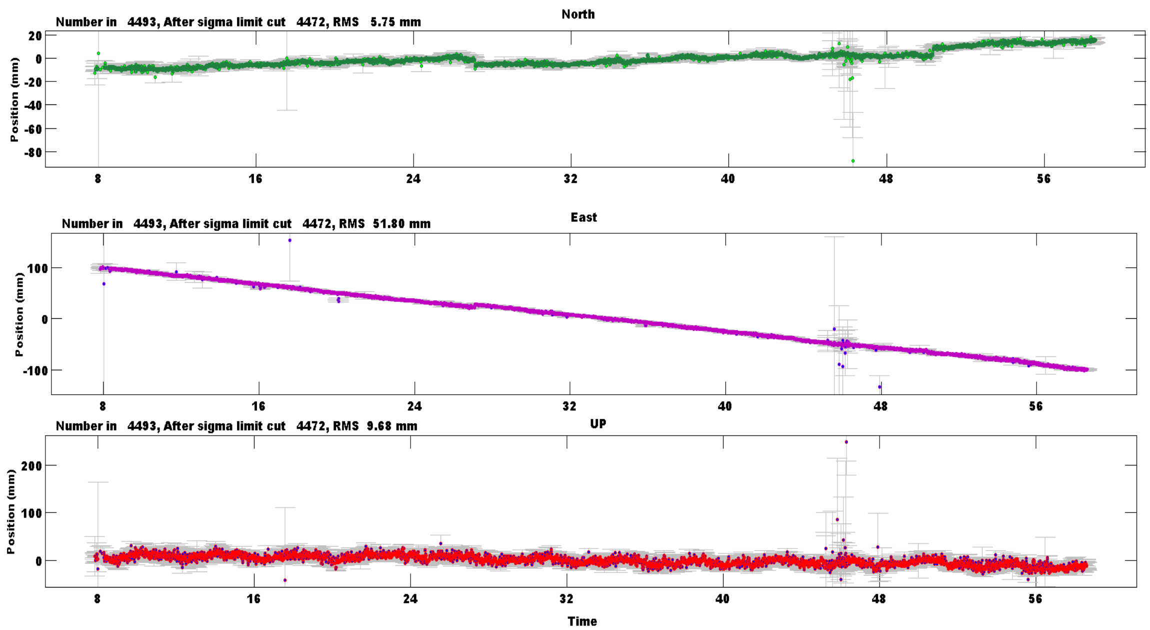

Fig. 1 Daily observation

From the plot we can be observe that the rms is 5.75 mm in the North part, in the East part it is 51.80 mm and 9.68 mm vertical.

To continue the analysis process we will deterend the daily observation and apply the Gauss-Markov noise mode and at this stage we didn't apply the seasonal variation. The results are presented in fig. 2.

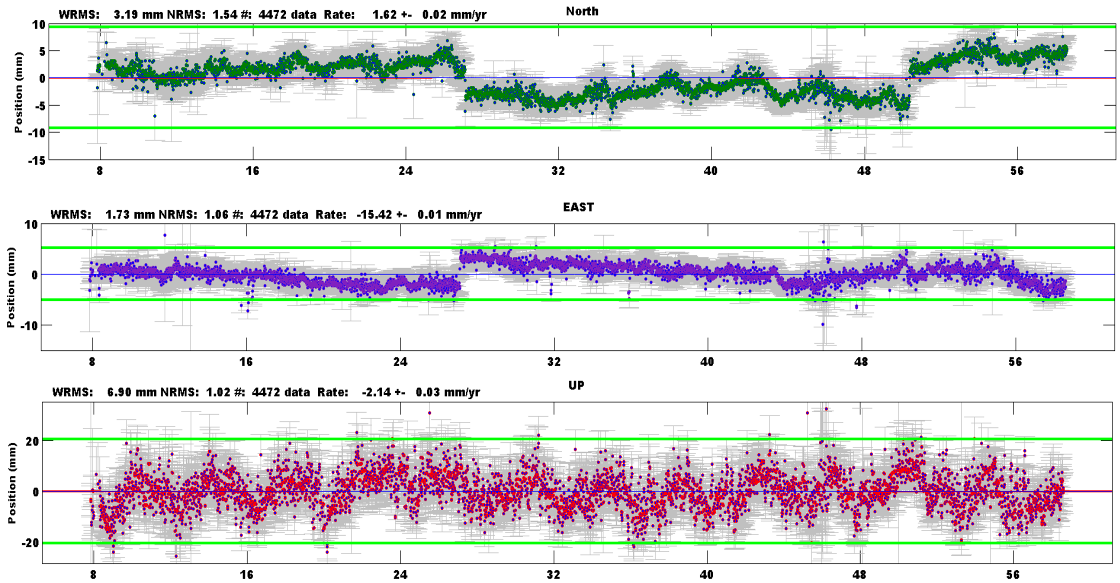

Fig. 2 Detrending daily observation without seasonal variation

We can observe from the plot that WRMS it around 2 mm in horizontal, with the normalized rms around 1mm. At this stage the noise model behaves very well. In the next phase of the experiment we have introduce in the detrending operation the influence of the seasonal variation. From the plot we can observe that the NRMS presents changes but also the uncertainty presents “interesting“ variation. The results are presented in fig .3.

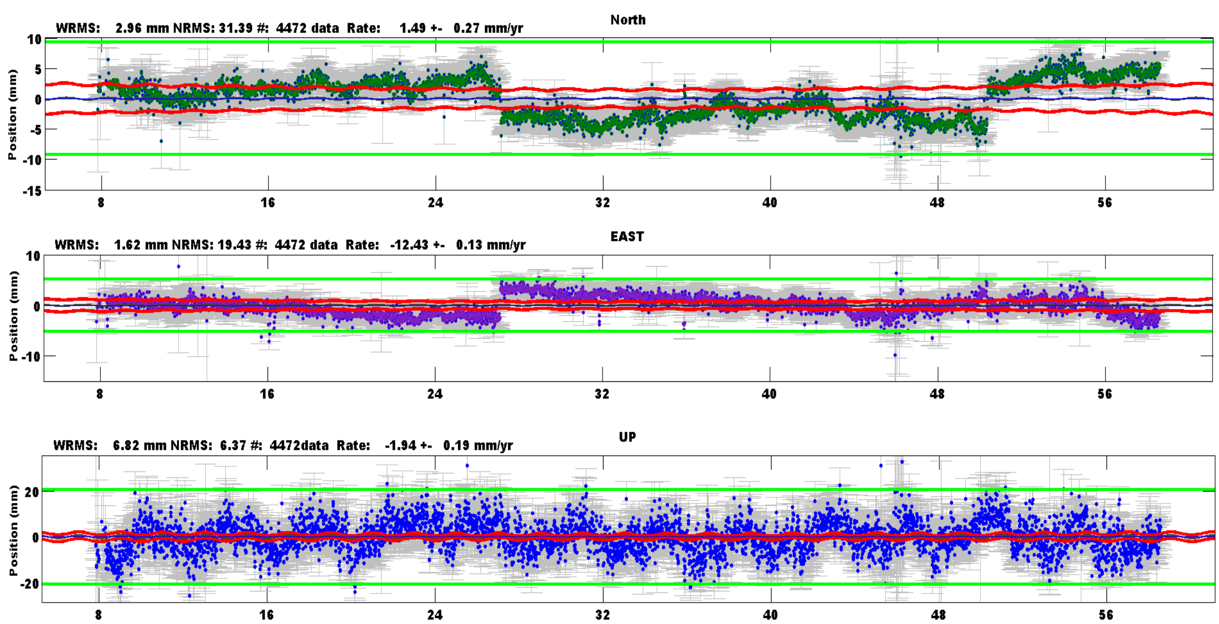

Fig. 3 Detrending daily observation including seasonal variation

The values that resulted by adding the seasonal variation are: the WRMS didn't have noticeable changes – in the North 2.96 mm, in the East 1.62 mm and in the Vertical 6.37 mm. The major difference was notice on NRMS where in the North was 31.39 mm, in the East 19.43 mm, and in Vertical 6.37 mm. The rate uncertainty presented the following values: 1.49 ± 0.27, -12.43 ± 0.13 and -1.94 ± 0.19 mm. The NRMS could be explained by the fact that at the middle of the experiment one of the point was destroyed and we tried to restore the point. To reevaluate the data it is recommended to use the log breaks to analyses the data at before and after the recomputation of the position of the point.

By using the spectral analysis we can identify the seasonal variation and the draconitic periods. The results are presented in fig. 4.

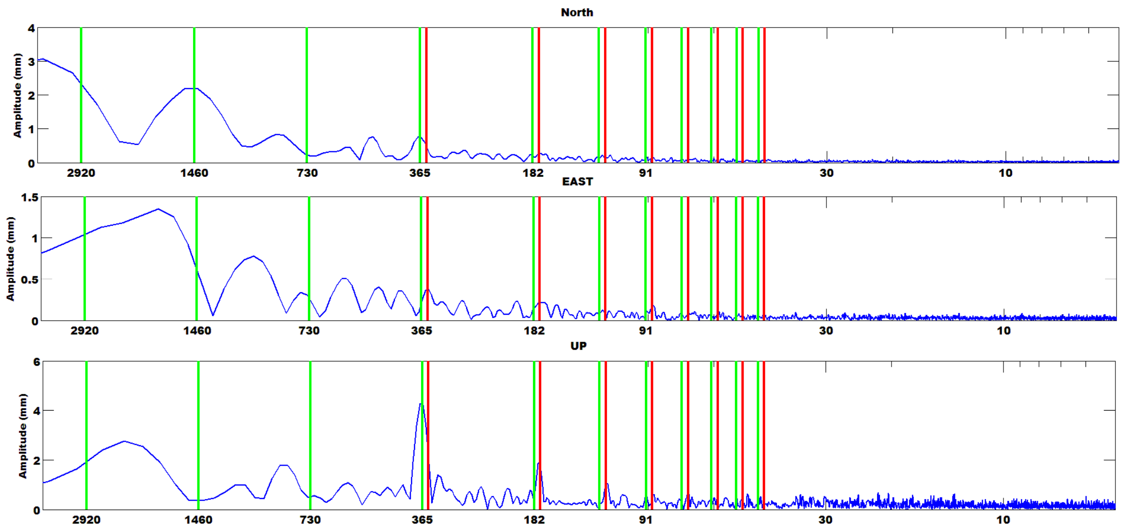

Fig. 4 Spectral analysis presenting the seasonal variationa and draconitic periods

The green line represents the annual variation and the reed lines represents the draconitic periods.

We have also plotted the power spectra density - the results are presented in fig. 6.

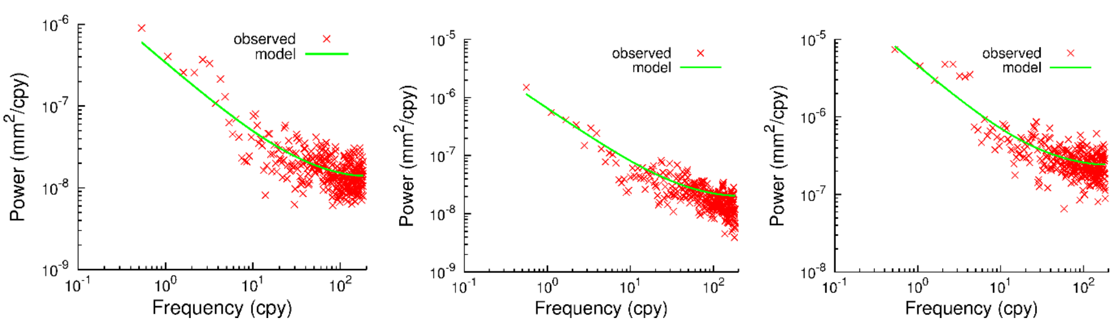

Fig. 5 Power spectra density

The data for time series analysis was obtain using double differentiate technique but we can also use data from PPP technique (Nistor & Buda, 2015) and VLBI data (Nistor & Buda, 2014a).

4. Conclusions

The main idea in this paper is that to be able to obtain realistic uncertainties we need not only to introduce into the computation part the seasonal variation but also to use proper method to estimate the noise. In time series analysis the colored noise has significant effect on the uncertainty of rate estimation

The recommended combination for estimate the noise and to obtain reliable rate uncertainties is the white noise plus flicker noise, because a pure white noise model underestimate total velocity error. Also we have to take into account the annual and semiannual seasonal variation and then to make an average for reducing the noise.

Other suggestion are increasing the sampling rate that it is able to reduce the presence of white and flicker noise - in the flicker noise there is not noticeable changes – but in the case of random walk the sampling has no effect.

The problem is that the model of the noise that has been chosen for estimating the uncertainties can bias the results.

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