Abstract: The article refers to the calculation of bottom-discharge conduits used in locally sourced dams. In such cases, the foundation terrain is usually elastic. The conduits themselves have either a circular or polygonal cross-section and are made up of reinforced concrete. The length of the conduit is split into sections that are joined together with sealant tape. The article showcases the calculation for the Ibaneasă dam bottom-discharge conduit using the finite element method. There are two scenarios being considered: one in which the conduit is split up into several shorter sections, and one in which only two sections are taken into account – one upstream and one downstream on the dam, with the two being joined in the dam’s axis. The bending momentum is smaller when the number of sections is greater (and their size is diminished).

Keywords: earth dam, bottom-discharge conduit, correlation bending moment-sectioning.

1. Introduction

The method of evacuating water in embankment dams through a bottom-discharge conduit of various cross sections is well known. However, this solution has a few big disadvantages that require great care and attention both during construction as well as later.

The conduit itself is a rigid concrete or steel structure that is usually set directly on soil if it is fit enough to be a sturdy foundation; otherwise, pillars are used. The earth filling that makes up the dam and the conduit interaction cause several difficult problems:

- overloading of the concrete structure caused by the filling
- uneven settlements throughout the dam, above and around the conduit
- uncontrolled infiltration caused by the uneven settling

In the case of steel or concrete structures traversing the dam, the stiffer foreign element phenomenon occurs [1]. These structures lay on firmer terrain so that eventual deformation that would appear during exploitation would not affect continuous functionality. This is why the load on the upper part of the conduit is greater than weight of the earth filling directly above it (due to friction forces acting vertically in planes tangent to the conduit) [6].

A certain overload phenomenon also appears in the clay adjacent to the structure which can not consolidate on the same level. This can lead to a detachment of the earth along the structure and an increase in permeability of the earth to either side of the conduit. For these reasons, the inclusion of massive concrete structures within the dam sealing ring should be avoided when possible. However, this solution is common place, especially in the case of lower height dams.

The vertical pressures on the conduit can be greater that the weight of the column of earth directly above it. The risk of detachment between the conduit and the filling around it
(which leads to infiltration from the lake) can be reduced by the use of adequate geometries for the concrete structure as well as the use of special constructive measures regarding the sealing ring.

Usually, the joints are sealed using rubber or plastic strips. If such systems are not in place, as it is in the case of work joints, high plasticity materials need to be used on the outside of the conduit to ensure water tightness.

2. Evaluating the load caused by earth filling

Further, Marston’s results regarding the load from embankment filling are showcased:(fig.1)

Total vertical load is [6]:

\[ P_v = C_v \gamma D_e^2, \]  

in which

\[ C_v = \frac{e^{2K\mu(H_e/D_e)}-1}{2K\mu} \]  sau \[ C_v = \frac{e^{2K\mu(H_e/D_e)}-1+\left(\frac{H}{D_e}-\frac{H_e}{D_e}\right)}{2K\mu}e^{2K\mu(H_e/D_e)} \]

in which

\[ K = \tan^2(45^\circ - \varphi/2) , \] \text{Rankine lateral earth pressure coefficient}
\[ \mu = \tan \varphi , \] \text{ground friction coefficient}
\[ H_e = \text{position of the plane of equal settlement} \]
Marston determined the existence of a horizontal plane above the pipe where the shearing forces are zero. This plane is called the \textit{plane of equal settlement}. Above this plane, the interior and exterior prisms of soil settle equally.

\[ D_e = \text{exterior diameter of the conduit} \]

Equation (1) works for $H_e > H$ (plane of equal settlement is imaginary), while equation (2) works for $H_e < H$.

The extra load (on top of the weight of the column of earth on top of the structure) depends on the friction forces that appear in the vertical planes tangent to the conduit. For example, for a clay damn with a filling thickness of 8m over a conduit 3.2m in width, the load given by the earth is (for each meter of conduit):

\[ P_v1 = \gamma H D_e = 19 \cdot 8 \cdot 3.2 = 486 \text{ kN/m} \]
\[ \gamma = 19 \text{ kN/m}^3; K \mu = 0.13 \] [4]
\[ H = 8 \text{m}; D_e = 1 \text{m} \]

According to Marston we have: (considering the plane of equal settlement on the surface $H = H_e$)

\[ C_e = \frac{e^{2.0.138/3.2} - 1}{2 \cdot 0.13} = 3.5 \]

\[ P_v2 = C_e \gamma D_e^2 = 3.5 \cdot 19 \cdot 3.2^2 = 680 \text{ kN/m} \]

According to [4] the working equation to determine vertical pressure is:

\[ P_v = C_e \gamma H D_e \]

in which

\[ C_e = \frac{e^{2K \mu H / D_e} - 1}{2K \mu H / D_e} \]

\[ C_e = \frac{e^{2.0.138 \mu H / D_e} - 1}{2 \cdot 0.13 \cdot 8 / 3.2} = 1.4 \]

therefore:

\[ P_v = 1.4 \cdot 19 \cdot 8 \cdot 3.2 = 680 \text{ kN/m} \]

We therefore have an extra 40% load on the foundation underneath the conduit and a diminished load on the areas adjacent to the structure thus preventing them from settling on the same level. When the conduit is set directly on the ground, the load on the conduit is greater than around it, while if pillars are used as a foundation solution, the situation is reversed, with the load being greater around and near the conduit than underneath it. This unevenness in load leads to detachment of the ground from the rigid structure and to an increase in permeability in the area. In many cases problems like this have led to dams giving in and collapsing.

3. Calculating the bottom discharge conduit at the Ibaneasa dam in Botosani County

Due to the complex nature of the relationship between the concrete structure and the filling, the only way to accurately model this phenomenon is with finite element analysis[3].
The bottom discharge conduit is made up of 5 reinforced concrete sections, each 9m long, with a final 11m section downstream. There are 2.5cm joints between sections that have been sealed (fig. 2).

The calculations will be done for one of the 9m segments. (fig. 3). This segment will be split into 6 finite elements of 1.5m each (fig. 4). The ground reaction will be estimated using the Winkler model. Each node will be thought of as a spring with its elasticity determined by:

\[ k_s = B \cdot l \cdot k \]

in which

- \( B = 3.2 \text{ m} \) is the width of the conduit
- \( l \) is the length of the finite element

![Fig. 2. Dam section through the bottom discharge](image)

The marginal nodes will have the same coefficient of subgrade reaction as the other ones according to [2] (Bowles 1995)

Coefficient of subgrade reaction according to Vesić apud Bowles [2]

\[
k = 0.65\sqrt{\frac{E_p \cdot B^4}{E_b \cdot I_b}} \cdot \frac{E_p}{B(1-\mu_p^2)}
\]  

(4)

Ground parameters are (silty clay):

- \( E_p = 35 \text{ MPa} \)
- \( \mu_p = 0.35 \)
- \( \gamma_p = 19 \text{ kN/m}^3 \)

The conduit parameters are:

- \( A = 5.36 \text{ m}^2 \)
- \( I_b = 6.67 \text{ m}^4 \)
- \( E_b = 26 \text{ GPa} \) (for C12/15)

\[
k = 0.65\sqrt{\frac{35 \cdot 3.2^4}{26000 \cdot 6.67 \cdot 3.2(1-0.35^2)}} = 5875 \text{ kN/m}^3
\]

\[ k_s = 3.2 \cdot 1.5 \cdot 5875 = 28.200 \text{ kN/m} \]

In Romanian standards, the coefficient of subgrade reaction (Winkler) is: [7]

\[
k = k_m \cdot \frac{E_p}{\alpha(1-\mu_p^2)}
\]

(5)
$k_m=0.338$ (from the table K1 depends on $\alpha$)

$\alpha = L/B = 9/3.2 = 2.8$

Therefore:

$$k = 0.338 \frac{35000}{2.8(1-0.35^2)} = 4814 \text{ kN/m}^3$$

The coefficient of subgrade reaction according to Bowles which simplifies the Vesić equation is:

$$k \approx \frac{E_p}{B(1-\mu_p^2)}$$

With the ground parameters we have $k=12464 \text{ kN/m}^3$

From (6) we have a coefficient of subgrade reaction over twice as big as in the first two equations (4) and (5). For further calculations we shall use the coefficient given by the Vesić equation.
Calculations done using Beam 2D[9]

The load on the conduit is made up of both its own weight as well as the weight of the ground above it. The gravitational load caused by the filling ($p=\gamma h$) will be increased by 40% (due to the rigid foreign element phenomenon previously described).

The conduit parameters are:

\[ A=5.36 \text{ m}^2; \quad I_b=6.67 \text{ m}^4; \quad E_b=26 \text{ GPa (for C12/15)} \]

The first calculation scheme is (fig. 5)

Conduit deformation (a maximum of 37 cm in node 1; minimum 31 cm in node 7)

Momentum diagram ($M_{\text{max}}=1.139.131$ Nm)

The second calculation scheme considers the conduit as being made up of two segments joined together at the dam axis. The calculation is done for the downstream section:

Load scheme in Beam 2D
Fig. 6 Input data for Beam 2 second calculation model

![Input data for Beam 2 second calculation model](image)

**Fig. 7. Structural model for second calculation scheme**

Results:

Conduit deformation (maximum of 37 cm in node 1; minimum 10 cm in node 11)

![Structural model for second calculation scheme](image)

Momentum diagram ($M_{\text{max}} = 4743937 \text{ Nm}$)
4. Results and Conclusions:
- the maximum compaction in both cases is 37cm and is roughly equal to the estimate given when the dam was built (35cm). Even if settlement is inadequate by Winkler model
- the bending momentum is 4 times greater in the case of the monolith conduit;
- the maximum tensions in the monolith conduit are:
  \[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{I} \]
  \[ \sigma_{c} = \frac{M_{\text{max}}}{I} y_{c} = \frac{4743937}{6.67} = 1.71 \, \text{N/m}^2 = 1.2 \, \text{N/mm}^2 = 1.2 \, \text{MPa} \]
  \[ \sigma_{t} = \frac{M_{\text{max}}}{I} y_{r} = \frac{4743937}{6.67} = 1.49 \, \text{N/m}^2 = 1.05 \, \text{N/mm}^2 = 1.05 \, \text{MPa} \]
- the maximum tensions in the monolith conduit are:
  \[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{I} \]
  \[ \sigma_{c} = \frac{M_{\text{max}}}{I} y_{c} = \frac{4743937}{6.67} = 1.71 \, \text{N/m}^2 = 1.2 \, \text{N/mm}^2 = 1.2 \, \text{MPa} \]
  \[ \sigma_{t} = \frac{M_{\text{max}}}{I} y_{r} = \frac{4743937}{6.67} = 1.49 \, \text{N/m}^2 = 1.05 \, \text{N/mm}^2 = 1.05 \, \text{MPa} \]

The average compressive strength of concrete is: (EN 1992-1-1)
\[ f_{\text{cm}} = 20 \, \text{MPa}, \text{ and the average tensile strength is } f_{\text{ctm}} = 1.6 \, \text{MPa} \]

Therefore, the conduit section withstands the load without reinforcement during maximum momentum. This is possible due to the overall rigidity of the structure. The rigidity of one section is \[ [5,8] \]
\[ t = \frac{1 - \mu_{p}^{2}}{1 - \mu_{b}^{2}} \frac{E_{p}}{E_{b}} \frac{\pi (B/2)^{3}}{4I} \approx 10 \frac{E_{p}}{E_{b}} \frac{(B/2)^{3}}{h^{3}} \]
\[ E_{p} = 35 \, \text{MPa} \]
\[ \mu_{p} = 0.35 \text{ (silty clay)} \]
\[ E_{b} = 26,000 \, \text{MPa} \text{ (concrete class C12/15)} \]
\[ \mu_{b} = 0.2 \]
\[ B = 3.20 \, \text{m} \]
\[ I = 6.67 \, \text{m}^{4} \]

Given this data, we have \( t = 0.0007 << 1 \) so the conduit is rigid.

In conclusion, the bottom discharge conduit can be made up of a maximum of two sections joined at the dam axis. This method circumvents the problems caused by seepage at the section joints.

5. References
7. NP 112-2014 Normativ privind proiectarea fundațiilor de suprafață