DEVELOPMENT OF A DIRECT CALCULATION ALGORITHM OF THE HORIZONTAL STRAIN VECTOR OF THE STUDIED CONSTRUCTION, USING CONDITIONAL GEODETIC MEASUREMENTS

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Abstract: *Obtaining an improved construction quality, choosing optimal solutions from the socio-economic point of view, can be attained by completing the strength and stability calculation with laboratory experimental research on models and with measurements and observations on constructions at full scale, made by means of machines and high-tech methods. Geodetic methods, by the high precision of measurements as well as the methods for data processing and estimation of the quality of results, represent a basic system in the extensive process of experimental study of constructions, in many respects being irreplaceable. The paper introduces a direct calculation algorithm of the horizontal deformation vector of the studied construction, using precision geodetic measurements, the method of conditional measurements and a mathematical model to estimate accuracy results.*

Keywords: *conditional measurements, observed constructions, horizontal deformations, accuracy assessment.*

1. Introduction

Choosing the optimal solutions for the design and rational operation of buildings implies a thorough and highly complex system, that is required to be performed both at the beginning of the construction process, in the design and planning phase, and finally, in the verification phase of the construction the implementation and operation (Nistor Gh., 1993). Experimental research of a building, aiming to establish its behavior under the action of a specific trial system, allows determining at any building point or area, all or the main values that typically are obtained by computing: stresses, strains, static and dynamic displacements, rotations, axial forces, etc. Of these, only a part can result directly from statements or records of meters and control instruments (AMC).

Unless otherwise obtained through laboratory testing of construction, other elements can be known from observations and measurements over a long time, based on the study and analysis of behaviour in time (*in situ*), under operational conditions, using geodetic and photogrammetric methods. Concurrent analysis and interpretation of laboratory experimental research results with design and calculation results and with the results obtained from processing the measurements made at monitoring the behaviour *in situ*, is a precious material to achieve optimal construction solutions.

Geodetic methods, through high precision measurements, and by data processing methods and estimation of the quality of results represent a basic system in the extensive and difficult process of studying the in situ construction behaviour. Geodetic methods, included in the geometric methods group, report the position of certain points (control points), fixed on the construction, to fixed points outside the construction, in non-deformable grounds and outside the construction influence area, both making up the geodetic network to track.

For large constructions (dams, locks, bridges, etc.) determining the deformation and horizontal displacement vector is performed by the trigonometric-microtriangulation method. Determination of deformation and horizontal displacement vector of control points fixed on the construction under study involves repeated / cyclical measurements of the microtriangulation network with the same precision with which it was originally built. Compensation calculations must be performed rigorously by the least squares method in order to obtain the most probable values of the changes in the positions of the control points in plan, with the possibility of assessing the accuracy of the compensation results, and thus, the accuracy of the horizontal deformation vector of control points and by this the position change of the building under the influence of several factors.

Reference network compensation is made rigorous by the indirect measurements method or conditional measurements method. In the classical sense, the X, Y plane rectangular coordinates of control points are obtained after which by analysis of the distribution of control points coordinates obtained in each cycle the sizes of absolute, partial and total deformations and horizontal displacements can be determined.

Since a part of the compensation data remain unchanged in all measurements cycles, the paper will present a direct calculation algorithm of ΔX_k, ΔY_k deformation components, depending on measured elements changes, such as the azimuth angles, sides, due to changes in the studied construction position. The presented method eliminates network compensation operation in every cycle except for the initial / zero cycle. For the developed algorithm the mathematical model for assessing the precision of results is alsopresented.

2. Establishing the calculation algorithm

It is supposed that in order to determine the deformations and horizontal displacements vector of a number of *N* control points on the studied construction in relation to a *P* number of microtriangulation network fixed points, an *n* number of direct measurements of the same accuracy (angles, distances) were performed in the field, in the initial / zero cycle.

In developing the calculation algorithm for the deformation and displacement vector of the studied construction, in direct function of the cyclic variation of the elements measured in the field, it will be proceeded as follows: first, direct measurements of direct measurements resulting from the initial / zero cycle is made, in order to obtain the most likely values of direct measurements bound by conditions by means of the matrix relationship

$$X^0_{n1} = M^0_{n1} + V^0_{n1}\ , \tag{1}$$

where X^0_{n1} – most likely values of direct measurements bound by conditions; M^0_{n1} – average values of measured sizes in the field; V^0_{n1} – the values of corrections made to values measured in the field, obtained by the condition of minimum.

In the system of condition equations

$$G_j(x^0_i) = 0,\ unde\ i = \overline{1,n}\ \ şi\ \ j = \overline{1,r} \tag{2}$$

the replacement in (1) being made, the system of the condition equations of correction is reached, having a non-linear form

$$G_j(M_i^0 + V_i^0) = 0 \,. \tag{3}$$

Compensation measurements being made under the condition of minimum, first the linearization operation will be made, resulting the matrix equation

$$A_{rn} V_{n1}^0 + W_{r1}^0 = 0 \,, r < n \,, \tag{4}$$

where the notations were made: n – the number of direct measurements; r – the number of linear condition equations of corrections / unknowns; W_{r1}^0 – free terms vector / non-closures.

The developed equation matrix is as follows:

$$A_{rn} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \\ b_1 & b_2 & \dots & b_n \\ . & . & \dots & . \\ r_1 & r_2 & \dots & r_n \end{bmatrix} ; \; V_{n1}^0 = \begin{bmatrix} v_1^0 \\ v_2^0 \\ \dots \\ v_n^0 \end{bmatrix} ; \; W_{r1}^0 = \begin{bmatrix} w_1^0 \\ w_2^0 \\ \dots \\ w_r^0 \end{bmatrix} . \tag{5}$$

Geodetic measurements compensation is made under the condition of minimum

$$(V^0)_{1n}^T V_{n1}^0 = \min . \tag{6}$$

In the case of conditional measurements compensation the sizes of corrections / unknowns V^0 should simultaneously fulfill bothe the condition of minimum (6), and the system of condition equations (4); instead of condition (6) the equivalent Lagrange function will written, by introducing a number of unknown parameters representing Gauss correlates or Lagrange multipliers, *K*, whose number equals the number of equations of condition, *r*. The Lagrangian will be

$$G = (V^0)_{1n}^T V_{n1}^0 - 2(K^0)_{1r}^T (A_{rn} V_{n1}^0 + W_{r1}^0) = \min . \tag{7}$$

The function minimum will be obtained by canceling the first partial derivatives related to the corrections

$$\frac{1}{2}\frac{\partial G}{\partial (V^{0T} V^0)} = (V^0)_{1n}^T - (K^0)_{1r}^T A_{rn} = 0 \,, \tag{8}$$

hence the correction equations system, according to correlates

$$V_{n1}^0 = A_{nr}^T K_{r1}^0 \,, \tag{9}$$

which, having n – equations and $(n+r)$ unknowns, V_{n1}^0 *şi* K_{r1}^0, cannot be solved. Therefore, the solution is possible by re-uniting equations (4) si (9)

$$\begin{cases} V_{n1}^0 = A_{nr}^T K_{r1}^0 \\ A_{rn} V_{n1}^0 + W_{r1}^0 = 0 \,. \end{cases} \tag{10}$$

The solving operation will be made by substitution of unknowns (9), in the second equation of the system (10), resulting

$$A_{rn} A_{nr}^T K_{r1}^0 + W_{r1}^0 = 0 \Rightarrow N_{rr} K_{r1}^0 + W_{r1}^0 = 0 \,. \tag{11}$$

This is the normal system of correlates, hence correlates vector will result

$$K_{r1}^0 = -N_{rr}^{-1} W_{r1}^0 \Rightarrow K_{r1}^0 = -\overline{Q_{rr}} W_{r1}^0 \,. \tag{12}$$

The $\overline{Q_{rr}}$ matrix is the matrix of weight coefficients of correlates, also called co-actors matrix.

By replacing the correlates vector in equation (9) the corrections vector will be obtained. In their turn, they are replaced in equation (1), to give the most probable values of direct measurements related by conditions X^0. Finally, based on compensated sizes, the elements entering the calculation of rectangular coordinates of control points embedded in the studied construction are calculated X_k^0 *şi* Y_k^0, where *k = 2N.*

In the conventional sense, measurements compensation in each cycle of observations *t* will have to be done, where $t = \overline{1,T}$, resulting the coordinates of X_k^t *şi* Y_k^t control points. The differences in coordinates willlead to components on the two-axes

$$\Delta X_k^t = X_k^t - X_k^0\ ,\ \Delta Y_k^t = Y_k^t - Y_k^0\ ,\ k = 2N\ . \tag{13}$$

Based on components, the deformation and horizontal displacements vector of the construction in the control point $L_{\frac{k}{2}}^t$ will be calculated, and its orientation, $\theta_{L_{\frac{k}{2}}^t}$, compared to the *X* axis of the chosen system:

$$L_{\frac{k}{2}}^t = \sqrt{(\Delta X_k^t)^2 + (\Delta Y_k^t)^2}\ ;\ \theta_{L_{\frac{k}{2}}^t} = arctg\left(\frac{\Delta Y_k^t}{\Delta X_k^t}\right). \tag{14}$$

When the results of each measurement cycle are referred to the zero / initial cycle results, total deformations are obtained. When the results of two cycles of conjugate observations are analyzed partial deformations are obtained. This conventional way of computing is difficult in practice. Therefore, a less sophisticated method will be shown, with a specific and advantageous calculation algorithm, since it eliminates the laborious rigorous network compensation operations, from the first cycle of proper measurements, when studied construction behaviour is actually highlighted.

The algorithm to be shown will allow determination of variations / changes in component sizes on the two axes, in direct function of variations between the average values of measurements taken in cycle *t*, M_i^t, and the most probable sizes of compensated measurements in the zero / initial cycle, X_i^0.

The solving starts by applying the known method of expressing the correlates of the normal network reference equations depending on the free terms of normal equations. Next, the successive elimination of correlates will be done, yielding formulas by means of which the most probable values of the components of the deformation vector of each control point will be calculated, as based on the differences / changes between the measurement values in the field of cycle *t*, $t = \overline{1,T}$, and the values resulting from the compensationof the microtriangulation network in the zero cycle.

In terms of the above shown, the order of operations is as follows:

The linear system of condition equations of corrections (4) is considered, shown in the developed form

$$\begin{bmatrix} a_1 & a_2 & \dots & a_n \\ b_1 & b_2 & \dots & b_n \\ . & . & \dots & . \\ r_1 & r_2 & \dots & r_n \end{bmatrix} \begin{bmatrix} v_1^t \\ v_2^t \\ \dots \\ v_n^t \end{bmatrix} + \begin{bmatrix} w_1^t \\ w_2^t \\ \dots \\ w_r^t \end{bmatrix} = 0\ , \tag{15}$$

and the corresponding system of normal equations of correlates (11) under the developed form

$$\begin{bmatrix} [aa] & [ab] & \dots & [ar] \\ [ab] & [bb] & \dots & [br] \\ . & . & \dots & . \\ [ar] & [br] & \dots & [rr] \end{bmatrix} \begin{bmatrix} k_1^t \\ k_2^t \\ \dots \\ k_r^t \end{bmatrix} + \begin{bmatrix} w_1^t \\ w_2^t \\ \dots \\ w_r^t \end{bmatrix} = 0\ . \tag{16}$$

As based on these, the expression of correlates will be made depending on the free terms of the condition equations, with equation (12) with the general form

$$\begin{bmatrix} k_1^t \\ k_2^t \\ \dots \\ k_r^t \end{bmatrix} = -\begin{bmatrix} Q_{11} & Q_{12} & \dots & Q_{1r} \\ Q_{21} & Q_{22} & \dots & Q_{2r} \\ . & . & \dots & . \\ Q_{r1} & Q_{r2} & \dots & Q_{rr} \end{bmatrix} \begin{bmatrix} w_1^t \\ w_2^t \\ \dots \\ w_r^t \end{bmatrix} . \tag{17}$$

The matrix is the matrix of weight coefficients of correlates, also called co-factors matrix, and its elements are obtained either by inverting the matrix of normal equations coefficients of correlates, $\overline{Q_{rr}} = N_{rr}^{-1}$, or in additional columns to the solving schemes of normal equations of correlates, Gauss-Doolittle or Cholesky-Banachiewicz (Nistor Gh., 1996).

Corresponding to the condition equations system (15), in the correction equations system according to correlates (17), the corrections vector is expressed

$$\begin{bmatrix} v_1^t \\ v_2^t \\ \dots \\ v_n^t \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & \dots & r_1 \\ a_2 & b_2 & \dots & r_2 \\ . & . & \dots & . \\ a_n & b_n & \dots & r_n \end{bmatrix} \begin{bmatrix} k_1^t \\ k_2^t \\ \dots \\ k_r^t \end{bmatrix} . \tag{18}$$

In the obtained system the correlates are replaced in equation (17), yielding the corrections equations system, expressed according to the free terms of the condition equations

$$V_{n1} = A_{nr}^T K_{r1}^t = -A_{nr}^T \overline{Q_{rn}} W_{r1}^t \ , \tag{19}$$

or developed

$$\begin{bmatrix} v_1^t \\ v_2^t \\ \dots \\ v_n^t \end{bmatrix} = -\begin{bmatrix} a_1 & b_1 & \dots & r_1 \\ a_2 & b_2 & \dots & r_2 \\ . & . & \dots & . \\ a_n & b_n & \dots & r_n \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & \dots & Q_{1r} \\ Q_{21} & Q_{22} & \dots & Q_{2r} \\ . & . & \dots & . \\ Q_{r1} & Q_{r2} & \dots & Q_{rr} \end{bmatrix} \begin{bmatrix} w_1^t \\ w_2^t \\ \dots \\ w_r^t \end{bmatrix} . \tag{20}$$

Marking the product of matrices

$$Z_{nr} = -A_{nr}^T \overline{Q_{rn}} \ , \tag{21}$$

or

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1r} \\ Z_{21} & Z_{22} & \dots & Z_{2r} \\ . & . & \dots & . \\ Z_{n1} & Z_{n2} & \dots & Z_{nr} \end{bmatrix} = -\begin{bmatrix} a_1 & b_1 & \dots & r_1 \\ a_2 & b_2 & \dots & r_2 \\ . & . & \dots & . \\ a_n & b_n & \dots & r_n \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & \dots & Q_{1r} \\ Q_{21} & Q_{22} & \dots & Q_{2r} \\ . & . & \dots & . \\ Q_{r1} & Q_{r2} & \dots & Q_{rr} \end{bmatrix} ,$$

whose elements remain constant for all cycles of observations of the network, the vector of compensated corrections will be expressed by the relation

$$V_{n1}^t = Z_{nr} W_{r1}^t , \tag{22}$$

or

$$\begin{bmatrix} v_1^t \\ v_2^t \\ \dots \\ v_n^t \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \dots Z_{1r} \\ Z_{21} & Z_{22} \dots Z_{2r} \\ . & . \quad \dots \quad . \\ Z_{n1} & Z_{n2} \dots Z_{nr} \end{bmatrix} \begin{bmatrix} w_1^t \\ w_2^t \\ \dots \\ w_r^t \end{bmatrix} .$$

Next, perform the differentiation of formulas for calculating the coordinates of the control points is done, they representing the coordinates changes due to changes of measured elements of the network. Marking these changes of the coordinates of a network control point *k (k = 2N)*, its result from re-compensation will be reached. Formulas resulting from the coordinates differentiation, will be shown in the form

$$\begin{cases} \Delta X_k^t = m_1 v_1^{'} + m_2 v_2^{'} + \dots + m_n v_n^{'} \ , \\ \Delta Y_k^t = n_1 v_1^{'} + n_2 v_2^{'} + \dots + n_n v_n^{'} \ , \end{cases} \tag{23}$$

where $v_1^{'}, v_2^{'}, \dots , v_n^{'}$ are the modified network elements, determined by comparing the new elements of the compensated network with the corresponding elements of the values which were obtained after the first network compensation. Note the differences between the new measured elements in the *t* cycle and their compensated sizes, obtained in the result of the first network compensation and with v_1 , v_2, ... , v_n , as the above corrections have been named in the new compensation. In this way the relations can be written

$$\begin{cases} v_1^{'} = v_1^t + v_1 \\ v_2^{'} = v_2^t + v_2 \\ \dots\dots\dots\dots\dots\dots \\ v_n^{'} = v_n^t + v_n \end{cases} \tag{24}$$

These expressions are introduced in (23) and by replacing v_1 , v_2, ... , v_n with the expressions (22) for the correction coordinates, will result the equations

$$\begin{cases} \Delta X_k^t = m_1 v_1^t + m_2 v_2^t + \dots + m_n v_n^t + \\ \qquad + p_1 w_1^t + p_2 w_2^t + \dots + p_r w_r^t \ , \\ \Delta Y_k^t = n_1 v_1^t + n_2 v_2^t + \dots + n_n v_n^t + \\ \qquad + q_1 w_1^t + q_2 w_2^t + \dots + q_r w_r^t \ . \end{cases} \tag{25}$$

Further on, the relations (25) the free terms w_j^t are replaced with correspondents of these parts in equation (15), by previously replacing the v_i^t corrections by $-v_1^t, -v_2^t, \dots , -v_n^t$ will lead to the corrections / components equations system.
Thus:

$$\begin{cases} \Delta X_k^t = \varphi_1 v_1^t + \varphi_2 v_2^t + \dots + \varphi_n v_n^t \ , \\ \Delta Y_k^t = \psi_1 v_1^t + \psi_2 v_2^t + \dots + \psi_n v_n^t \ . \end{cases} \tag{26}$$

The obtained system (26), made for the microtriangulation network, allows direct calculation of deformation and displacement vector components of all control points, in direct function of the average size variations measured directly in cycle *t*, v_i^t , relative to their values obtained by compensating only the zero cycle. Coefficients φ_i şi ψ_i, $i = \overline{1,n}$, remain constant for all subsequent measurements cycles.

3. Evaluating the results precision

As based on the deformation vector components forms (26), it is possible to assess the accuracy of the results. Considering the components forms as functions of the independent sizes measured directly, the sizes of the components of the square error can be written, resulting

$$\begin{cases} s^2_{\Delta X^t_k} = \varphi_1^2 s_1^2 + \varphi_2^2 s_2^2 + \ldots + \varphi_n^2 s_n^2 \ , \\ s^2_{\Delta Y^t_k} = \psi_1^2 s_1^2 + \psi_2^2 s_2^2 + \ldots + \psi_n^2 s_n^2 \ , \end{cases} \tag{27}$$

where s_i, $i = \overline{1,n}$, are mean square errors of the network direct measurements. In case the measurements are of the same accuracy, $s_1 \approx s_2 \approx \ldots \approx s_n = s_0$, the components errors will be:

$$s_{\Delta X_k} = \pm s_0 \sqrt{[\varphi^2]} \ ; \ \ s_{\Delta Y_k} = \pm s_0 \sqrt{[\psi^2]} \ . \tag{28}$$

With forms (27) and (28) respectively, the mean square errors of horizontal deformation vectors can be calculated for each control point. For example, for point *K* will result:

$$s_{L^t_{\frac{k}{2}}} = \sqrt{s^2_{\Delta X^t_k} + s^2_{\Delta Y^t_k}} \tag{29}$$

The confidence interval within which the true vector size with a probability *P = 67.3%,* corresponding to the mean error will range, is expressed by the double inequality

$$L^t_{\frac{k}{2}} - s_{L^t_{\frac{k}{2}}} \leq \overline{L}^t_{\frac{k}{2}, \, \mathrm{p}=67.3\%} \leq L^t_{\frac{k}{2}} + s_{L^t_{\frac{k}{2}}} \tag{30}$$

The precision of determining the horizontal deformation vector will be inversely proportional to the length of the interval.

4. Conclusions

From the presentation of the calculation algorithm and the mathematical model for assessing the precision of results, the following conclusions come out:

a. The algorithm presented, to be processed in a microtriangulation network using the conditional direct measurements enables fast and reliable calculation of horizontal deformation vectors in direct function of the cyclic variations of sizes measured directly in the field (angles, directions, sides) and subjected to thorough compensation operation.

b. Making the algorithm was possible because a number of compensation elements remain unchanged.

c. The algorithm involves conventional compensation of the whole network through conditional measurements, of the same precision, or weighted, only the zero / initial cycle, to yield the most probable values of the measured sizes X^0_i, $i = \overline{1,n}$. The compensated sizes of these measurements will be reference elements, to which direct measurements performed in all other cycles will report.

d. In all other cycles *t*, (*t = 1,T*) conventional compensation of the entire network will be avoided, by establishing the computing relationships of the components of horizontal deformation vectors in the control points ΔX_k^t *şi* ΔY_k^t, where *k = 2N*, in direct function of v_i^t changes / differences, among the mean sizes of cycle *t* measurements and the compensated sizes of the same measurement in the zero / initial cycle. The results are similar to those obtained in the case where the network would have been re-compensated in the cycle *t*.

e. The *X, Y* system of axes in relation to which determinations are made will be chosen so that they are directed in relation to the axis of the construction, on the directions of maximal and minimal deformation.

f. The mathematical model (27) ... (30), enables evaluation of results precision as based on the mean square error of the components of the horizontal deformations vectors, as well as writing the confidence intervals within which the true sizes will range. It is worth mentioning that in case that there is no zero value within the interval, it means that there is a change in the position of the considered control point, including the studied construction.

5. References

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