MATHEMATICAL MODEL FOR PROCESSING TWO-DIMENSIONAL GEODETIC NETWORKS USING CLASSICAL AND SATELLITE TECHNOLOGIES

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***Abstract**: GNSS technology has increasingly gained traction in the field of geodesy due to its numerous advantages. This paper proposes a case study where this technology was used to perform measurements on a geodetic network at the Râușor Dam between certain points where there were obstacles to performing measurements using the classical method (due to the positioning of the points, existing vegetation, or even the dam itself).*

To achieve the best possible ratio between the true value of the measured quantity and the obtained value, the data acquired from the network was processed to adjust for the inevitable errors that occur during the measurement process. Using the initial coordinates, the distances and orientations between the measured points were calculated. To formulate the correction equations and establish weights, coefficients for directions and distances were calculated, along with the variation in orientation and distance as a function of variations in planar coordinates and the free term, which are necessary elements. For the normalization of the linear equation system and solving the normal system, the matrix of the normal system N and its inverse Q were calculated, followed by calculating the vector of unknown parameters. With these elements, the necessary corrections for the measurements were calculated, resulting in the most probable coordinates of the new points.

The novelty of this mathematical model lies in the combination of classical and satellite measurements by introducing a transformation parameter regarding rotation into the adjustment process. To verify if the adjusted elements meet the conditions of the functional model, a control of the results is performed after the adjustment. To conclude the observation processing, calculations for evaluating precision indicators are carried out, followed by the graphical representation of the error ellipse, which indicates the confidence domain in the 2D position of a point. Finally, the results obtained after processing the network are presented, including the most probable coordinates of the new points, the precision estimates of the unknown parameters, and the elements of the obtained ellipses.

The case study demonstrated how GNSS technology offers advantages in this field (such as the lack of necessity for visibility between points), but also disadvantages, such as the lower precision of GNSS measurements compared to those made using classical methods. GNSS measurements are redundant, providing, in some cases, the ability to detect errors in the adjustment of classical measurements.

***Keywords:** GNSS; processing; measure; geodetic network; azimuth*

1. Introduction

From the beginning of the world until the present, the world has always been surrounded by natural phenomena. Driven by the desire to understand and find the purpose of these phenomena, various studies and research have been created, revealing that most of these phenomena were directly related to the shape and dimensions of the Earth.

This article shows how technology has led to a significant leap in geodesy since the launch of the first artificial satellites of Earth on October 4, 1957. In order to obtain the most accurate data through a method that simplifies the work involved, the possibility of using GNSS (Global Navigation Satellite System) observations was established.

The paper proposes a case study in which, with the help of GNSS technology, measurements could be made at a geodetic network at the Râușor Dam between certain points where measurements could not be conducted using classical methods due to various unavoidable obstacles.

Figure 1 – *Anchor barrage*

Figure 2 – *Making measurements*

2. Materials and Methods

The processing of planar geodetic networks aims to determine the most probable values of the Cartesian coordinates (N and E) of new points within these networks. Another goal of the processing is to determine the precision indicators associated with these points. To process the measurements taken within a geodetic network, several steps are followed, such as: preliminary processing of geodetic observations and reducing the observations to the chosen reference surface, calculating provisional elements, forming the functional-stochastic model, transforming the correction equations according to the rules of equivalence, normalizing the system of linear equations of corrections, and solving the normal system of equations, calculating the compensated elements (and if necessary, controlling the compensation), performing precision assessment calculations, error ellipse calculations, and presenting the results. [1]

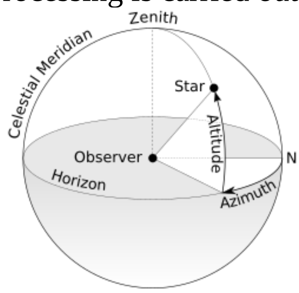
To achieve the best possible relationship between the true value of the measured quantity and the obtained value, the initial data collected in the field have been processed to compensate for the inevitable errors that occur during the measurement process. Thus, having the initial coordinates, the distances and orientations between the measured points—whether determined by classical methods or satellite methods—could be calculated.

P.S.	P.V.	Direcție orizontală	P.S.	P.V.	Distanțe
PS4	PD4	0.00001	PS4	PD4	445.1883
	PD5	344.42441		PD5	262.7851
	PS6	279.05528		PS6	108.4416
	PD3P	392.96129		PD3P	420.8114

P.S.	P.V.	Azimut	P.S.	P.V.	Distanță GNSS
PS4	PD5	264.7314	PS4	PD5	262.7865
	PS6	199.3747		PS6	108.4372

Figure 3 – *Data obtained for a station point*

In the case of GNSS measurements, the processing of the network utilizes the azimuth and distances obtained in the field, calculating the rotation of the system and the rotation coefficient. Using these elements, the processing is carried out.



$$A_{ij} \approx \theta_{ij} = atan\frac{\Delta E_{ij}}{\Delta N_{ij}}$$

P.S.	P.V.	Azimut
PS4	PD5	264.7314
	PS6	199.3747
PS6	PD5	291.592
PS5	PD5	252.5981
	PS4	155.1283
	PS6	185.7391

P.S.	P.V.	Distanță GNSS
PS4	PD5	262.7865
	PS6	108.4372
PS6	PD5	226.5197
PS5	PD5	260.0047
	PS4	49.8013
	PS6	150.1167

Figure 4 – *Azimuth and GNSS distances*

To perform observations on the dam, a local coordinate system is created, transforming the points based on the calculation relationship of the azimuth between the orientation of the points in the local system and the rotation angle. [2]

$$A^{*} = \theta + R$$

$$R = A^{*} - \theta$$

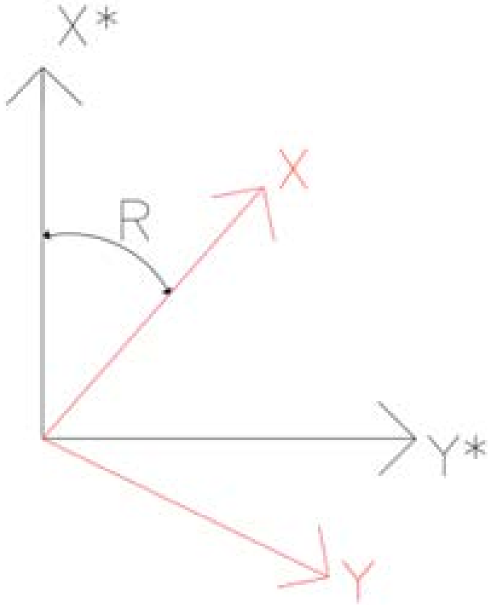


Figure 5 – *Rotation applied to the local system*

The next step in order to carry out the processing of the network is to create the functional-stochastic model, which involves formulating the correction equations and establishing weights for the measurements taken in the geodetic network. For this purpose, coefficients for directions and distances were calculated, along with the variations in orientation and distance based on the variations in the planar coordinates and the free term, as these are the necessary elements.

$$l_a = (\theta^{\circ} - R^{\circ} - A^{*})$$

$$R^{\circ} = \sum \frac{R}{n}$$

To normalize the system of linear equations and solve it, the elements of the system are found by calculating the normal system matrix (N_0) and its inverse (Q_0). Using these, the parameter vector (x) is calculated, which is expressed in decimetres, as this is the derived unit of measure for the direction coefficients.

$$\mathrm{N} = \mathrm{A}^{t} \times \mathrm{P} \times \mathrm{A} \qquad x = -\mathrm{N}^{-1} \times \mathrm{A}^{t} \times \mathrm{P} \times l = \begin{pmatrix} dN_i \\ dE_i \\ dN_j \\ dE_j \end{pmatrix} (dm)$$

$$\mathrm{N}^{-1} = \mathrm{Q}$$

The novelty of this mathematical model lies in the combination of classical and satellite measurements by introducing the transformation parameter regarding rotation into the compensation process.

These calculated elements formulate corrections that are applied to the directions and distances. When checking the correctness of the obtained results, the variation in orientation must be equal for both forward and backward directions, and the sum of the corrections for horizontal directions in one direction must equal zero. Thus, after applying the corrections, the most probable coordinates of the new points are obtained. In the case of GNSS measurements, corrections for azimuth are calculated using the rotation coefficient, while for distances, they are calculated in the same way as in the classical method.

$$v_{ij}^{\alpha} = -dz_i + d\theta_{ij} + l_{ij}^{\alpha}$$

$$v_{ij}^{D} = dD_{ij} + l_{ij}^{D}$$

$$v_a = -dR + a_{ij} \times dN_j + b_{ij} \times dE_j - a_{ij} \times dN_i - b_{ij} \times dE_i + l_a$$

For one of the station points (PS4), the corrections obtained for horizontal directions, distances, azimuth, and GNSS distances are presented in the table.

P.S.	P.V.	dN_{PS5}	dE_{PS5}	dN_{PS6}	dE_{PS6}	dN_{PD3P}	dE_{PD3P}	dN_{PD5}	dE_{PD5}	lijα	dθ / dD	dzi(cc) / dR	vijα	Control
		0.000	-0.101	0.004	-0.072	-2.454	0.198	-0.004	0.029					
PS4	PD4	0	0	0	0	0	0	0	0	-72.19254	0	-83.066454	10.87391	0.00000
	PD5	0	0	0	0	0	0	185.7718513	-155.4797191	-91.84723	-5.309823		-14.09060	
	PS6	0	0	-89.43436628	-580.19632	0	0	0	0	-129.54576	41.4434752		-5.03584	
	PD3P	0	0	0	0	150.94071	10.00060376	0	0	293.58554	-368.39947		8.25253	
PS4	PD5	0	0	0	0	0	0	-0.641814811	-0.766859667	0.09396	-0.0199519		0.07401	
	PS6	0	0	-0.988327265	0.1523457	0	0	0	0	0.02540	-0.0149763		0.01042	
	PD3P	0	0	0	0	0.0661102	-0.99781233	0	0	0.34173	-0.3600752		-0.01834	
PS4	PD5	0	0	0	0	0	0	185.7718513	-155.4797191	-52.99040	-5.309823	-16.197288	-42.10293	0.00000
	PS6	0	0	-89.43436628	-580.19632	0	0	0	0	-214.98892	41.4434752		-157.34816	
PS4	PD5	0	0	0	0	0	0	-0.641814811	-0.766859667	0.07996	-0.0199519		0.06001	
	PS6	0	0	-0.988327265	0.1523457	0	0	0	0	0.06940	-0.0149763		0.05442	

Compensation control represents the stage in which it is verified whether the compensated elements, specifically the compensated measurements and the most probable values of the unknowns, meet the conditions of the unlinearized functional model. This stage is performed for both classical measurements and satellite measurements.

P.S.	P.V.	αij*(gon)	vijα(cc)	αij comp(gon)	zi0	dzi	zi comp	θij comp (gon)	θij coord (gon)	Control
		Dij*(m)	vijD(dm)	Dij comp(m)	(gon)	(cc)	(gon)	Dij comp (m)	Dij coord (m)	(cc)/(mm)
PS4	PD4	0.00001	10.87391	0.001097391	311.22113	-83.06645	311.2128249	311.2139223	311.2139223	0.0000
	PD5	344.42441	-14.09060	344.4230009				255.6358258	255.6358258	0.0000
	PS6	279.05528	-5.03584	279.0547764				190.2676013	190.2676014	-0.0006
	PD3P	392.96129	8.25253	392.9621153				304.1749402	304.174937	0.0314
PS4	PD5	262.78510	0.07401	262.7851074				262.79250	262.7925013	0.0000
	PS6	108.44160	0.01042	108.441601				108.44264	108.4426423	-0.0002
	PD3P	420.81140	-0.01834	420.8113982				420.80957	420.8096361	-0.0705
PS4	PD5	264.73140	-42.10293	264.7271897	390.91026	-16.19729	390.9086361	255.6358258	255.6358258	0.0000
	PS6	199.37470	-157.34816	199.3589652				190.2676013	190.2676014	-0.0006
PS4	PD5	262.78650	0.06001	262.786506				262.79250	262.7925013	0.0000
	PS6	108.43720	0.05442	108.4372054				108.44264	108.4426423	-0.0002

The conclusion of the processing involves calculations to evaluate the precision indicators, followed by the graphical representation of the error ellipse.

Punct	qNN	qEE	S0	SN(dm)	SE(dm)	SP(dm)	S R (cc)
PS5	3.85E-06	1.85E-06	25.14498778	0.049	0.034	0.060	28.769977
PS6	3.05E-06	1.48E-06		0.044	0.031	0.054	
PD3P	9.57881E-06	0.000002		0.078	0.034	0.085	
PD5	2.00822E-06	0.000002		0.036	0.034	0.049	

The error ellipse represents the confidence domain for the 2D position of a point determined by the method of least squares. This applies to positioning in two-dimensional space, such as in the case of positioning on a projection plane or on the reference ellipsoid. [3]

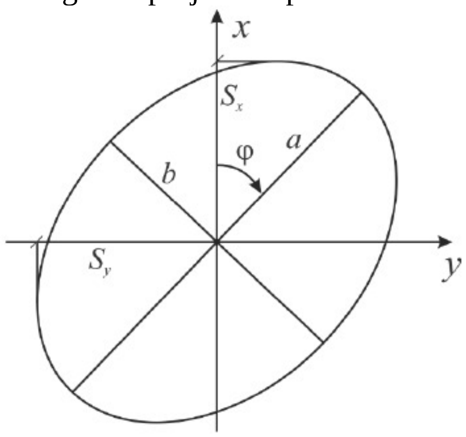


Figure 6 – *The error elllipse*

Punct	Blocul bidimensional		λ1	λ2	a(dm)	b(dm)	φ(gon)
PS5	3.85E-06	-8.85E-07	4.19E-06	1.52E-06	0.051	0.031	76.9258
	-8.85E-07	1.85E-06					
PS6	3.05E-06	3.94E-07	3.15E-06	1.39E-06	0.045	0.030	114.7795
	3.94E-07	1.48E-06					
PD3P	9.57881E-06	1.575E-06	9.89E-06	1.48E-06	0.079	0.031	112.2266
	1.575E-06	1.785E-06					
PD5	2.00822E-06	-3.73E-07	2.31011E-06	1.55E-06	0.038	0.031	56.6855
	-3.73016E-07	1.849E-06					

The processing algorithm for GNSS measurements follows the same pattern as that of classical measurements, with new elements such as system rotation, the rotation coefficient, and rotation deviation emerging.

Rm	dR(${}^{g\ cc\ c}$)	S R (cc)
390.91	-16.20	28.77

3. Results and Discussion

To present the final results, they are formatted as textual and graphical reports.

For the textual reports, a table will be created that includes the names of the measured points, the Cartesian coordinates (N and E), the standard deviation of the position unknowns, the total position error of the point, the semi-major axis, the semi-minor axis, and the orientation of the semi-major axis of the ellipse.

In the case of graphical reports, a sketch of the network is prepared for each compensation case, which must include the following elements: title, scale, area, projection plan/reference system and coordinates, scale of the error ellipses (which differs from the scale of the plan), axes with graduations and labels/grid with graduations, and the north direction (N).

Punct	*N(m)*	*E(m)*	*SN(dm)*	*SE(dm)*	*SP(dm)*	*a(dm)*	*b(dm)*	*φ(gon)*
PS5	1879.9069	854.4306	0.049	0.034	0.060	0.051	0.031	76.9258
PS6	1739.782	908.25999	0.044	0.031	0.054	0.045	0.030	114.7795
PD3P	1874.5367	471.84113	0.078	0.034	0.085	0.079	0.031	112.2266
PD5	1678.2941	690.22264	0.036	0.034	0.049	0.038	0.031	56.6855

Figure 7 – *Presentation of the results*

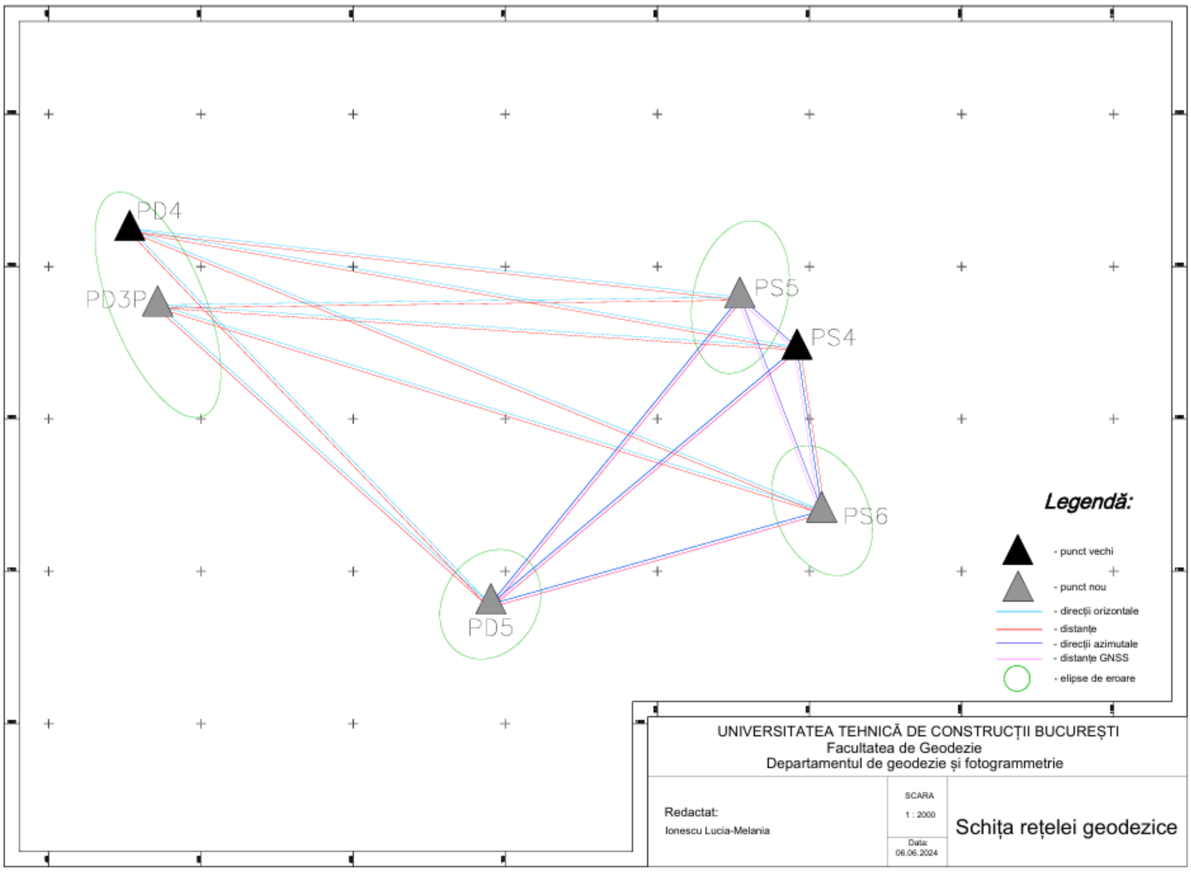


Figure 8 – *Sketch of the geodetic network*

4. Conclusions

As a result of the case study, it was demonstrated how GNSS technology provides advantages in this field, one of which is finding a solution in the absence of visibility between points. Although GNSS technology offers several benefits, there are also certain disadvantages compared to classical measurement methods, such as lower precision, as can be seen in the case study, where classical measurements have better accuracy than those conducted using GNSS.

Most often, GNSS measurements are carried out in the monitoring of dams, and the data obtained through GNSS technology can provide information about the behavior of the dam over time.

Following the compensation, we obtained the most probable values of the new points and their precision, with the compensation controls showing very good values. GNSS measurements are redundant; in some cases, they provide the opportunity to detect the occurrence of errors in the compensation of classical measurements.

5. Acknowledgments

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6. References

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