STUDY ON THE INFLUENCE OF REFERENCE DATA ON THE PROCESSING OF GEODETIC MONITORING NETWORKS

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Abstract: *The purpose of this work is twofold. Initially, it started with the desire to study potential displacements of characteristic points located on the Pecineagu Dam. Subsequently, we realized that determining the generalized inverse is an important and current topic, so we decided to pursue both aspects in parallel: monitoring the behavior of the characteristic points on the dam and determining the generalized inverse using three methods. The motivation for this study stems from the importance of dams for both the population and nature, serving multiple roles such as flood protection, irrigation, electricity production, and more.*

The initial data consisted of the provisional coordinates of the new points and measurements of distances and directions conducted within a local network. This network is formed by six new points (6, 7, 8, Ro, 12, 13) included in the geodetic network located at the Pecineagu Dam, a 107-meter-high rockfill dam on the Dâmbovița River in Argeș. Initially, we calculated the coefficients for distances and directions as well as the orientation angle of each station, which were later used in developing the stochastic functional model. The next step was outlining the three methods we used for determining the generalized inverse.

The first method involved determining the generalized inverse using the "pinv" function integrated into the Octave application. To extract the A, P, and l matrices from the functional model, we applied the first equivalence rule, aiming to eliminate the unknowns associated with the stations orientation angles, replacing each unknown with a sum equation.

The second method involved applying the S transformation with a partial minimum condition. To determine the generalized inverse using the transformation matrix S, it was necessary to first apply a reduction to the network's center of gravity regarding the coordinates. At this stage, the initial functional model was used without applying the equivalence rule.

The third method was applying the "pinv" function using the initial functional model without applying the equivalence rule.

After determining the generalized inverse, we proceeded with determining the corrections, applying them, and calculating the precisions and the elements of the error ellipses. As can be observed from the results, methods 1 and 2 yielded the same final matrices (both Q and x), while applying the "pinv" function without eliminating the orientation unknowns of the stations resulted in different outcomes that cannot be further used in the continuation of the adjustment. After calculating the characteristic elements of the ellipses, the precision was significantly better in the 2020 stage compared to the 2023 stage. Regarding the monitoring of the characteristic points between 2020-2023, the Fischer test was passed marginally, prompting me to also apply the Student's test. The result indicated that point 12 had displaced in both directions, while point 13 displaced only in the East direction.

Keywords: *network; compensation; transformation matrix; pseudoinverse; monitoring*

1. Introduction

The geoid is defined as the average reference surface of calm seas and oceans extended under the continents.[1]

The reference ellipsoid is the ellipsoid used at a given moment to solve geodetic problems.[1]

Orientation is the horizontal angle formed by the North direction and the chosen reference direction, measured clockwise from the North direction to the reference direction.[1]

The weight of a measurement/equation represents the positive quantity that expresses the confidence assigned to that measurement.[1]

The meridian of a point is defined by the plane section that passes through the normal to the ellipsoid at the considered point and through the axis of the poles.[1]

The geodetic network is an infrastructure of points located in the area where a work is being conducted, and whose position is known within a unified reference system.[1]

Positioning refers to determining the position of stationary or moving objects through static positioning (used in geodetic measurements) or kinematic positioning (used in navigation). [1]

A model is understood as a simplified representation of a real phenomenon or process. Gravity represents the component of all forces acting on a point located on the Earth's surface.[1]

2. Materials and Methods

If, after normalization, two systems of equations lead to the same solutions, we can state that they are equivalent systems of equations. The purpose of applying equivalence rules is to reduce the number of unknowns and equations. In our field, we know three important situations of equivalence, which are referred to as Schreiber's rules of equivalence.

In my situation, for processing this free geodetic network, only Rule 1 of equivalence was applied, which states that for all horizontal angular directions, the unknown dz disappears and is replaced by a sum equation. Consequently, for the functional model, the four unknowns related to the orientation angles of the stations vanish, and four sum equations appear. The application of this equivalence rule does not alter the equations related to distances; they remain the same as those in the functional-stochastic model. Following the application of Rule 1 of equivalence, we obtain the following elements: matrix A, which represents the matrix of coefficients for the correction equations; matrix l, which represents the vector of free terms; and matrix P, which represents the weights matrix. [4]

The calculation of the pseudoinverse using the Octave computing program begins by manually entering the coefficient matrix A and the weights matrix P into the application, using the following functions:

```
GNU Octave, version 7.3.0
Copyright (C) 1993-2022 The Octave Project Developers.
This is free software; see the source code for copying conditions.
There is ABSOLUTELY NO WARRANTY; not even for MERCHANTABILITY or
FITNESS FOR A PARTICULAR PURPOSE.  For details, type 'warranty'.

Octave was configured for "x86_64-w64-mingw32".

Additional information about Octave is available at https://www.octave.org.

Please contribute if you find this software useful.
For more information, visit https://www.octave.org/get-involved.html

Read https://www.octave.org/bugs.html to learn how to submit bug reports.
For information about changes from previous versions, type 'news'.

>> A=0
A = 0
>> P=0
P = 0
>> |
```

Figure 1 *Entering matrices into the program*

To determine the values of matrix N, the relationship for calculating it will be written in the workspace of the application, which will compute it automatically. We also check the rank and the deficiency of rank of matrix N. After determining matrix N, we can proceed to the final step in the application, namely determining the generalized inverse using the numerical function ***pinv***. [2]

```
A=0
P=0
AT=A'
N=AT*P*A
Q=pinv(N)
```

Figure 2 *Applying the pinv function*

Q [12x12 double]

	1	2	3	4	5	6	7	8	9	10	11	12
1	3.3644e-05	-6.0584e-06	-2.1874e-06	-5.7345e-06	-1.8507e-05	3.4485e-05	3.0347e-06	-5.0378e-06	-1.4302e-05	-4.6437e-05	-1.2828e-06	2.8783e-05
2	-6.0584e-06	6.5251e-05	6.9005e-06	6.5859e-05	-4.007e-06	5.7364e-05	-1.284e-05	-0.00013382	-5.5182e-06	6.2644e-06	2.1523e-05	-6.0923e-05
3	-2.1874e-06	6.9005e-06	5.5231e-05	3.5799e-06	-3.0844e-05	2.3548e-05	8.447e-07	-3.7799e-06	-2.1853e-05	-5.4944e-05	-1.1913e-06	2.4695e-05
4	-5.7345e-06	6.5859e-05	3.5799e-06	6.6949e-05	-2.3549e-06	5.7024e-05	-1.3129e-05	-0.00013604	-4.3414e-06	9.644e-06	2.198e-05	-6.3433e-05
5	-1.8507e-05	-4.007e-06	-3.0844e-05	-2.3549e-06	4.699e-05	-4.8784e-05	-5.1405e-06	-2.889e-06	1.2803e-05	2.5757e-05	-4.9024e-06	3.2277e-05
6	3.4485e-05	5.7364e-05	2.3548e-05	5.7024e-05	-4.8784e-05	0.00012384	-2.3682e-06	-0.00012296	-2.9222e-05	-6.0103e-05	2.234e-05	-5.5166e-05
7	3.0347e-06	-1.284e-05	8.447e-07	-1.3129e-05	-5.1405e-06	-2.3682e-06	4.0861e-05	9.5239e-05	-2.4171e-05	-2.8195e-05	-1.5428e-05	-3.8707e-05
8	-5.0378e-06	-0.00013382	-3.7799e-06	-0.00013604	-2.889e-06	-0.00012296	9.5239e-05	0.00081469	-5.3267e-05	-0.00017516	-3.0266e-05	-0.00024671
9	-1.4302e-05	-5.5182e-06	-2.1853e-05	-4.3414e-06	1.2803e-05	-2.9222e-05	-2.4171e-05	-5.3267e-05	5.7068e-05	9.2086e-05	-9.5453e-06	2.621e-07
10	-4.6437e-05	6.2644e-06	-5.4944e-05	9.644e-06	2.5757e-05	-6.0103e-05	-2.8195e-05	-0.00017516	9.2086e-05	0.00028278	1.1734e-05	-6.3417e-05
11	-1.2828e-06	2.1523e-05	-1.1913e-06	2.198e-05	-4.9024e-06	2.234e-05	-1.5428e-05	-3.0266e-05	-9.5453e-06	1.1734e-05	3.235e-05	-4.731e-05
12	2.8783e-05	-6.0923e-05	2.4695e-05	-6.3433e-05	3.2277e-05	-5.5166e-05	-3.8707e-05	-0.00024671	2.621e-07	-6.3417e-05	-4.731e-05	0.00048965

Figure 3 *The values of the pseudoinverse Q determined using the numerical function 'pinv'*

After determining the generalized inverse, we continue with the calculation of the unknown vector.

To compute the generalized inverse using the transformation matrix S, it is first necessary to apply a reduction to the centroid of the network on the coordinates.

Pct	ni	ei
6	-30.7914	-132.6762
7	-50.9463	-132.0438
8	57.9227	-195.1620
12	0.6741	155.1202
13	72.2642	145.2917
Ro	-49.1232	159.4702

Figure 4 *The coordinates of the new points reduced to the centroid.*

The processed geodetic network is a free one that has a rank deficiency, an element that was highlighted in the previous method using Octave software. In this case, the datum deficiency of the free geodetic network is 3, which results in one rotation and two translations of the network in the North and East directions, with the scale factor being the only one that remains unaffected.

Next, it is necessary to obtain the elements of the cofactor matrix Q_D . For this step, the matrix $N^{-1}$is padded with zero elements for rows and columns equal to the rank deficiency. We have removed the first three columns from matrix A; therefore, when padding matrix N^{-1}, the first three rows and columns were filled with zeros to maintain symmetry. Subsequently, the pseudoinverse was calculated. After determining it, we will use it to continue calculating the unknown vector X. The goal of this processing step is to compare the pseudoinverse and the unknown vector obtained through this method with the other elements obtained using different methods.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	3.364e-05	-6.06e-06	-2.19e-06	-5.73e-06	-1.891e-05	3.449e-05	3.03e-06	-5.04e-06	-1.43e-05	-4.644e-…	-1.28e-06	2.878e-05	-0.36021	-0.3607	-0.36057	-0.36079
2	-6.06e-06	6.525e-05	6.9e-06	6.586e-05	-4.01e-06	5.736e-05	-1.284e-05	-0.00013…	-5.52e-06	6.26e-06	2.152e-05	-6.092e-05	0.13009	0.13014	0.13021	0.1304
3	-2.19e-06	6.9e-06	5.523e-05	3.58e-06	-3.084e-05	2.355e-05	8.4e-07	-3.78e-06	-2.185e-05	-5.494e-…	-1.19e-06	2.47e-05	0.57803	0.57897	0.57824	0.57771
4	-5.73e-06	6.586e-05	3.58e-06	6.695e-05	-2.35e-06	5.702e-05	-1.313e-05	-0.00013…	-4.34e-06	9.64e-06	2.198e-05	-6.343e-05	0.093378	0.093362	0.093505	0.093729
5	-1.891e-…	-4.01e-06	-3.084e-…	-2.35e-06	4.699e-05	-4.878e-…	-5.14e-06	-2.89e-06	1.28e-05	2.576e-05	-4.9e-06	3.228e-05	-0.12045	-0.1207	-0.12027	-0.11976
6	3.449e-05	5.736e-05	2.355e-05	5.702e-05	-4.878e-05	0.000123…	-2.37e-06	-0.00012…	-2.922e-05	-6.01e-05	2.234e-05	-5.517e-05	-0.10974	-0.11007	-0.11013	-0.11023
7	3.03e-06	-1.284e-…	8.4e-07	-1.313e-05	-5.14e-06	-2.37e-06	4.086e-05	9.524e-05	-2.417e-05	-2.82e-05	-1.543e-05	-3.871e-05	-0.021918	-0.021952	-0.021673	-0.021742
8	-5.04e-06	-0.00013…	-3.78e-06	-0.00013604	-2.89e-06	-0.00012…	9.524e-05	0.000814…	-5.327e-05	-0.00017…	-3.027e-05	-0.00024671	0.013207	0.013515	0.013388	0.013515
9	-1.43e-05	-5.52e-06	-2.185e-…	-4.34e-06	1.28e-05	-2.922e-…	-2.417e-05	-5.327e-…	5.707e-05	9.209e-05	-9.55e-06	2.6e-07	-0.076257	-0.076359	-0.076721	-0.076254
10	-4.644e-…	6.26e-06	-5.494e-…	9.64e-06	2.576e-05	-6.01e-05	-2.82e-05	-0.00017…	9.209e-05	0.000282…	1.173e-05	-6.342e-05	-0.086526	-0.086693	-0.086365	-0.086124
11	-1.28e-06	2.152e-05	-1.19e-06	2.198e-05	-4.9e-06	2.234e-05	-1.543e-05	-3.027e-…	-9.55e-06	1.173e-05	3.235e-05	-4.731e-05	0.00080468	0.00074431	0.0010033	0.00083123
12	2.878e-05	-6.092e-…	2.47e-05	-6.343e-05	3.228e-05	-5.517e-…	-3.871e-05	-0.00024…	2.6e-07	-6.342e-…	-4.731e-05	0.00048965	-0.040407	-0.040249	-0.040612	-0.041287
13	-0.36021	0.13009	0.57803	0.093378	-0.12045	-0.10974	-0.021918	0.013207	-0.076257	-0.086526	0.00080468	-0.040407	9440.3	9454.7	9446	9442.9
14	-0.3607	0.13014	0.57897	0.093362	-0.1207	-0.11007	-0.021952	0.013515	-0.076359	-0.086693	0.00074431	-0.040249	9454.7	9469.1	9460.4	9457.2
15	-0.36057	0.13021	0.57824	0.093505	-0.12027	-0.11013	-0.021673	0.013388	-0.076721	-0.086365	0.0010033	-0.040612	9446	9460.4	9451.8	9448.6
16	-0.36079	0.1304	0.57771	0.093729	-0.11976	-0.11023	-0.021742	0.013515	-0.076254	-0.086124	0.00083123	-0.041287	9442.9	9457.2	9448.6	9445.5

Figure 5 *The pseudoinverse Q_0 determined using the transformation matrix $\mathcal{S}$*

To apply the numerical function "pinv" to the matrix N_0, which contains the unknown orientation of the stations (dz), we follow the same steps as in the first method. However, this time, the matrix A, matrix P, and matrix l are extracted from the initial functional-stochastic model, which is unaffected by Rule 1 of equivalence. At a glance, we observe that although we follow the same stages as in the first method, the dimensions of the matrices are different because we have 4 additional unknowns, as the orientation angles of the stations are also part of the functional model. After obtaining the pseudoinverse, we calculate the unknown vector X so that we can compare the elements obtained through all three methods. Subsequently, the compensating elements were calculated, and the main elements of the error ellipses were determined, along with the precision calculations.

I also conducted the Global Congruence Test for the two configurations from the years 2020 and 2023. For calculating the discrepancy vector, the correction vectors of the coordinates of the points in the free network from each measurement stage are used. The difference between X_{2023} and X_{2020} is referred to as the discrepancy vector, with its values measured in centimeters (cm) in our case.[3]

Table 1 *Calculation of the discrepancy vector*

	0.686
	-0.576
	1.065
	-0.949
	-1.382
d=	0.896
	-0.147
	-0.345
	0.034
	-0.745
	-0.256
	1.718

At the same time, the empirical standard deviation of the deformation model is calculated using the standard deviations obtained in the two processing stages, expressed in mm. [3]

Table 2 *Empirical standard deviation of the deformation model*

$S_{0,2020}$=	68.05
$S_{0,2023}$=	55.22
S_0	87.64
h	13.00

NOTE: The pseudoinverses obtained in the two processing stages (both for 2020 and 2023) are identical.

To identify any possible displacements, the Student test was performed, the results of which will be presented in the next chapter. [3]

3. Results and Discussion

α=	5
h=	13.00
f=	30
f0=	15
f1=	15
F_{lim}=	2.38

Concluzie	F=	2.070	<	F_{lim}=	2.380	=>	Nu avemdeplasări

Figure 6 *Decision of the Fischer Test*

At first glance, it can be observed that the difference between F and F_{lim} is very small; thus, the Fischer test was passed at the limit. Therefore, we decided to proceed with the localization of the displacements using the Student's test. [3]

s_j = individual empirical standard deviation (for each point j)

t_j = calculated value of the deformation localization test

t_{lim}= theoretical value of the deformation localization test
f = degrees of freedom

Table3 *Results of the Student's test*

Point	$Q_{jj\ [X]}$	s_{xj} [mm]	t_{xj}	t_{lim}	Conclusions
6	0.0001	0.7189	1.05		STABLE
7	0.0001	0.9211	0.86		STABLE
8	0.0001	0.8496	-0.61	3.01	STABLE
12	0.0016	3.5375	-24.07		DISPLACED
13	0.0006	2.0841	61.15		DISPLACED
Ro	0.0001	0.7049	-2.76		STABLE
Point	$Q_{jj\ [y]}$	s_{yj} [mm]	t_{yj}	$t_{yj\text{-}lim}$	Conclusions
6	0.0001	1.0011	-1.7		STABLE
7	0.0001	1.0141	-1.1		STABLE
8	0.0002	1.3792	1.5	3.01	STABLE
12	0.0016	3.5375	-10.3		DISPLACED
13	0.0006	2.0841	-2.8		STABLE
Ro	0.0010	2.7425	1.6		STABLE

From the perspective of monitoring the dam during the period from 2020 to 2023, we can observe that the Fischer test was passed at the limit, which made me reconsider, and we decided to apply the Student's test. The results indicated that point 12 is displaced in both directions, while point 13 is displaced only to the East.

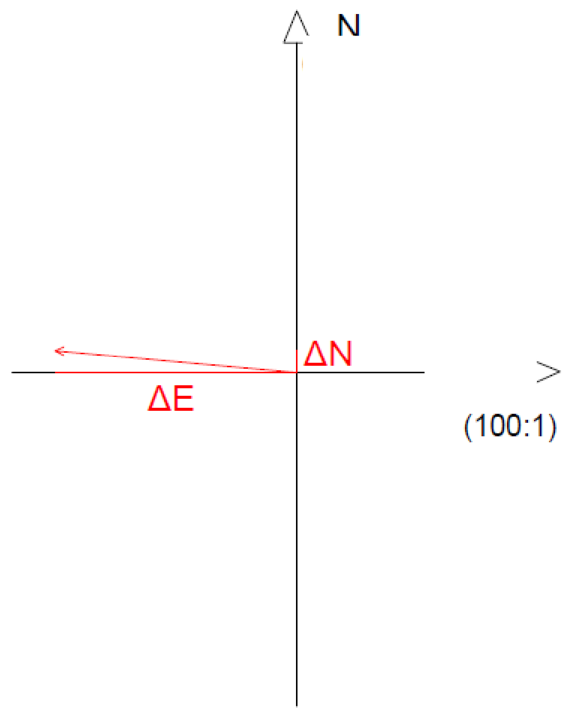

Figure 7 *Displacements of point 12*

Table 4 *Observations and discussions*

Elements	Method 1	Method 2	Method 3	Observations
Q	Identical	Identical	Different	$Q_{met3}$is different from the other two.
dN_6	1.290	1.290	0.920	method1=method2, method3 provides different values
dE_6	0.519	0.519	0.605	method1=method2, method3 provides different values
dN_7	2.347	2.347	1.979	method1=method2, method3 provides different values
dE_7	-0.049	-0.049	0.093	method1=method2, method3 provides different values
dN_8	-3.593	-3.593	-4.137	method1=method2, method3 provides different values
dE_8	3.399	3.399	3.238	method1=method2, method3 provides different values
dN_{Ro}	0.043	0.043	0.475	method1=method2, method3 provides different values
dE_{Ro}	-0.909	-0.909	-0.911	method1=method2, method3 provides different values
dN_{12}	-0.036	-0.036	0.369	method1=method2, method3 provides different values
dE_{12}	-0.945	-0.945	-1.147	method1=method2, method3 provides different values
dN_{13}	-0.051	-0.051	0.393	method1=method2, method3 provides different values
dE_{13}	-2.015	-2.015	-1.879	method1=method2, method3 provides different values
dz_6	-	10681.451	25.055	The values obtained here are completely different
dz_7	-	10714.882	58.485	The values obtained here are completely different
dz_{12}	-	10633.331	-23.065	The values obtained here are completely different
dz_8	-	10595.992	-60.405	The values obtained here are completely different

4. Conclusions

This type of network is the most commonly used in geodetic work. For processing the measurements taken, we used three different methods of processing and obtaining the pseudoinverse, with the aim of addressing the influence of reference data on the processing of monitoring geodetic networks. This also facilitated the actual compensation and determination of the most probable values of the points that define the network.

To begin with, we established the three methods we used, which aimed to compare the resulting elements (the pseudoinverse Q and the correction vector of the provisional coordinates X) to determine whether the unknowns of the orientations influence the final result or not.

It all started started with the functional-stochastic model, which was classically composed in all three methods, with minor differences arising from this point onward. For the first method, we decided to apply the first equivalence rule and eliminate the station unknowns, replacing them with the specific summation equations of this rule. To determine the pseudoinverse, we used Octave, applying the "pinv" function. For Method 2, we chose to use the transformation S with a partial minimum condition; however, this time we worked with the original functional model, including the unknowns dz, ultimately resulting in the generalized inverse. In the last case, we used the Octave application again, but this time we opted not to eliminate dz from the functional-stochastic model, thus obtaining a different matrix Q in this instance as well.

As can be seen in the table above, the values obtained through Methods 1 and 2 yield the same final matrices (both Q and x), while applying the "pinv" function without eliminating the unknowns related to the station orientations resulted in different outcomes that cannot be used further in the compensation process. The explanation for the results obtained is that, in the case of the first method, there is a minimum condition on all the unknowns associated with the coordinates; in the second method, we have a partial minimum condition; and in the third case, there is a minimum condition on all the unknowns related to the orientations of the stations.

After calculating the characteristic elements of the ellipses, we can observe that the precision is significantly better in the 2020 phase compared to the phase in 2023.

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